Fuzzy Soft W-Hausdorff Spaces

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***Abstract**: In this paper the definition of W-Hausdorff space introduced by Warren, R.H. [2] is extended to fuzzy soft topological spaces in two different ways. It is shown that these two concepts are hereditary and productive.*

Keywords : *Fuzzy topology, Fuzzy W-Hausdorff space, Fuzzy soft topology, Fuzzy soft W-Hausdorff space.*

I. INTRODUCTION

Most of the real life problems have various uncertainties. Traditional mathematical tools are unable to solve uncertain problems. A number of theories have been proposed for dealing with uncertainties in an efficient way.

In 1965, Zadeh, L.A. [12] introduced the concept of fuzzy set theory which provides us with an intuitively pleasing method of representing one form of uncertainty. In 1968, Chang, C.L. [1] defined fuzzy topology and later in 1976, Lowen R. [3] redefined fuzzy topology in a different way. In 1999, Molodstov [6] initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainty. In recent years researchers have contributed a lot towards fuzzification of soft set theory. In 2001, Maji et al. [4] initiated the concept of fuzzy soft set with some properties regarding union, intersection and complement of a fuzzy soft set, De Morgan's law etc. In 2011, Shabir and Naz [9] defined soft topological spaces and studied separation axioms. In 2011, Tanay and Kandemir [10] initially gave the concept of fuzzy soft topology using fuzzy soft sets and gave the basic notions of it by following Chang [1]. Pazar Varol and Aygun [7] defined fuzzy soft topology in Lowen's sense. In section II of this paper, preliminary definitions regarding fuzzy set, soft set and fuzzy soft set are given. In section III of this paper, the definition of W-Hausdorff space introduced by Warren, R.H. [2] is extended to fuzzy soft topological space in two different ways. It is shown that these two concepts are hereditary and productive.

Throughout this paper, X denotes initial universe and E denotes the set of parameters for the universe X.

II. PRELIMINARY DEFINITIONS

Definition : 2.1 [12]

A fuzzy set in X is a map $f : X \to [0, 1] = I$. The family of fuzzy sets in X is denoted by I^X. Following are some basic operations on fuzzy sets. For the fuzzy sets f and g in X,

(1) $f = g \Leftrightarrow f(x) = g(x)$ for all $x \in X$.
(2) $f \le g \Leftrightarrow f(x) \le g(x)$ for all $x \in X$.
(3) $(f \vee g)(x) = \max\{f(x), g(x)\}$ for all $x \in X$.
(4) $(f \wedge g)(x) = \min\{f(x), g(x)\}$ for all $x \in X$.
(5) $f^C(x) = 1 - f(x)$ for all $x \in X$. Here f^C denotes the complement of f.
(6) For a family $\{f_\lambda / \lambda \in \Lambda\}$ of fuzzy sets defined on a set X.
 (i) $(V_{\lambda\in\Lambda} f_\lambda)(x) = V_{\lambda\in\Lambda} f_\lambda(x)$
 (ii) $(\Lambda_{\lambda\in\Lambda} f_\lambda)(x) = \Lambda_{\lambda\in\Lambda} f_\lambda(x)$
(7) For any $\alpha \in I$, the constant fuzzy set $\boldsymbol{\alpha}$ in X is a fuzzy set in X defined by $\boldsymbol{\alpha}(x) = \alpha$ for all $x \in X$ and is denoted by α_X. 0_X denotes null fuzzy set in X and 1_X denotes universal fuzzy set in X.

Definition : 2.2 [12]

Let f and g be fuzzy sets in X and Y respectively. The Cartesian product $f \times g$ of f and g is a fuzzy set on $X \times Y$ defined by $(f \times g)(x, y) = f(x) \wedge g(x)$ for each $(x, y) \in X \times Y$.

Definition : 2.3 [1]

A fuzzy topological space is a pair (X, τ) where X is a non empty set and τ is a family of fuzzy sets on X satisfying the following properties :

(1) the constant functions 0_X and 1_X belongs to τ.
(2) $f, g \in \tau$ implies $f \wedge g \in \tau$
(3) $f_\lambda \in \tau$ for each $\lambda \in \Lambda$ implies $V_{\lambda\in\Lambda} f_\lambda \in \tau$.

Then τ is called a fuzzy topology on X. Every member of τ is called fuzzy open. g is called fuzzy closed in (X, τ) if $g^C \in \tau$.

Definition : 2.4 [11]

Let (X, τ_1) and (Y, τ_2) be two fuzzy topological spaces. Then the product topology on $\tau_1 \times \tau_2$ on $X \times Y$ is the fuzzy topology having the collection $\{f \times g / f \in \tau_1, g \in \tau_2\}$ as a basis.

Definition : 2.5 [11]

Let $\{(X_\lambda, \tau_\lambda) \;/\; \lambda \in \Lambda\}$ be a family of fuzzy topological spaces and $X = \Pi_{\lambda\in\Lambda} X_\lambda$. The product topology on X is the one with basic fuzzy open sets of the form $\Pi_{\lambda\in\Lambda} f_\lambda$ where $f_\lambda \in \tau_\lambda$ and $f_\lambda = 1_X$ except for finitely many λ's.

Definition : 2.6 [2]

A fuzzy topological space (X, τ) is said to be fuzzy W-Hausdorff if $\forall$ x, y ∈ X, x ≠ y, there exist f, g ∈ τ such that f(x) = 1, g(x) = 1 and $f \wedge g = 0_X$.

Definition : 2.7 [6]

Let A ⊆ E. A soft set f_A over X is a mapping from E to P(X) i.e., $f_A : E \to P(X)$ where P(X) is the power set of X.

Definition : 2.8 [4,8]

Let A ⊆ E. A fuzzy soft set $\tilde{f}_A$ over X is a mapping from E to I^X i.e., $\tilde{f}_A : E \to I^X$ where $\tilde{f}_A$ (e) ≠ 0_X if e ∈ A ⊆ E and $\tilde{f}_A(e) = 0_X$ if e ∉ A. The family of fuzzy soft sets over X is denoted by FS(X, E).

Definition : 2.9 [8]

The fuzzy soft set $\tilde{f}_\phi \in$ FS(X, E) is called null fuzzy soft set denoted by $\tilde{0}_E$ if for all e ∈ E, $\tilde{f}_\phi$ (e) = 0_X.

Definition : 2.10 [8]

The fuzzy soft set $\tilde{f}_E \in$ FS(X, E) is called universal fuzzy soft set denoted by $\tilde{1}_E$ if for all e ∈ E, $\tilde{f}_E(e) = 1_X$.

Definition : 2.11 [8]

Let $\tilde{f}_A$, $\tilde{g}_B \in$ FS(X, E). $\tilde{f}_A$ is called a fuzzy soft subset of $\tilde{g}_B$ if $\tilde{f}_A(e) \le \tilde{g}_B(e)$ for every e ∈ E and we write $\tilde{f}_A \mathrel{\tilde{\subseteq}} \tilde{g}_B$.

Definition : 2.12 [8]

Let $\tilde{f}_A$, $\tilde{g}_B \in$ FS(X, E). $\tilde{f}_A$ and $\tilde{g}_B$ are said to be equal denoted by $\tilde{f}_A = \tilde{g}_B$ if $\tilde{f}_A \mathrel{\tilde{\subseteq}} \tilde{g}_B$ and $\tilde{g}_B \mathrel{\tilde{\subseteq}} \tilde{f}_A$.

Definition : 2.13 [8]

Let $\tilde{f}_A$, $\tilde{g}_B \in$ FS(X, E). The union of $\tilde{f}_A$ and $\tilde{g}_B$ is also a fuzzy soft set $\tilde{h}_C$ defined by $\tilde{h}_C(e) = \tilde{f}_A(e) \vee \tilde{g}_B(e)$ for all e ∈ E where C = A ∪ B. Here we write $\tilde{h}_C = \tilde{f}_A \mathbin{\tilde{\cup}} \tilde{g}_B$.

Definition : 2.14 [8]

Let $\tilde{f}_A$, $\tilde{g}_B \in$ FS(X, E). The intersection of $\tilde{f}_A$ and $\tilde{g}_B$ is also a fuzzy soft set $\tilde{h}_C$ defined by $\tilde{h}_C(e) = \tilde{f}_A(e) \wedge \tilde{g}_B(e)$ for all e ∈ E where C = A ∩ B. Here we write $\tilde{h}_C = \tilde{f}_A \mathbin{\tilde{\cap}} \tilde{g}_B$.

Definition : 2.15 [8]

Let $\tilde{f}_A \in$ FS(X, E). The complement of $\tilde{f}_A$ denoted by $\tilde{f}_A^C$ is a fuzzy soft set defined by $\tilde{f}_A^C(e) = 1_X - \tilde{f}_A(e)$ for every e ∈ E. Clearly $(\tilde{f}_A^C)^C = \tilde{f}_A$, $\tilde{1}_E^C = \tilde{0}_E$ and $\tilde{0}_E^C = \tilde{1}_E$.

Definition : 2.16 [10]

A fuzzy soft topological space is a triple (X, E, $\tilde{\tau}$) where X is a nonempty set, E is a parameter set and $\tilde{\tau}$ is a family of fuzzy soft sets over X satisfying the following properties :

(1) $\tilde{0}_E$, $\tilde{1}_E \in \tilde{\tau}$

(2) $\tilde{f}_A$, $\tilde{g}_B \in \tilde{\tau}$ then $\tilde{f}_A \mathbin{\tilde{\cap}} \tilde{g}_B \in \tilde{\tau}$

(3) If $\tilde{f}_{A_i} \in \tilde{\tau}$ $\forall$ i ∈ Λ then $\tilde{\cup}_{\lambda\in\Lambda} \tilde{f}_{A_i} \in \tilde{\tau}$

Then $\tilde{\tau}$ is called a fuzzy soft topology over X. Every member of $\tilde{\tau}$ is called fuzzy soft open. $\tilde{g}_B$ is called fuzzy soft closed in (X, E, $\tilde{\tau}$) if $\tilde{g}_B^C \in \tilde{\tau}$.

Definition : 2.17 [7]

Let (X, E, $\tilde{\tau}$) be a fuzzy soft topological space and G ⊆ E. Then (X, G, $\tilde{\tau}_G$) is called a fuzzy soft subspace of (X, E, $\tilde{\tau}$) where $\tilde{\tau}_G = \{\tilde{f}_A/G : \tilde{f}_A \in \tilde{\tau}\}$ relative to the parameter set G.

Definition : 2.18 [7]

Let FS(X, E) and FS(Y, E′) be two families of fuzzy soft sets over X and Y with respect to E and E′ respectively. Let $\tilde{f}_E \in$ FS(X, E) and $\tilde{g}_{E'} \in$ FS(Y, E′). The Cartesian product of $\tilde{f}_E$ and $\tilde{g}_{E'}$ denoted by $\tilde{f}_E \otimes \tilde{g}_{E'}$ is a fuzzy soft set over X × Y with respect to the parameter set E × E′ defined as $\tilde{f}_E \otimes \tilde{g}_{E'} : E \times E' \to I^X \times I^Y$, $(\tilde{f}_E \otimes \tilde{g}_{E'})(e, e') = \tilde{f}_E(e) \times \tilde{g}_{E'}(e')$ such that $\tilde{f}_E(e) \times \tilde{g}_{E'}(e')$ is a fuzzy product of fuzzy sets $\tilde{f}_E(e)$ and $\tilde{g}_{E'}(e')$ where $\tilde{f}_E(e) \times \tilde{g}_{E'}(e') : X \times Y \to I$ and $(\tilde{f}_E(e) \times \tilde{g}_{E'}(e'))\,(x, y) = (\tilde{f}_E(e))\,(x) \wedge (\tilde{g}_{E'}(e'))\,(y)$.

Definition : 2.19 [7]

Let FS(X, E) and FS(Y, E′) be two families of fuzzy soft sets over X and Y with respect to E and E′

respectively. Let $\tilde{f}_E \in FS(X, E)$ and $\tilde{g}_{E'} \in FS(Y, E')$. Then

(1) $\tilde{1}_E \otimes \tilde{1}_{E'} = \tilde{1}_{E \times E'}$ denotes the universal fuzzy soft set over $X \times Y$ with respect to $E \times E'$.

(2) $\tilde{f}_E \otimes \tilde{0}_E = \tilde{0}_E \otimes \tilde{g}_{E'} = \tilde{0}_{E \times E'} = \tilde{0}_E \otimes \tilde{0}_{E'}$

(3) $\forall$ e $\in$ E, e′ $\in$ E′, $((\tilde{f}_E \otimes \tilde{1}_{E'})(e, e'))(x, y) = (\tilde{f}_E(e))(x)$, $((\tilde{1}_E \otimes \tilde{g}_{E'})(e, e'))(x, y) = (\tilde{g}_{E'}(e'))(y)$ where $x \in X$ and $y \in Y$.

Definition : 2.20 [7]

Let $(X_1, E_1, \tilde{\tau}_1)$ and $(X_2, E_2, \tilde{\tau}_2)$ be two fuzzy soft topological spaces. The fuzzy soft product topology $\tilde{\tau}_1 \otimes \tilde{\tau}_2$ over $X_1 \times X_2$ with respect to $E_1 \times E_2$ is the fuzzy soft topology having the collection $\{(\tilde{f}_{E_1} \otimes \tilde{g}_{E_2}) / \tilde{f}_{E_1} \in \tilde{\tau}_1, \tilde{g}_{E_2} \in \tilde{\tau}_2\}$ as a basis.

III.FUZZY SOFT W-HAUSDORFF SPACES

Definition : 3.1

A fuzzy soft topological space $(X, E, \tilde{\tau})$ is said to be fuzzy soft W-Hausdorff space of type 1 denoted by $(FW\text{-}H)_1$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(e_1) = 1_X$, $\tilde{g}_B(e_2) = 1_X$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}_E$.

Definition : 3.2

Fix e $\in$ E. A fuzzy soft topological space $(X, E, \tilde{\tau})$ is said to be fuzzy soft W-Hausdorff space of type 2 denoted by $(FW\text{-}H)_2$ if for every $x, y \in X$, $x \neq y$ there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(e)(x) = 1$, $\tilde{g}_B(e)(x) = 1$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}_E$.

Theorem : 3.3

Subspace of a $(FW\text{-}H)_1$ space is $(FW\text{-}H)_1$.

Proof :

Let $(X, E, \tilde{\tau})$ is a $(FW\text{-}H)_1$ space. Let $G \subseteq E$. Let $(X, G, \tilde{\tau}_G)$ be a fuzzy soft subspace of $(X, E, \tilde{\tau})$ where $\tilde{\tau}_G = \{\tilde{f}_A / G : \tilde{f}_A \in \tilde{\tau}\}$ relative to the parameter set G. Consider $g_1, g_2 \in G$ such that $g_1 \neq g_2$ then $g_1, g_2 \in E$ there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(g_1) = 1_X$, $\tilde{g}_B(g_2) = 1_X$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}_E$. Therefore $\tilde{f}_A / G, \tilde{g}_B / G \in \tilde{\tau}_G$.

Also $(\tilde{f}_A / G)(g_1) = 1_X$, $(\tilde{g}_B / G)(g_2) = 1_X$ and $(\tilde{f}_A / G) \tilde{\cap} (\tilde{g}_B / G) = (\tilde{f}_A \tilde{\cap} \tilde{g}_B) / G = \tilde{0}_E / G = \tilde{0}_G$. Hence $(X, G, \tilde{\tau}_G)$ is $(FW\text{-}H)_1$.

Theorem : 3.4

Subspace of a $(FW\text{-}H)_2$ space is $(FW\text{-}H)_2$.

Proof :

Let $(X, E, \tilde{\tau})$ is a $(FW\text{-}H)_2$ space. Let $G \subseteq E$. Let $(X, G, \tilde{\tau}_G)$ be a fuzzy soft subspace of $(X, E, \tilde{\tau})$ where $\tilde{\tau}_G = \{\tilde{f}_A / G : \tilde{f}_A \in \tilde{\tau}\}$ relative to the parameter set G. Fix $g \in G \Rightarrow g \in E$.

Consider $x, y \in X$ such that $x \neq y$, then there exist $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(g)(x) = 1$, $\tilde{g}_B(g)(y) = 1$ and $(\tilde{f}_A / G) \tilde{\cap} (\tilde{g}_B / G) = (\tilde{f}_A \tilde{\cap} \tilde{g}_B) / G = \tilde{0}_E / G = \tilde{0}_G$. Hence $(X, G, \tilde{\tau}_G)$ is $(FW\text{-}H)_2$.

Theorem: 3.5

Product of two $(FW\text{-}H)_1$ spaces is $(FW\text{-}H)_1$.

Proof :

Let $(X_1, E_1, \tilde{\tau}_1)$ and $(X_2, E_2, \tilde{\tau}_2)$ be two $(FW\text{-}H)_1$ spaces. Consider two distinct points (e_1, k_1), $(e_2, k_2) \in E \times k$. Either $e_1 \neq e_2$ or $k_1 \neq k_2$. Assume $e_1 \neq e_2$. Then $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}_1$ such that $\tilde{f}_A(e_1) = 1_X$, $\tilde{g}_B(e_2) = 1_X$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}_E$. Therefore $\tilde{f}_A \otimes \tilde{1}_K, \tilde{g}_B \otimes \tilde{1}_K \in \tilde{\tau}_1 \otimes \tilde{\tau}_2$. $(\tilde{f}_A \otimes \tilde{1}_K)(e_1, k_1) = \tilde{f}_A(e_1) \times \tilde{1}_K(k_1) = 1_X \times 1_Y = 1_{X \times Y}$. $(\tilde{g}_B \otimes \tilde{1}_K)(e_2, k_2) = \tilde{g}_B(e_2) \times \tilde{1}_K(k_2) = 1_X \times 1_Y = 1_{X \times Y}$. For any $(e, k) \in E \times K$, $(\tilde{f}_A \otimes \tilde{1}_K)(e, k) \neq 0_{X \times Y}$.

$\Rightarrow$ $(\tilde{f}_A \otimes \tilde{1}_K)(e, k)(x, y) \neq 0$ for all $(x, y) \in X \times Y$.

$\Rightarrow$ $\tilde{f}_A(e)(x) \wedge \tilde{1}_K(k)(y) \neq 0$ for all $x \in X$ and $y \in Y$.

$\Rightarrow$ $\tilde{f}_A(e)(x) \neq 0$ for all $x \in X$.

$\Rightarrow$ $\tilde{g}_B(e)(x) = 0$ for all $x \in X$.

$\Rightarrow$ $\tilde{g}_B(e)(x) \wedge \tilde{1}_K(k)(y) = 0$ for all $x \in X$ and $y \in Y$.

$\Rightarrow$ $(\tilde{g}_B \otimes \tilde{1}_K)(e, k)(x, y) = 0$ for all $(x, y) \in X \times Y$.

$\therefore$ $(\tilde{f}_A \otimes \tilde{1}_K) \tilde{\cap} (\tilde{g}_B \otimes \tilde{1}_K) = \tilde{0}_{E \times K}$.

Hence $(X \times Y, E \times K, \tilde{\tau}_1 \otimes \tilde{\tau}_2)$ is a $(FW\text{-}H)_1$.

Theorem: 3.6

Product of two $(FW\text{-}H)_2$ spaces is $(FW\text{-}H)_2$.

Proof :

Let $(X_1, E_1, \tilde{\tau}_1)$ and $(X_2, E_2, \tilde{\tau}_2)$ be two $(FW\text{-}H)_2$ spaces. Fix $(e, k) \in E \times K$. Consider two distinct points (x_1, y_1), $(x_2, y_2) \in X \times y$.
Either $x_1 \neq x_2$ or $y_1 \neq y_2$. Assume $x_1 \neq x_2$.
Then $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau}$ such that $\tilde{f}_A(e)(x_1) = 1$, $\tilde{g}_B(e)(x_2) = 1$ and $\tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}_E$.
Therefore $\tilde{f}_A \otimes \tilde{1}_K, \tilde{g}_B \otimes \tilde{1}_K \in \tilde{\tau}_1 \otimes \tilde{\tau}_2$.

$(\tilde{f}_A \otimes \tilde{1}_K)(e,k)(x,y) = \tilde{f}_A(e)(x) \wedge \tilde{1}_K(k)(y) = 1\wedge 1 = 1.$

$(\tilde{g}_B \otimes \tilde{1}_K)(e,k)(x,y) = \tilde{g}_B(e)(x) \wedge \tilde{1}_K(k)(y) = 1\wedge 1 = 1.$

For any $(x, y) \in X \times Y$, $(\tilde{f}_A \otimes \tilde{1}_K)(e, k)(x, y) \neq 0$.

$\Rightarrow \tilde{f}_A(e)(x) \wedge \tilde{1}_K(k)(y) \neq 0$ for all $(x, y) \in X \times Y$.

$\Rightarrow \tilde{f}_A(e)(x) \neq 0$ for all $x \in X$.

$\Rightarrow \tilde{g}_B(e)(x) = 0$ for all $x \in X$.

$\Rightarrow \tilde{g}_B(e)(x) \wedge \tilde{1}_K(k)(y) = 0$ for all $x \in X$ and $y \in Y$.

$\Rightarrow (\tilde{g}_B \otimes \tilde{1}_K)(e, k)(x, y) = 0$ for all $(x, y) \in X \times Y$.

Therefore $(\tilde{f}_A \otimes \tilde{1}_K) \tilde{\cap} (\tilde{g}_B \otimes \tilde{1}_K) = \tilde{0}_{E\times K}$.

Hence $(X \times Y, E \times K, \tilde{\tau}_1 \otimes \tilde{\tau}_2)$ is a $(FW\text{-}H)_2$ space.

Definition : 3.7

Let $\{(X_\lambda, E_\lambda, \tilde{\tau}_\lambda) / \lambda \in \Lambda\}$ be families of fuzzy soft topological spaces. Let $X = \Pi_{\lambda\in\Lambda} X_\lambda$ and $E = \Pi_{\lambda\in\Lambda} E_\lambda$. The fuzzy soft product topology $\tilde{\tau}$ over (X, E) is the one with basic fuzzy soft open sets of the form $\Pi_{\lambda\in\Lambda} \tilde{f}_{A_\lambda}$ where $\tilde{f}_{A_\lambda} \in \tilde{\tau}_\lambda$ and $\tilde{f}_{A_\lambda} = \tilde{1}_{E_\lambda}$ except for finitely many λ's.

Here $((\Pi_{\lambda\in\Lambda} \tilde{f}_{A_\lambda})((e_\lambda)_{\lambda\in\Lambda}))(x_\lambda)_{\lambda\in\Lambda} = \Lambda_{\lambda\in\Lambda} \tilde{f}_{A_\lambda}(e_\lambda)(x_\lambda)$ for every $(x_\lambda)_{\lambda\in\Lambda} \in \Pi_{\lambda\in\Lambda} X_\lambda$ and for every $(e_\lambda)_{\lambda\in\Lambda} \in \Pi_{\lambda\in\Lambda} E_\lambda$.

Theorem : 3.8

Arbitrary product of $(FW\text{-}H)_1$ spaces is $(FW\text{-}H)_1$.

Proof :

Let $(X_\lambda, E_\lambda, \tilde{\tau}_\lambda) / \lambda \in \Lambda\}$ be a collection of $(FW\text{-}H)_1$ spaces. Let $X = \Pi_{\lambda\in\Lambda} X_\lambda$ and $E = \Pi_{\lambda\in\Lambda} E_\lambda$ and $\tilde{\tau} = \Pi_{\lambda\in\Lambda} \tilde{\tau}_\lambda$. Consider two distinct points $(e_\lambda)_{\lambda\in\Lambda}$, $(k_\lambda)_{\lambda\in\Lambda} \in \Pi_{\lambda\in\Lambda} E_\lambda$. Assume $e_\mu \neq k_\mu$ for some $\mu \in \Lambda$, there exist $\tilde{f}_{A_\mu}, \tilde{g}_{B_\mu} \in \tilde{\tau}_\mu$ such that $\tilde{f}_{A_\mu}(e_\mu) = 1_{X_\mu}$, $\tilde{g}_{B_\mu}(k_\mu) = 1_{X_\mu}$ and $\tilde{f}_{A_\mu} \tilde{\cap} \tilde{g}_{B_\mu} = \tilde{0}_{E_\mu}$. Let $\tilde{f}_A = \Pi_{\lambda\in\Lambda} \tilde{f}_{A_\lambda}$ where $\tilde{f}_{A_\lambda} = \tilde{1}_{E_\lambda}$ for $\lambda \neq \mu$ and $\tilde{g}_B = \Pi_{\lambda\in\Lambda} \tilde{g}_{B_\lambda}$ where $\tilde{g}_{B_\lambda} = \tilde{1}_{E_\lambda}$ for $\lambda \neq \mu$.

Then $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau} = \Pi_{\lambda\in\Lambda} \tilde{\tau}_\lambda$.

Also

$$\begin{aligned}(\tilde{f}_A((e_\lambda)_{\lambda\in\Lambda}))(x_\lambda)_{\lambda\in\Lambda} &= ((\Pi_{\lambda\in\Lambda} \tilde{f}_{A_\lambda})(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda} \\ &= \Lambda_{\lambda\in\Lambda} \tilde{f}_{A_\lambda}(e_\lambda)(x_\lambda) \\ &= \tilde{1}_{X_\mu}(x_\mu) \\ &= 1 \\ &= 1_X((x_\lambda)_{\lambda\in\Lambda})\end{aligned}$$

$\Rightarrow \tilde{f}_A((e_\lambda)_{\lambda\in\Lambda}) = 1_X$.

$$\begin{aligned}(\tilde{g}_B((k_\lambda)_{\lambda\in\Lambda}))(x_\lambda)_{\lambda\in\Lambda} &= ((\Pi_{\lambda\in\Lambda} \tilde{g}_{B_\lambda})(k_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda} \\ &= \Lambda_{\lambda\in\Lambda} \tilde{g}_{B_\lambda}(k_\lambda)(x_\lambda) \\ &= \tilde{1}_{X_\mu}(x_\mu) \\ &= 1_X(x_\lambda)_{\lambda\in\Lambda}\end{aligned}$$

$\Rightarrow \tilde{g}_B((k_\lambda)_{\lambda\in\Lambda}) = 1_X$.

Now $((\tilde{f}_A \tilde{\cap} \tilde{g}_B)(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda}$

$= (\tilde{f}_A(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda} \wedge (\tilde{g}_B(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda}$

$= ((\Pi_{\lambda\in\Lambda} \tilde{f}_{A_\lambda})(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda} \wedge ((\Pi_{\lambda\in\Lambda} \tilde{g}_{B_\lambda})(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda}$

$= (\Lambda_{\lambda\in\Lambda} \tilde{f}_{A_\lambda}(e_\lambda)(x_\lambda)) \wedge (\Lambda_{\lambda\in\Lambda} \tilde{g}_{B_\lambda}(e_\lambda)(x_\lambda))$

$= \tilde{f}_{A_\mu}(e_\mu)(x_\mu) \wedge \tilde{g}_{B_\mu}(e_\mu)(x_\mu)$

$= ((\tilde{f}_{A_\mu} \tilde{\cap} \tilde{g}_{B_\mu})(e_\mu))(x_\mu)$

$= (\tilde{0}_{E_\mu}(e_\mu))(x_\mu)$

$= 0$

$\Rightarrow \tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}_E$

Hence $(X, E, \tilde{\tau})$ is $(FW\text{-}H)_1$ space.

Theorem : 3.9

Arbitrary product of $(FW\text{-}H)_2$ spaces is $(FW\text{-}H)_2$.

Proof :

Let $\{(X_\lambda, E_\lambda, \tilde{\tau}_\lambda) / \lambda \in \Lambda\}$ be a collection of $(FW\text{-}H)_2$ spaces. Let $X = \Pi_{\lambda\in\Lambda} X_\lambda$, $E = \Pi_{\lambda\in\Lambda} E_\lambda$ and $\tilde{\tau} = \Pi_{\lambda\in\Lambda} \tilde{\tau}_\lambda$.

Fix $(e_\lambda)_{\lambda\in\Lambda} \in E = \Pi_{\lambda\in\Lambda} E_\lambda$.

Consider two distinct points $(x_\lambda)_{\lambda\in\Lambda}$, $(y_\lambda)_{\lambda\in\Lambda} \in \Pi_{\lambda\in\Lambda} X_\lambda$.

Assume $X_\mu \neq y_\mu$ for some $\mu \in \Lambda$, there exist $\tilde{f}_{A_\mu}$, $\tilde{g}_{B_\mu} \in \tilde{\tau}_\mu$ such that $\tilde{f}_{A_\mu}(e_\mu)(x_\mu) = 1$, $\tilde{g}_{B_\mu}(e_\mu)(y_\mu) = 1$ and $\tilde{f}_{A_\mu} \tilde{\cap} \tilde{g}_{B_\mu} = \tilde{0}_{E_\mu}$.

Let $\tilde{f}_A = \Pi_{\lambda\in\Lambda} \tilde{f}_{A_\lambda}$ where $\tilde{f}_{A_\lambda} = \tilde{1}_{E_\lambda}$ for $\lambda \neq \mu$ and $\tilde{g}_B = \Pi_{\lambda\in\Lambda} \tilde{g}_{B_\lambda}$ where $\tilde{g}_{B_\lambda} = \tilde{1}_{E_\lambda}$ for $\lambda \neq \mu$.

Then $\tilde{f}_A, \tilde{g}_B \in \tilde{\tau} = \Pi_{\lambda\in\Lambda} \tilde{\tau}_\lambda$.

Also

$$\begin{aligned}(\tilde{f}_A(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda} &= ((\Pi_{\lambda\in\Lambda} \tilde{f}_{A_\lambda})(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda} \\ &= \Lambda_{\lambda\in\Lambda} \tilde{f}_{A_\lambda}(e_\lambda)(x_\lambda) \\ &= \tilde{f}_{A_\mu}(e_\mu)(x_\mu) \\ &= 1\end{aligned}$$

$$\begin{aligned}(\tilde{g}_B(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda} &= ((\Pi_{\lambda\in\Lambda} \tilde{g}_{B_\lambda})(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda} \\ &= \Lambda_{\lambda\in\Lambda} \tilde{g}_{B_\lambda}(e_\lambda)(x_\lambda) \\ &= \tilde{g}_{B_\mu}(e_\mu)(x_\mu) \\ &= 1\end{aligned}$$

Now $((\tilde{f}_A \tilde{\cap} \tilde{g}_B)(e_\lambda)_{\lambda\in\Lambda})(x_\lambda)_{\lambda\in\Lambda}$

$= ((\tilde{f}_A (e_\lambda)_{\lambda\in\Lambda}) (x_\lambda)_{\lambda\in\Lambda} \wedge (\tilde{g}_B (e_\lambda)_{\lambda\in\Lambda}) (x_\lambda)_{\lambda\in\Lambda}$

$= ((\Pi_{\lambda\in\Lambda} \tilde{f}_{A_\lambda}) (e_\lambda)_{\lambda\in\Lambda}) (x_\lambda)_{\lambda\in\Lambda} \wedge ((\Pi_{\lambda\in\Lambda} \tilde{g}_{B_\lambda})$
$(e_\lambda)_{\lambda\in\Lambda}) (x_\lambda)_{\lambda\in\Lambda}$

$= (\wedge_{\lambda\in\Lambda} \tilde{f}_{A_\lambda} (e_\lambda)(x_\lambda)) \wedge (\wedge_{\lambda\in\Lambda} \tilde{g}_{B_\lambda} (e_\lambda)(x_\lambda))$

$= \tilde{f}_{A_\mu} (e_\mu) (x_\mu) \wedge \tilde{g}_{B\mu} (e_\mu) (x_\mu)$

$= ((\tilde{f}_{A_\mu} \tilde{\cap} \tilde{g}_{B\mu}) (e_\mu)) (x_\mu)$

$= (\tilde{0}_{E_\mu} (e_\mu)) (x_\mu)$

$= 0$

$\Rightarrow \tilde{f}_A \tilde{\cap} \tilde{g}_B = \tilde{0}_E$

Hence (X, E, $\tilde{\tau}$) is $(FW\text{-}H)_2$ space.

IV. CONCLUSIONS

In this paper, the concept of fuzzy soft W-Hausdorff space is introduced and some basic properties regarding this concept are proved.

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