An Optimal Solution For intuitionistic Fuzzy Assignment Problem Using Genetic Algorithm

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Abstract *In this paper, Fuzzy interval method is proposed to find an optimal solution for an intuitionistic fuzzy assignment problem. The fuzzy objective functions have fuzzy demand and supply coefficients, which are represented as an intuitionistic fuzzy numbers. The solution procedure is illustrated with a numerical example. The genetic algorithm can serve as an important tool for the decision making.*

Keywords— *Assignment Problem, Genetic Algorithm, Reproduction, Ranking of Intuitionistic triangular fuzzy numbers.*

I. Introduction

Assignment Problem (AP) is used worldwide in solving real world problems. An assignment problem plays an important role in an assigning of persons to jobs, or classes to rooms, operators to machines, drivers to trucks, trucks to routes, or problems to research teams, etc. The assignment problem is a special type of linear programming problem (LPP) in which our objective is to assign n number of jobs to n number of machines (persons) at a minimum cost. To find solution to assignment problems, various algorithms such as linear programming, Hungarian algorithm, neural network, Genetic Algorithm as a worthy alternative (see [2], [3], [4], [5]) for generalities on Genetic Algorithm). This Algorithm, in spite of its unfamiliar and peculiar accessories, is a much faster and more efficient tool to handle the Assignment problem than the Hungarian Method. In section 2, we recall some definitions from an intuitionistic fuzzy numbers in Genetics so that the Algorithm in question may make sense. Section 3 Mathematical form of intuitionistic fuzzy Assignment problem. Our section 4 is the central one the describes how to apply Genetic Algorithm to the Assignment Problem. In Section 5, we illustrate the Algorithm by working out the test example. In section 6 conclusions.

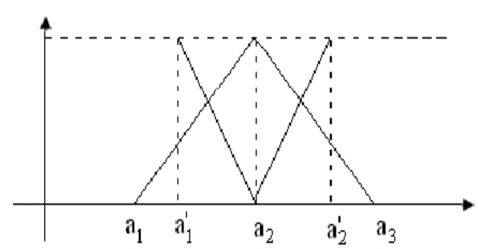
II. Definitions

A. (Triangular Intuitionistic fuzzy number (TIFN))

Type I : A (TIFN) $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membefunction $\mu_{\tilde{A}^I}$ (x), and non membership function $V_{\tilde{A}^I}$ (x), :

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & for\, a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & for\, a_2 \le x \le a_3 \\ 0 & for\, x > a_3 \end{cases} \quad V_{\tilde{A}^I}(x)$$

$$= \begin{cases} \frac{a_2-x}{a_2-a_1'} & for\, a_1 \le x \le a_2 \\ \frac{a_3'-x}{a_3'-a_2} & for\, a_2 \le x \le a_3 \\ 0 & for\, x > a_3 \end{cases}$$



B. Genetic Algorithm: The terminology of a genetic algorithm is an odd mixture of Computer Science and Genetics. Genetic Algorithms are search algorithms for finding optimal or near optimal solutions .For an excellent introduction to Genetic Algorithms, we refer the reader to [2], [3] and [5].

C. Chromosomes: A structure in the core containing a direct string of DNA, which transmits hereditary data and connected with RNA and his tones. An individual's hereditary structure is portrayed by bit strings as a rundown of 1's and 0's. These strings are called chromosomes.

D. Alleles: One of two or more option types of a quality at comparing site (loci) of homogeneous chromosomes, which decides elective characters in legacy Chromosomes strings containing bits are called alleles.

E. Genotype: The whole hereditary constitution of an individual, additionally, the alleles present at one or more particular locus. The bit string connected with a given individual is known as the individual's genotype.

F. Generation:
(i) The process of reproduction
(ii) A class composed of all individuals removed by the same number of successive ancestors from common predecessors.

G. Reproduction: The productions of offspring by organized bodies, individuals from one generation are selected for the next generation.

H. Crossover: The exchanging of material between homologous chromosomes during the first meiotic division resulting in new combinations of genes. Genetic material from one individual is exchanged with genetic material of another individual.

	Job1	Job2	Job3	.Jobj	JobN
Person1	$\tilde{a}_{11}$	$\tilde{a}_{12}$	$\tilde{a}_{13}$	.. $\tilde{a}_{ij}$	$\tilde{a}_{1n}$
Person2	$\tilde{a}_{21}$	$\tilde{a}_{22}$	$\tilde{a}_{23}$	... $\tilde{a}_{ij}$	$\tilde{a}_{2n}$
. . Personi .	- - $\tilde{a}_{i1}$	- - $\tilde{a}_{i2}$	- - $\tilde{a}_{i3}$	- - .. $\tilde{a}_{ij}$.	- - -
PersonN	$\tilde{a}_{n1}$	$\tilde{a}_{n2}$	$\tilde{a}_{n3}$	.. $\tilde{a}_{nj}$ ..	$\tilde{a}_{nn}$

I. Mutation: A permanent transmissible change in genetic material.

J. Ranking of Triangular Intuitionistic Fuzzy numbers :
The Ranking of a triangular intuitionistic fuzzy number $\tilde{A}^I$ =(a_1 , a_2 , a_3;$a_1{}'$,a_2 ,$a_3{}'$) is defined by [4]

$$R(\grave{A}^I)=\frac{1}{3}\left[\frac{(a_3'-a_1')(a_2-2a_3'-2a_1')+(a_3-a_1)(a_1+a_2+a_3)+3(a_3'^2-a_1'^2)}{a_3'-a_1'+a_3-a_1}\right]$$
,

The ranking technique [4] is:
If R($\tilde{A}^I$) $\leq$ R($\tilde{B}^I$), then $\tilde{A}^I \leq \tilde{B}^I$ i.e., min { $\tilde{A}^I$, $\tilde{B}^I$ }= $\tilde{A}^I$

Example: Let $\tilde{A}^I$ =(8,10,12 ;6,10,14) and $\tilde{B}^I$ = (3,5,8 ;1,5,10) be any two TrIFN, then its rank is defined by R($\tilde{A}^I$) =10, R($\tilde{B}^I$)=5.33 this implies $\tilde{A}^I \succ \tilde{B}^I$

III. INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM

Consider the situation of assigning n machines to n jobs and each machine is capable of doing any job at different costs. Let c_{ij} be an intuitionistic fuzzy cost of assigning the jth job to the ith machine. Let x_{ij} be the decision variable denoting the assignment of the machine i to the job j. The objective is to minimize the total intuitionistic fuzzy cost of assigning all the jobs to the available machines (one machine per job) at the least total cost. This situation is known as balanced intuitionistic fuzzy assignment problem.

(IFAP)Minimize $\tilde{A}^I$ $c_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{n} c_{ij}\, x_{ij}$

Subject to

$$\sum_{i=1}^{n} c_{ij} = 1 \quad \text{for} \quad i=1,2,\ldots\ldots n$$

$$\sum_{j=1}^{n} c_{ij} = 1 \quad \text{for} \quad j=1,2,3,\ldots\ldots.n$$

Where

$$x_{ij}=\begin{cases}1, if \quad the \quad i^{th} \quad machine \quad is \quad assigned \quad to \quad j^{th} \quad job\\ 0, if \quad the \quad i^{th} \quad machine \quad is \quad not \quad assigned \quad to \quad j^{th} \quad job\end{cases} \quad x_{ij} \in \{0,1\}$$

$$\grave{C}^I_{ij} = (C^1_{ij}, C^2_{ij}, C^3_{ij})(C^{1'}_{ij}, C^2_{ij}, C^{3'}_{ij})$$

IV. SOLUTION OF ASSIGNMENT PROBLEM USING GENETIC ALGORITHM

1. The string ij| c_{ij} is defined as where i and j are row and column and C is cost.

2. Write ij| c_{ij} in bit string, a list of's and 's.

3. The selection of individual's genotype depends on the largest value of cost and genotype is allowed two off springs and the smallest value of cost is not fit, put this right adjacent to highest value.

4. Put the remaining genotypes to right and left according to value of cost in decreasing order.

5. Three genetic operators are applied to produce the next generation solution (chromosome).These operators are selection / reproduction, crossover and

mutation. After reproduction, crossover and mutate the new individual by a small number of bits, arrows with dotted line indicate mutation and arrows with smooth lines indicate copying. We have generation n+1 Choose highest value of bits to make the circle| c_{ij} in the cost matrix and draw the vertical line of column of matrix. If there are two same highest values, consider the lowest cost.

6.Again take the genotype for next generation of second row except 2 above return to step 5.

7. Put the remaining genotypes of third and fourth row for reproduction, after completion of all steps, choose the highest value of bits to make the circle| c_{ij} Cost and draw vertical line ofj^{th} column. Now take value of cost of remaining row.

V. NUMERICAL EXAMPLES

A company has four machines and four jobs to be completed. Each machine must be assigned to complete one job. The required to set up each machine for completing each job is shown in table below. M stand for Machine and J stands for jobs.

Solution: The above intuitionistic fuzzy assignment problem can be formulated in the following mathematical programming

Mim(2,4,7;1,4,8)x_{11}+(1,3,4;0,3,5)x_{12}+(2,3,6;1,3,7)x_{13}+(3,5,8;2,5,13) x_{14}+(3,6,10;2,6,11)x_{21} +(2,3,5;1,3,6) x_{22}+(3,7,10;2,7,12) x_{23}+(6,7,11;3,7,12) x_{24}+(1,3,4;0,3,5)x_{31}+(2,3,5;1,3,6)x_{32}+(3,6,10;2,6,11) x_{33}+(2,3,6;1,3,7) x_{34}+ (3,7,10;2,7,12)x_{41}+ (2,3,4;-1,3,5)x_{42}+ (2,3,5;1,3,6)x_{43} + (3,6,10;2,6,11)x_{44}]

Subjectto $x_{11}+x_{12}+x_{13}+x_{14}=1$

$x_{11}+x_{21}+x_{31}+x_{41}=1$

$x_{21}+x_{22}+x_{23}+x_{24}=1$ $x_{12}+x_{22}+x_{32}+x_{42}=1$

$x_{31}+x_{32}+x_{33}+x_{34}=1$ $x_{13}+x_{23}+x_{33}+x_{43}=1$

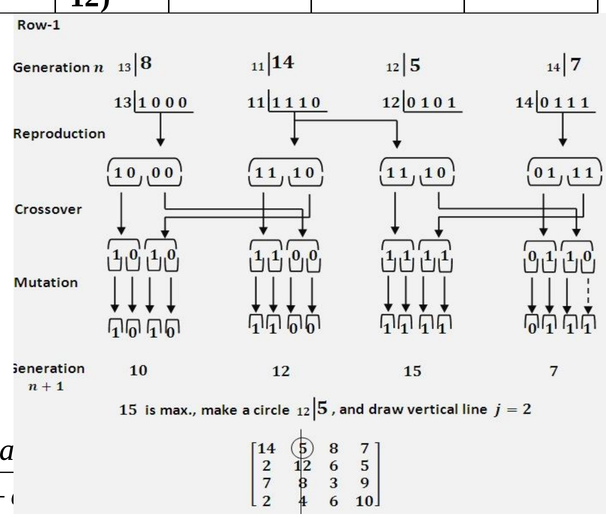
$x_{41}+x_{42}+x_{43}+x_{44}=1$ $x_{14}+x_{24}+x_{34}+x_{44}=1$ where $x_{ij} \in [0,1]$

$$R(\grave{A}^{I}) = \frac{1}{3}[\frac{(a_3' - a_1')(a_2 - 2a_3' - 2a_1') + (a_3 - a_1)(a}{a_3' - a_1' + a_3 - }$$

We replace nearest integer these values for their corresponding c_{ij} in which result in a convenient assignment problem in the linear programming problem

$$\begin{bmatrix} 5 & 3 & 6 & 8 \\ 7 & 4 & 9 & 10 \\ 3 & 4 & 7 & 6 \\ 9 & 2 & 11 & 7 \end{bmatrix}$$

	J_1	J_2	J_3	J_4
M_1	(2,4,7;1,4,8)	(1,3,4;0,3,5)	(2,3,6;1,3,7)	(3,5,8;2,5,13)
M_2	(3,6,10;2,6,11)	(2,3,5;1,3,6)	(3,7,10;2,7,12)	(6,7,11;3,7,12)
M_3	(1,3,4;0,3,5)	(2,3,5;1,3,6)	(3,6,10;2,6,11)	(2,3,6;1,3,7)
M_4	(3,7,10;2,7,12)	(2,3,4;-1,3,5)	(2,3,5;1,3,6)	(3,6,10;2,6,11)



calculated R($\tilde{C}_{11}$) =5.11≈ (5)R($\tilde{C}_{12}$) = 3.04 R($\tilde{C}_{13}$) = 6.16 R($\tilde{C}_{14}$) =8.08

R($\tilde{C}_{21}$) =7.08R($\tilde{C}_{22}$) = 4.16R($\tilde{C}_{23}$) = 9.12

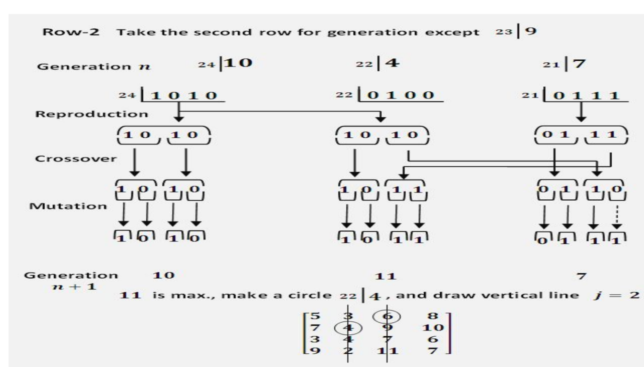
R($\tilde{C}_{24}$) = 10.01

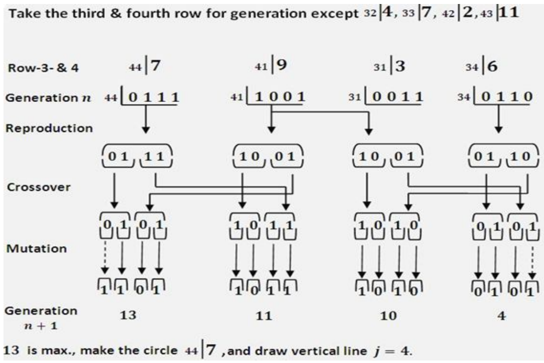
R($\tilde{C}_{31}$) = 3.04 R($\tilde{C}_{32}$) =4.16 R($\tilde{C}_{33}$) =7.08

R($\tilde{C}_{34}$) = 6.16

R($\tilde{C}_{41}$) = 9.12 R($\tilde{C}_{42}$) = 2.24 R($\tilde{C}_{43}$) = 11.04

R($\tilde{C}_{44}$) = 7.08





$$\begin{bmatrix} 5.11 & 3.04 & \mathbf{6.16} & 8.08 \\ 7.08 & \mathbf{4.16} & 9.12 & 10.01 \\ \mathbf{3.04} & 4.16 & 7.08 & 6.13 \\ 9.12 & 2.27 & 11.04 & \mathbf{7.08} \end{bmatrix}$$

Now take remaining value of third row 3.04 X_{13}=6.16, X_{22}=4.16, X_{31}=3.04, X_{44}=7.08 optimal solution=20.44

VI. CONCLUSIONS

In this paper the new algorithm is proposed to Intuitionistic fuzzy optimal solution to a Intuitionistic fuzzy assignment problem. First An Intuitionistic trapezoidal fuzzy numbers transformed in to Crisp form using ranking procedure of Annie Varghese and Sunny Kuiakose [1] next Genetic Algorithm has been applied to find an optimal solution. Numerical example has been shown that the total cost obtain that total cost is optimal. Finally, we deduce that the proposed method provides the better optimum toInstuitionsticfuzzy assignment problem and it can serve an important tool in decision making problem.

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