Interior Operators over Bipolar Intuitionistic M- Fuzzy Prime Group and Anti M- Fuzzy Prime Group

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Abstract. *In this paper the concept of a bipolar intuitionistic M fuzzy prime group and anti M fuzzy prime group is a new algebraic structure of a bipolar intuitionistic M fuzzy prime subgroup of a M fuzzy prime group and anti M fuzzy prime group are defined, related operators and interior operators are investigated. The purpose of the study is to implement the fuzzy set theory and group theory of bipolar intuitionistic M fuzzy prime subgroup of a M fuzzy prime group and anti M fuzzy prime group. The relation between operation of operators and interior operators of bipolar intuitionistic M fuzzy prime group and bipolar intuitionistic anti M fuzzy prime group are established.*

Keywords. *Bipolar fuzzy set, bipolar M fuzzy prime group, bipolar anti M fuzzy prime group, bipolar intuitionistic M fuzzy prime group, bipolar intuitionistic anti M fuzzy prime group.*

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1. Introduction

The concept of fuzzy sets was initiated by L.A. Zadeh [16] then it has becomes a vigorous area of research in engineering , medical science, graph theory. Rosenfeld [15] gave the idea of fuzzy subgroups. The author W.R. Zhang [17] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. In case of bipolar valued fuzzy sets membership degree is enlarged from the interval [0,1] to [-1,1]. In a bipolar valued fuzzy sets, the membership degree '0' means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements satisfy the property and the membership degree [-1, 0) indicates that elements satisfy the implicit counter property. Atanassov. K.T [1-7] introduced intuitionistic fuzzy sets and new operations defined over intuitionistic fuzzy sets. He was investigated operators over interval valued intuitionistic fuzzy sets and interior operators on intuitionistic fuzzy sets. He introduced necessity and possibility operators on intuitionistic fuzzy sets are defined in some logics. This fact that intuitionistic fuzzy sets are proper extension of ordinary fuzzy sets. R. Muthuraj [11,12] introduced the concept of bipolar fuzzy subgroup and bipolar anti M fuzzy group. M. Palanivelrajan and S. Nandakumar [13,14] introduced some operations of intuitionistic fuzzy primary and semi primary ideal. We discuss some of its properties and some operations of operator and interior operator of bipolar intuitionistic M fuzzy prime group and bipolar intuitionistic anti M fuzzy prime group are established.

2. preliminaries

In this section the fundamental definitions that will be used in the sequel. Throughout this paper, $G = (G, *)$ is a finite groups, e is the identity element of G, and xy mean x*y.

Definition.2.1. *[1]* Let A be an bipolar intuitionistic fuzzy sets of E then the " necessity" operator $\Box$ is defined by

$$\Box A^{+} = \left\{ < x, \mu_A^{+}(x),\ 1 - \mu_A^{+}(x) > / \ x \in E \right\} \ and$$

$$\Box A^{-} = \left\{ < x, \mu_A^{-}(x),\ -1 - \mu_A^{-}(x) > / \ x \in E \right\}$$

Definition.2.2. ***[1]*** Let A be an bipolar intuitionistic fuzzy sets of E then the " possibility" operator ◊ is defined by

$$\Diamond A^{+}=\left\{< x,1-\upsilon_{A}^{+}(x),\upsilon_{A}^{+}(x)>/x\in E\right\}\ and$$
$$\Diamond A^{-}=\left\{< x,-1-\upsilon_{A}^{-}(x),\upsilon_{A}^{-}(x)>/x\in E\right\}$$

Definition.2.3. *[9]* Let G be an M group and A be a bipolar intuitionistic fuzzy subgroup of G then A is called a bipolar intuitionistic M fuzzy group of G, if for all $x\in G$ and $m\in M$ then

$$i)\ \mu_{A}^{+}(mx)\geq\mu_{A}^{+}(x)\ and\ \upsilon_{A}^{+}(mx)\leq\upsilon_{A}^{+}(x)$$
$$ii)\ \mu_{A}^{-}(mx)\leq\mu_{A}^{-}(x)\ and\ \upsilon_{A}^{-}(mx)\geq\upsilon_{A}^{-}(x)$$

Example 2.4.

Consider $1\in M$ $\mu_{A}^{+}(x)=\begin{cases}0.7\ if\quad x=1\\ 0.6\ if\ x=-1\\ 0.4\ if\ x=i,-i\end{cases}$; $\upsilon_{A}^{+}(x)=\begin{cases}0.2\ if\quad x=1\\ 0.3\ if\ x=-1\\ 0.5\ if\ x=i,-i\end{cases}$ and

$$\mu_{A}^{-}(x)=\begin{cases}-0.8\ if\quad x=1\\ -0.5\ if\ x=-1\\ -0.3\ if\ x=i,-i\end{cases};\ \upsilon_{A}^{-}(x)=\begin{cases}-0.1\ if\quad x=1\\ -0.4\ if\ x=-1\\ -0.6\ if\ x=i,-i\end{cases}$$

Definition.2.5. [9] Let G be an M group and A be a bipolar intuitionistic anti fuzzy subgroup of G then A is called a bipolar intuitionistic anti M fuzzy group of G, if for all $x\in G$ and $m\in M$ then

$$i)\ \mu_{A}^{+}(mx)\leq\mu_{A}^{+}(x)\ and\ \upsilon_{A}^{+}(mx)\geq\upsilon_{A}^{+}(x)$$
$$ii)\ \mu_{A}^{-}(mx)\geq\mu_{A}^{-}(x)\ and\ \upsilon_{A}^{-}(mx)\leq\upsilon_{A}^{-}(x)$$

Example 2.6.

Consider $1\in M$ $\mu_{A}^{+}(x)=\begin{cases}0.4\ if\quad x=1\\ 0.6\ if\ x=-1\\ 0.7\ if\ x=i,-i\end{cases}$; $\upsilon_{A}^{+}(x)=\begin{cases}0.5\ if\quad x=1\\ 0.3\ if\ x=-1\\ 0.2\ if\ x=i,-i\end{cases}$ and

$$\mu_{A}^{-}(x)=\begin{cases}-0.3\ if\quad x=1\\ -0.5\ if\ x=-1\\ -0.8\ if\ x=i,-i\end{cases};\ \upsilon_{A}^{-}(x)=\begin{cases}-0.6\ if\quad x=1\\ -0.4\ if\ x=-1\\ -0.1\ if\ x=i,-i\end{cases}$$

Definition.2.7.[1]Let A be an bipolar intuitionistic fuzzy set then the interior operator I is defined by

$$I(A^{+})=\{< x,\ \min(\mu_{A}^{+}(y)),\ \max(\upsilon_{A}^{+}(y))>/x\in E,y\in E\}\ and$$
$$I(A^{-})=\{< x,\ \max(\mu_{A}^{-}(y)),\ \min(\upsilon_{A}^{-}(y))>/x\in E,y\in E\}$$

Definition.2.8. Let A be a fuzzy subgroup of μ of a group G is called a fuzzy prime if for all $a,b\in G$ and $a,b\in A$ either $\mu_{A}(ab)=\mu_{A}(a)$ or else $\mu_{A}(ab)=\mu_{A}(b)$

***Definition. 2.9.*.** A bipolar fuzzy subgroup A of a M fuzzy group of G is said to be bipolar intuitionistic M fuzzy prime group or bipolar intuitionistic anti M fuzzy prime group of G if for all $x,m\in G$ either

$$\mu_A^+(mx) = \mu_A^+(m),\ \upsilon_A^+(mx) = \upsilon_A^+(m)\ \ and\ \ \mu_A^-(mx) = \mu_A^-(m),\ \upsilon_A^-(mx) = \upsilon_A^-(m) \quad \text{else}$$

$$\mu_A^+(mx) = \mu_A^+(x),\ \upsilon_A^+(mx) = \upsilon_A^+(x)\ \ and\ \ \mu_A^-(mx) = \mu_A^-(x),\ \upsilon_A^-(mx) = \upsilon_A^-(x)$$

Example.2.10.

Let G={1,2,3,5,7} and A={1,2,5,7}

$$\mu_A^+(x) = \begin{cases} 1 \ if \ \ x \in <7> \\ 0.5 \ if \ \ x \in <2> \square <7> \end{cases} \qquad \upsilon_A^+(x) = \begin{cases} 0 \ if \ \ x \in <7> \\ 0.4 \ if \ \ x \in <2> \square <7> \end{cases} \quad and$$

$$\mu_A^-(x) = \begin{cases} -1 \ if \ \ x \in <7> \\ -0.5 \ if \ \ x \in <2> \square <7> \end{cases} \qquad \upsilon_A^+(x) = \begin{cases} 0 \ if \ \ x \in <7> \\ -0.4 \ if \ \ x \in <2> \square <7> \end{cases}$$

Theorem.2.11. If A is an bipolar intuitionistic M fuzzy prime group of G then I(A) is an bipolar intuitionistic M fuzzy prime group of G.

Proof. Let $x, m \in G \ then \ x, m \in A$

$$Consider \quad \mu_{I(A)}^+(mx) = \min(\mu_A^+(ab)) \geq \min(\mu_A^+(b)) = \mu_{I(A)}^+(x)$$
$$Therefore \quad \mu_{I(A)}^+(mx) \geq \mu_{I(A)}^+(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon_{I(A)}^+(mx) = \max(\upsilon_A^+(ab)) \leq \max(\upsilon_A^+(b)) = \upsilon_{I(A)}^+(x)$$
$$Therefore \quad \upsilon_{I(A)}^+(mx) \leq \upsilon_{I(A)}^+(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \mu_{I(A)}^-(mx) = \max(\mu_A^-(ab)) \leq \max(\mu_A^-(b)) = \mu_{I(A)}^-(x)$$
$$Therefore \quad \mu_{I(A)}^-(mx) \leq \mu_{I(A)}^-(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon_{I(A)}^-(mx) = \min(\upsilon_A^-(ab)) \geq \min(\upsilon_A^-(b)) = \upsilon_{I(A)}^-(x)$$
$$Therefore \quad \upsilon_{I(A)}^-(mx) \geq \upsilon_{I(A)}^-(x) \ \ for \ some \ a, m \in M$$

Therefore I(A) is an bipolar intuitionistic M fuzzy prime group of G.

Theorem.2.12. If A is an bipolar intuitionistic M fuzzy prime group of G then I (I(A))=I(A) is an bipolar intuitionistic M fuzzy prime group of G.

Proof. Let $x, m \in G \ then \ x, m \in A$

$$Consider \quad \mu_{I(I(A))}^+(mx) = \min(\mu_{I(A)}^+(ab)) = \min(\min(\mu_A^+(mx))) = \min(\mu_A^+(mx))$$
$$\geq \min(\mu_A^+(x)) = \mu_{I(A)}^+(x)$$
$$Therefore \quad \mu_{I(I(A))}^+(mx) \geq \mu_{I(A)}^+(x) \ \ for \ some \ m \in M$$

$$Consider \quad \upsilon_{I(I(A))}^+(mx) = \max(\upsilon_{I(A)}^+(ab)) = \max(\max(\upsilon_A^+(mx))) = \max(\upsilon_A^+(mx))$$
$$\leq \max(\upsilon_A^+(x)) = \upsilon_{I(A)}^+(x)$$
$$Therefore \quad \upsilon_{I(I(A))}^+(mx) \leq \upsilon_{I(A)}^+(x) \ \ for \ some \ m \in M$$

$$\begin{aligned} &Consider \quad \mu^-_{I(I(A))}(mx) = \max(\mu^-_{I(A)}(ab)) = \max(\max(\mu^-_A(mx))) = \max(\mu^-_A(mx)) \\ &\qquad\qquad \le \max(\mu^-_A(x)) = \mu^-_{I(A)}(x) \\ &Therefore \quad \mu^-_{I(I(A))}(mx) \le \mu^-_{I(A)}(x) \;\; for\;\; some\;\; m \in M \end{aligned}$$

$$\begin{aligned} &Consider \quad \upsilon^-_{I(I(A))}(mx) = \min(\upsilon^-_{I(A)}(ab)) = \min(\min(\upsilon^-_A(mx))) = \min(\upsilon^-_A(mx)) \\ &\qquad\qquad \ge \min(\upsilon^-_A(x)) = \upsilon^-_{I(A)}(x) \\ &Therefore \quad \upsilon^-_{I(I(A))}(mx) \ge \upsilon^-_{I(A)}(x) \;\; for\;\; some\;\; m \in M \end{aligned}$$

Therefore I (I(A)) = I(A) is an bipolar intuitionistic M fuzzy prime group of G.

Theorem.2.13. If A and B are bipolar intuitionistic M fuzzy prime group of G then $I(A \cap B) = I(A) \cap I(B)$ is an bipolar intuitionistic M fuzzy prime group of G.

Proof. Let $x, m \in G$ *then* $x, m \in A \cap B$ *implies* $x, m \in A$ *and* $x, m \in B$

$$\begin{aligned} &Consider \quad \mu^+_{I(A\cap B)}(mx) = \min(\mu^+_{A\cap B}(ab)) = \min(\min(\mu^+_A(ab), \mu^+_B(ab))) \\ &\qquad\qquad \ge \min(\min(\mu^+_A(b), \mu^+_B(b))) = \min(\min(\mu^+_A(b), \min(\mu^+_B(b)) \\ &\qquad\qquad = \min(\mu^+_{I(A)}(x), \mu^+_{I(B)}(x)) = \mu^+_{I(A)\cap I(B)}(x) \\ &Therefore \quad \mu^+_{I(A\cap B)}(mx) \ge \mu^+_{I(A)\cap I(B)}(x) \;\; for\;\; some\;\; a, m \in M \end{aligned}$$

$$\begin{aligned} &Consider \quad \upsilon^+_{I(A\cap B)}(mx) = \max(\upsilon^+_{A\cap B}(ab)) = \max(\max(\upsilon^+_A(ab), \upsilon^+_B(ab))) \\ &\qquad\qquad \le \max(\max(\upsilon^+_A(b), \upsilon^+_B(b))) = \max(\max(\upsilon^+_A(b), \max(\upsilon^+_B(b))) \\ &\qquad\qquad = \max(\upsilon^+_{I(A)}(x), \upsilon^+_{I(B)}(x)) = \upsilon^+_{I(A)\cap I(B)}(x) \\ &Therefore \quad \upsilon^+_{I(A\cap B)}(mx) \le \upsilon^+_{I(A)\cap I(B)}(x) \;\; for\;\; some\;\; a, m \in M \end{aligned}$$

$$\begin{aligned} &Consider \quad \mu^-_{I(A\cap B)}(mx) = \max(\mu^-_{A\cap B}(ab)) = \max(\max(\mu^-_A(ab), \mu^-_B(ab))) \\ &\qquad\qquad \le \max(\max(\mu^-_A(b), \mu^-_B(b)) = \max(\max(\mu^-_A(b), \max(\mu^-_B(b))) \\ &\qquad\qquad = \max(\mu^-_{I(A)}(x), \mu^-_{I(B)}(x)) = \mu^-_{I(A)\cap I(B)}(x) \\ &Therefore \quad \mu^-_{I(A\cap B)}(mx) \le \mu^-_{I(A)\cap I(B)}(x) \;\; for\;\; some\;\; a, m \in M \end{aligned}$$

$$\begin{aligned} &Consider \quad \upsilon^-_{I(A\cap B)}(mx) = \min(\upsilon^-_{A\cap B}(ab)) = \min(\min(\upsilon^-_A(ab), \upsilon^-_B(ab))) \\ &\qquad\qquad \ge \min(\min(\upsilon^-_A(b), (\upsilon^-_B(b))) = \min(\min(\upsilon^-_A(b), \min(\upsilon^-_B(b))) \\ &\qquad\qquad = \min(\upsilon^-_{I(A)}(x), \upsilon^-_{I(B)}(x)) = \upsilon^-_{I(A)\cap I(B)}(x) \\ &Therefore \quad \upsilon^-_{I(A\cap B)}(mx) \ge \upsilon^-_{I(A)\cap I(B)}(x) \;\; for\;\; some\;\; a, m \in M \end{aligned}$$

Therefore $I(A \cap B) = I(A) \cap I(B)$ is an bipolar intuitionistic M fuzzy prime group of G.

Theorem.2.14. If A is an bipolar intuitionistic M fuzzy prime group of G then $\square(I(A)) = I(\square(A))$ is an bipolar intuitionistic M fuzzy prime group of G.

Proof. Let $x, m \in G \ then \ x, m \in A$

$$Consider \quad \mu^{+}_{\Box(I(A))}(mx) = \mu^{+}_{(I(A))}(mx) = \min(\mu^{+}_{A}(ab)) \geq \min(\mu^{+}_{A}(b))$$
$$= \min(\mu^{+}_{\Box A}(b)) = \mu^{+}_{I(\Box A))}(x)$$
$$Therefore \quad \mu^{+}_{\Box(I(A))}(mx) \geq \mu^{+}_{I(\Box A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon^{+}_{\Box(I(A))}(mx) = 1 - \mu^{+}_{\Box(I(A))}(mx) = 1 - (\mu^{+}_{(I(A))}(mx)) = 1 - \min(\mu^{+}_{A}(ab))$$
$$\leq 1 - \min(\mu^{+}_{A}(b)) = 1 - \min(\mu^{+}_{\Box A}(b)) = 1 - (\mu^{+}_{I(\Box(A))}(x))$$
$$= \upsilon^{+}_{I(\Box(A))}(x)$$
$$Therefore \quad \upsilon^{+}_{\Box(I(A))}(mx) \leq \upsilon^{+}_{I(\Box(A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \mu^{-}_{\Box(I(A))}(mx) = \mu^{-}_{(I(A))}(mx) = \max(\mu^{-}_{A}(ab)) \leq \max(\mu^{-}_{A}(b))$$
$$= \max(\mu^{-}_{\Box A}(b)) = \mu^{-}_{I(\Box A))}(x)$$
$$Therefore \quad \mu^{-}_{\Box(I(A))}(mx) \leq \mu^{-}_{I(\Box A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon^{-}_{\Box(I(A))}(mx) = -1 - \mu^{-}_{\Box(I(A))}(mx) = -1 - (\mu^{-}_{(I(A))}(mx)) = -1 - \max(\mu^{-}_{A}(ab))$$
$$\geq -1 - \max(\mu^{-}_{A}(b)) = -1 - \max(\mu^{-}_{\Box A}(b)) = -1 - (\mu^{-}_{I(\Box(A))}(x))$$
$$= \upsilon^{-}_{I(\Box(A))}(x)$$
$$Therefore \quad \upsilon^{-}_{\Box(I(A))}(mx) \geq \upsilon^{-}_{I(\Box(A))}(x) \ \ for \ some \ a, m \in M$$

Therefore $\Box(I(A)) = I(\Box(A))$ is an bipolar intuitionistic M fuzzy prime group of G.

Theorem.2.15. If A is an bipolar intuitionistic M fuzzy prime group of G then $\Diamond(I(A)) = I(\Diamond(A))$ is an bipolar intuitionistic M fuzzy prime group of G.

Proof. Let $x, m \in G \ then \ x, m \in A$

$$Consider \quad \mu^{+}_{\Diamond(I(A))}(mx) = 1 - \upsilon^{+}_{(I(A))}(mx) = 1 - \max(\upsilon^{+}_{A}(ab)) \geq 1 - \max(\upsilon^{+}_{A}(b))$$
$$= 1 - \max(\upsilon^{+}_{\Diamond A}(b)) = 1 - (\upsilon^{+}_{I(\Diamond A))})(x) = \mu^{+}_{I(\Diamond A)}(x)$$
$$Therefore \quad \mu^{+}_{\Diamond(I(A))}(mx) \geq \mu^{+}_{I(\Diamond A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon^{+}_{\Diamond(I(A))}(mx) = \upsilon^{+}_{I(A)}(mx) = \max(\upsilon^{+}_{A}(ab)) \leq \max(\upsilon^{+}_{A}(b))$$
$$= \max(\upsilon^{+}_{\Diamond A}(b)) = \upsilon^{+}_{I(\Diamond(A))}(x)$$
$$Therefore \quad \upsilon^{+}_{\Diamond(I(A))}(mx) \leq \upsilon^{+}_{I(\Box(A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \mu^{-}_{\Diamond(I(A))}(mx) = -1 - \upsilon^{-}_{I(A)}(mx) = -1 - \min(\upsilon^{-}_{A}(ab)) \leq -1 - \min(\upsilon^{-}_{A}(b))$$
$$= -1 - \min(\upsilon^{-}_{\Diamond A}(b)) = -1 - (\upsilon^{-}_{I(\Diamond A))})(x) = \mu^{-}_{I(\Diamond A)}(x)$$
$$Therefore \quad \mu^{-}_{\Diamond(I(A))}(mx) \leq \mu^{-}_{I(\Diamond A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon^{-}_{\Diamond(I(A))}(mx) = \upsilon^{-}_{I(A)}(mx) = \min(\upsilon^{-}_{A}(ab)) \geq \min(\upsilon^{-}_{A}(b))$$
$$= \min(\upsilon^{-}_{\Diamond A}(b)) = \upsilon^{-}_{I(\Diamond A)}(x)$$
$$Therefore \quad \upsilon^{+}_{\Diamond(I(A))}(mx) \geq \upsilon^{-}_{I(\Box(A))}(x) \;\; for \;\; some \;\; a, m \in M$$

Therefore $\Diamond(I(A)) = I(\Diamond(A))$ is an bipolar intuitionistic M fuzzy prime group of G.

3.Bipolar Intuitionistic Anti M fuzzy prime group of G.

Theorem.3.1. If A is an bipolar intuitionistic anti M fuzzy prime group of G then I(A) is an bipolar intuitionistic anti M fuzzy prime group of G.

Proof. Let $x, m \in G \; then \; x, m \in A$

$$Consider \quad \mu^{+}_{I(A)}(mx) = \min(\mu^{+}_{A}(ab)) \leq \min(\mu^{+}_{A}(b)) = \mu^{+}_{I(A)}(x)$$
$$Therefore \quad \mu^{+}_{I(A)}(mx) \leq \mu^{+}_{I(A)}(x) \;\; for \;\; some \;\; a, m \in M$$

$$Consider \quad \upsilon^{+}_{I(A)}(mx) = \max(\upsilon^{+}_{A}(ab)) \geq \max(\upsilon^{+}_{A}(b)) = \upsilon^{+}_{I(A)}(x)$$
$$Therefore \quad \upsilon^{+}_{I(A)}(mx) \geq \upsilon^{+}_{I(A)}(x) \;\; for \;\; some \;\; a, m \in M$$

$$Consider \quad \mu^{-}_{I(A)}(mx) = \max(\mu^{-}_{A}(ab)) \geq \max(\mu^{-}_{A}(b)) = \mu^{-}_{I(A)}(x)$$
$$Therefore \quad \mu^{-}_{I(A)}(mx) \geq \mu^{-}_{I(A)}(x) \;\; for \;\; some \;\; a, m \in M$$

$$Consider \quad \upsilon^{-}_{I(A)}(mx) = \min(\upsilon^{-}_{A}(ab)) \leq \min(\upsilon^{-}_{A}(b)) = \upsilon^{-}_{I(A)}(x)$$
$$Therefore \quad \upsilon^{-}_{I(A)}(mx) \leq \upsilon^{-}_{I(A)}(x) \;\; for \;\; some \;\; a, m \in M$$

Therefore I(A) is an bipolar intuitionistic anti M fuzzy prime group of G.

Theorem.3.2. If A is an bipolar intuitionistic anti M fuzzy prime group of G then I (I(A)) = I(A) is an bipolar intuitionistic anti M fuzzy prime group of G.

Proof. Let $x, m \in G \; then \; x, m \in A$

$$Consider \quad \mu^{+}_{I(I(A))}(mx) = \min(\mu^{+}_{I(A)}(ab)) = \min(\min(\mu^{+}_{A}(mx))) = \min(\mu^{+}_{A}(mx))$$
$$\leq \min(\mu^{+}_{A}(x)) = \mu^{+}_{I(A)}(x)$$
$$Therefore \quad \mu^{+}_{I(I(A))}(mx) \leq \mu^{+}_{I(A)}(x) \;\; for \;\; some \;\; m \in M$$

$$Consider \quad \upsilon^{+}_{I(I(A))}(mx) = \max(\upsilon^{+}_{I(A)}(ab)) = \max(\max(\upsilon^{+}_{A}(mx))) = \max(\upsilon^{+}_{A}(mx))$$
$$\geq \max(\upsilon^{+}_{A}(x)) = \upsilon^{+}_{I(A)}(x)$$
$$Therefore \quad \upsilon^{+}_{I(I(A))}(mx) \geq \upsilon^{+}_{I(A)}(x) \;\; for \;\; some \;\; m \in M$$

$$Consider \quad \mu^{-}_{I(I(A))}(mx) = \max(\mu^{-}_{I(A)}(ab)) = \max(\max(\mu^{-}_{A}(mx))) = \max(\mu^{-}_{A}(mx))$$
$$\geq \max(\mu^{-}_{A}(x)) = \mu^{-}_{I(A)}(x)$$
$$Therefore \quad \mu^{-}_{I(I(A))}(mx) \geq \mu^{-}_{I(A)}(x) \;\; for \; some \; m \in M$$

$$Consider \quad \upsilon^{-}_{I(I(A))}(mx) = \min(\upsilon^{-}_{I(A)}(ab)) = \min(\min(\upsilon^{-}_{A}(mx))) = \min(\upsilon^{-}_{A}(mx))$$
$$\leq \min(\upsilon^{-}_{A}(x)) = \upsilon^{-}_{I(A)}(x)$$
$$Therefore \quad \upsilon^{-}_{I(I(A))}(mx) \leq \upsilon^{-}_{I(A)}(x) \;\; for \; some \; m \in M$$

Therefore I(I(A)) = I(A) is an bipolar intuitionistic anti M fuzzy prime group of G.

Theorem.3.3. If A and B are bipolar intuitionistic anti M fuzzy prime group of G then $I(A \cap B) = I(A) \cap I(B)$ is an bipolar intuitionistic anti M fuzzy prime group of G.

Proof. Let $x, m \in G \; then \; x, m \in A \cap B \; implies \; x, m \in A and \; x, m \in B$

$$Consider \quad \mu^{+}_{I(A\cap B)}(mx) = \min(\mu^{+}_{A\cap B}(ab)) = \min(\min(\mu^{+}_{A}(ab), \mu^{+}_{B}(ab)))$$
$$\leq \min(\min(\mu^{+}_{A}(b), \mu^{+}_{B}(b))) = \min(\min(\mu^{+}_{A}(b), \min(\mu^{+}_{B}(b))$$
$$= \min(\mu^{+}_{I(A)}(x), \mu^{+}_{I(B)}(x)) = \mu^{+}_{I(A)\cap I(B)}(x)$$
$$Therefore \quad \mu^{+}_{I(A\cap B)}(mx) \leq \mu^{+}_{I(A)\cap I(B)}(x) \;\; for \; some \; a, m \in M$$

$$Consider \quad \upsilon^{+}_{I(A\cap B)}(mx) = \max(\upsilon^{+}_{A\cap B}(ab)) = \max(\max(\upsilon^{+}_{A}(ab), \upsilon^{+}_{B}(ab)))$$
$$\geq \max(\max((\upsilon^{+}_{A}(b), (\upsilon^{+}_{B}(b))) = \max(\max(\upsilon^{+}_{A}(b), \max(\upsilon^{+}_{B}(b)))$$
$$= \max(\upsilon^{+}_{I(A)}(x), \upsilon^{+}_{I(B)}(x)) = \upsilon^{+}_{I(A)\cap I(B)}(x)$$
$$Therefore \quad \upsilon^{+}_{I(A\cap B)}(mx) \geq \upsilon^{+}_{I(A)\cap I(B)}(x) \;\; for \; some \; a, m \in M$$

$$Consider \quad \mu^{-}_{I(A\cap B)}(mx) = \max(\mu^{-}_{A\cap B}(ab)) = \max(\max(\mu^{-}_{A}(ab), \mu^{-}_{B}(ab)))$$
$$\geq \max(\max(\mu^{-}_{A}(b), \mu^{-}_{B}(b))) = \max(\max(\mu^{-}_{A}(b), \max(\mu^{-}_{B}(b)))$$
$$= \max(\mu^{-}_{I(A)}(x), \mu^{-}_{I(B)}(x)) = \mu^{-}_{I(A)\cap I(B)}(x)$$
$$Therefore \quad \mu^{-}_{I(A\cap B)}(mx) \geq \mu^{-}_{I(A)\cap I(B)}(x) \;\; for \; some \; a, m \in M$$

$$Consider \quad \upsilon^{-}_{I(A\cap B)}(mx) = \min(\upsilon^{-}_{A\cap B}(ab)) = \min(\min(\upsilon^{-}_{A}(ab), \upsilon^{-}_{B}(ab)))$$
$$\leq \min(\min((\upsilon^{-}_{A}(b), (\upsilon^{-}_{B}(b))) \leq \min(\min(\upsilon^{-}_{A}(b), \min(\upsilon^{-}_{B}(b)))$$
$$= \min(\upsilon^{-}_{I(A)}(x), \upsilon^{-}_{I(B)}(x)) = \upsilon^{-}_{I(A)\cap I(B)}(x)$$
$$Therefore \quad \upsilon^{-}_{I(A\cap B)}(mx) \leq \upsilon^{-}_{I(A)\cap I(B)}(x) \;\; for \; some \; a, m \in M$$

Therefore $I(A \cap B) = I(A) \cap I(B)$ is an bipolar intuitionistic anti M fuzzy prime group of G.

Theorem.3.4. If A is an bipolar intuitionistic anti M fuzzy prime group of G then $\square(I(A)) = I(\square(A))$ is an bipolar intuitionistic anti M fuzzy prime group of G.

Proof. Let $x, m \in G \; then \; x, m \in A$

$$Consider \quad \mu^{+}_{\Box(I(A))}(mx) = \mu^{+}_{(I(A))}(mx) = \min(\mu^{+}_{A}(ab)) \leq \min(\mu^{+}_{A}(b))$$
$$= \min(\mu^{+}_{\Box A}(b)) = \mu^{+}_{I(\Box A))}(x)$$
$$Therefore \quad \mu^{+}_{\Box(I(A))}(mx) \leq \mu^{+}_{I(\Box A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon^{+}_{\Box(I(A))}(mx) = 1 - \mu^{+}_{\Box(I(A))}(mx) = 1 - (\mu^{+}_{(I(A))}(mx)) = 1 - \min(\mu^{+}_{A}(ab))$$
$$\geq 1 - \min(\mu^{+}_{A}(b)) = 1 - \min(\mu^{+}_{\Box A}(b)) = 1 - (\mu^{+}_{I(\Box(A))}(x))$$
$$= \upsilon^{+}_{I(\Box(A))}(x)$$
$$Therefore \quad \upsilon^{+}_{\Box(I(A))}(mx) \geq \upsilon^{+}_{I(\Box(A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \mu^{-}_{\Box(I(A))}(mx) = \mu^{-}_{(I(A))}(mx) = \max(\mu^{-}_{A}(ab)) \geq \max(\mu^{-}_{A}(b))$$
$$= \max(\mu^{-}_{\Box A}(b)) = \mu^{-}_{I(\Box A))}(x)$$
$$Therefore \quad \mu^{-}_{\Box(I(A))}(mx) \geq \mu^{-}_{I(\Box A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon^{-}_{\Box(I(A))}(mx) = -1 - \mu^{-}_{\Box(I(A))}(mx) = -1 - (\mu^{-}_{(I(A))}(mx)) = -1 - \max(\mu^{-}_{A}(ab))$$
$$\leq -1 - \max(\mu^{-}_{A}(b)) = -1 - \max(\mu^{-}_{\Box A}(b)) = -1 - (\mu^{-}_{I(\Box(A))}(x))$$
$$= \upsilon^{-}_{I(\Box(A))}(x)$$
$$Therefore \quad \upsilon^{-}_{\Box(I(A))}(mx) \leq \upsilon^{-}_{I(\Box(A))}(x) \ \ for \ some \ a, m \in M$$

Therefore $\Box(I(A)) = I(\Box(A))$ is an bipolar intuitionistic anti M fuzzy prime group of G.

Theorem.3.5. If A is an bipolar intuitionistic anti M fuzzy prime group of G then $\Diamond(I(A)) = I(\Diamond(A))$ is an bipolar intuitionistic anti M fuzzy prime group of G.

Proof. Let $x, m \in G \ then \ x, m \in A$

$$Consider \quad \mu^{+}_{\Diamond(I(A))}(mx) = 1 - \upsilon^{+}_{(I(A))}(mx) = 1 - \max(\upsilon^{+}_{A}(ab)) \leq 1 - \max(\upsilon^{+}_{A}(b))$$
$$= 1 - \max(\upsilon^{+}_{\Diamond A}(b)) = 1 - (\upsilon^{+}_{I(\Diamond(A))})(x) = \mu^{+}_{I(\Diamond(A))}(x)$$
$$Therefore \quad \mu^{+}_{\Diamond(I(A))}(mx) \leq \mu^{+}_{I(\Diamond(A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon^{+}_{\Diamond(I(A))}(mx) = \upsilon^{+}_{I(A)}(mx) = \max(\upsilon^{+}_{A}(ab)) \geq \max(\upsilon^{+}_{A}(b))$$
$$= \max(\upsilon^{+}_{\Diamond A}(b)) = \upsilon^{+}_{I(\Diamond A)}(x)$$
$$Therefore \quad \upsilon^{+}_{\Diamond(I(A))}(mx) \geq \upsilon^{+}_{I(\Box(A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \mu^{-}_{\Diamond(I(A))}(mx) = -1 - \upsilon^{-}_{I(A)}(mx) = -1 - \min(\upsilon^{-}_{A}(ab)) \geq -1 - \min(\upsilon^{-}_{A}(b))$$
$$= -1 - \min(\upsilon^{-}_{\Diamond A}(b)) = -1 - (\upsilon^{-}_{I(\Diamond A))})(x) = \mu^{-}_{I(\Diamond A)}(x)$$
$$Therefore \quad \mu^{-}_{\Diamond(I(A))}(mx) \geq \mu^{-}_{I(\Diamond A))}(x) \ \ for \ some \ a, m \in M$$

$$Consider \quad \upsilon^{-}_{\Diamond(I(A))}(mx) = \upsilon^{-}_{I(A)}(mx) = \min(\upsilon^{-}_{A}(ab)) \leq \min(\upsilon^{-}_{A}(b))$$

$$= \min(\upsilon^{-}_{\Diamond A}(b)) = \upsilon^{-}_{I(\Diamond A)}(x)$$

$$Therefore \quad \upsilon^{+}_{\Diamond(I(A))}(mx) \leq \upsilon^{-}_{I(\Box(A))}(x) \ for\ some\ a, m \in M$$

Therefore $\Diamond(I(A)) = I(\Diamond(A))$ is an bipolar intuitionistic anti M fuzzy prime group of G

4. Conclusion.

The concept of bipolar intuitionistic M fuzzy prime group and anti M fuzzy prime group are defined and new algebraic structure of a bipolar intuitionistic M fuzzy prime subgroup of a M fuzzy prime group and anti M fuzzy prime group are created and some related properties and some operations of Interior operators are investigated. The purpose of the study is to implement fuzzy set theory and group theory of bipolar intuitionistic M fuzzy prime subgroup of a M fuzzy prime group and anti M fuzzy prime group. We hope that our results can also be extended to other algebraic system.

5.References.

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