Pre Semi Homeomorphisms and Generalized Semi Pre Homeomorphisms in Topological Spaces

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Abstract - *In this paper, two new classes of pre semi homeomorphisms and psc homeomorphisms are introduced. Moreover, some properties of these two homeomorphisms are obtained.*

Keywords — *Put your keywords here, keywords are separated by comma.*

1. INTRODUCTION

Levine [11] has generalized the concept of closed sets to generalized closed sets. Bhattacharyya and Lahiri [23] have generalized the concept of closed sets to semi-generalized closed sets with the help of semi-open sets and obtained various topological properties. Devi, Balachandran and Maki [6] defined two classes of maps called semi-generalized homeomorphisms and generalized semi-homeomorphisms and also defined two classes of maps called sgc-homeomorphisms and gsc homeomorphism. M.K.R.S.Veera Kumar [24] have defined Pre semi closed set and obtained various topological properties. In this paper, we introduce two classes of maps called pre semi homeomorphism (briefly ps homeomorphism) and Pre semi closed homeomorphism (briefly psc homeomorphism) and study their properties.

Throughout the present paper, (X, τ) and (Y, σ) denote topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of X. We denote the interior of A (respectively the closure of A) with respect to τ by int (A) (respectively cl (A))

2. PRELIMINARIES

Since we use the following definitions and some properties, we recall them in this section.

Definition 2.1 - A subset A of a space (X, □) is called

1. a ***semi-open*** set[**12**] if A □ cl(int(A)) and a ***semi-closed*** set if int(cl(A)) □ A.
2. a ***preopen*** set[**16**] if A □ int(cl(A)) and a ***preclosed*** set if cl(int(A)) □ A.
3. an ***□-open set***[**18**] if A □ int(cl(int(A))) and a ***□-closed*** set if cl(int(cl(A))) □ A.
4. a ***semi-preopen*** set[**2**] (=***□-open***[**1**]) if A □ cl(int(cl(A))) and a ***semi-preclosed*** set[**2**] (=***□-closed*** [**1**]) if int(cl(int(A))) □ A.

The semi-closure (resp. □-closure, semi-preclosure) of a subset A of (X, □) is the intersection of all semi-closed (resp. □–closed, semi-preclosed) sets that contain A and is denoted by scl(A) (resp. □cl(A), spcl(A)).

Definition 2.2 – A subset A of a space (X, □) is called

1. a ***generalized closed*** (briefly ***g-closed***) set [**11**] if cl(A) □ U whenever A □ U and U is open in (X, □).
2. a ***semi-generalized closed*** set (briefly ***sg-closed***) [**5**] if scl(A)□ □U whenever A□U and U is semi-open in (X, □). The complement of a sg-closed set is called a sg-open set.
3. a ***generalized semi-closed*** set (briefly ***gs-closed***) [**3**] if scl(A) □ U whenever A □ U and U is open in (X, □).
4. a ***generalized □-closed*** set (briefly ***g□-closed***) [**15**] if □cl(A) □ U whenever A □ U and U is □-open in (X, □).
5. a ***generalized semi-preclosed*** (briefly ***gsp-closed***) set[**8**] if spcl(A)□U whenever A□U and U is open in (X, □).
6. a ***pre-semi-closed*** set[**24**] if spcl(A) □□U whenever A □ U and U is g-open in (X, □).
7. a ***ĝ-closed*** set[**20**] if cl(A) ⊆ G whenever A ⊆ G and G is semi-open in (X,□).

Definition 2.3 – A function f**:** (X, □) □ (Y, □) is said to be

1. ***semi- continuous*** [**12**] if f^{-1}(V) is semi-open in (X, □) for every open set V of (Y, □).
2. ***pre- continuous*** [**16**] if f^{-1}(V) is pre-closed in (X, □) for every closed set V of (Y, □).
3. ***□-continuous*** [**17**] if f^{-1}(V) is □-closed in (X, □) for every closed set V of (Y, □).
4. ***□-continuous*** [**1**] if f^{-1}(V) is semi-preopen in (X, □) for every open set V of (Y, □).
5. ***g- continuous*** [**4**] if f^{-1}(V) is g-closed in (X, □) for every closed set V of (Y, □).

6. ***sg- continuous*** [**19**] if $f^{-1}(V)$ is sg-closed in (X, τ) for every closed set V of (Y, σ).
7. ***gs - continuous*** [**6**] if $f^{-1}(V)$ is gs-closed in (X, τ) for every closed set V of (Y, σ).
8. ***gα- continuous*** [**15**] if $f^{-1}(V)$ is gα-closed in (X, τ) for every closed set V of (Y, σ).
9. ***gsp-continuous*** [**8**] if $f^{-1}(V)$ is gsp-closed in (X, τ) for every closed set V of (Y, σ).
10. ***pre-β-closed*** [**13**]if f(V) is semi-preclosed in (Y, σ) for every semi-preclosed set V of (X, τ).
11. ***pre-semi-continuous*** [**24**] if $f^{-1}(V)$ is a pre-semi-closed set of (X, τ) for every closed set V of (Y, σ).
12. ***ĝ-continuous*** [**20**] if $f^{-1}(V)$ is ĝ-closed in (X, τ) for every closed set V of (Y, σ).
13. ***gc-irresolute***[**4**] if $f^{-1}(V)$ is g-closed in (X, τ) for every g-closed set V of (Y, σ).
14. ***gs-irresolute***[**6**] if $f^{-1}(V)$ is gs-closed in (X, τ) for every gs-closed set V of (Y, σ).
15. ***irresolute*** [**14**] if $f^{-1}(V)$ is semi-open in (X, τ) for every semi-open set V of (Y, σ).
16. ***sg-irresolute*** [**19**] if $f^{-1}(V)$ is sg-closed in (X, τ) for every sg-closed set V of (Y, σ).
17. ***ĝ-irresolute*** [**20**] if $f^{-1}(V)$ is ĝ-closed in (X, τ) for every ĝ-closed set V of (Y, σ).
18. ***pre-semi-irresolute*** [**24**] if $f^{-1}(V)$ is a pre-semi-closed set of (X, τ) for every pre-semi-closed set V of (Y, σ).

Definition 2.4 – A function f**:** (X, τ) → (Y, σ) is said to be ***homeomorphism (topological mapping)*** if and only if the following conditions are satisfied:

i) f is bijective
ii) f is continuous
iii) f^{-1} is continious

A bijection f **:** (X, τ) → (Y, σ) is called a

1. a ***semi homeomorphism(B) (simply s.h. (B))*** [**9**] if f is continuous, f is semi-open (i.e.f(U) is semi-open for every open set U of (X, τ)) and f is bijective.
2. a ***g-homeomorphism***[**22**] if f is both g-continuous and g-open.
3. a ***gc-homeomorphism***[**22**] if f and it's inverse f^{-1} are gc-irresolute.
4. a ***sg-homeomorphism***[**6**] if f is both sg-continuous and sg-open.
5. a ***gs-homeomorphism***[**6**] if f is both gs-continuous and gs-open.
6. a ***ĝ-homeomorphism*** [**21**] if f is ĝ-continuous and ĝ-open.
7. a ***ĝc-homeomorphism*** [**21**] if f is and f^{-1} are ĝ-irresolute.

3. Pre Semi Homeomorphisms

In this section we introduce the concepts of pre semi open map and pre semi homeomorphisms (briefly ps-homeomorphism); we discuss some of their properties.

Definition 3.1 - A function f**:** (X, τ) → (Y, σ) is called pre semi open if f (V) is pre semi open in (Y, σ) for every open set V of (X, τ).

Definition 3.2 - A bijection f**:** (X, τ) → (Y, σ) is called a pre semi homeomorphisms (briefly ps-homeomorphism) if f is both pre semi continuous and pre semi open map.

Theorem 3.1 - Every homeomorphism is ps-homeomorphism but not conversely.

Proof - Let f be a homeomorphism from a topological space (X, τ) to (Y, σ). Since every continuous map is pre semi continuous and every open map is pre semi open, we conclude that f is ps-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.

Let X=Y= {a,b,c}, τ = {ϕ,X,{a},{a,c}}and σ = {ϕ,Y,{a,c}}. Then
Let f **:** (X, τ) → (Y, σ) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not an open map, f is not homeomorphism. However f is ps-homeomorphism.

Theorem 3.2 - Every α-homeomorphism is ps-homeomorphism but not conversely.

Proof - Let f be a α-homeomorphism from a topological space (X, τ) to (Y, σ). Since every α-continuous map is pre semi continuous and every α-open map is pre semi open, then f is ps-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.

Let X=Y= {a,b,c}, τ = {ϕ,X,{a, b}}and σ = {ϕ,Y,{a},{a, c}}. Then
Let f **:** (X, τ) → (Y, σ) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not α-continuous map, f is not α-homeomorphism.
However f is ps-homeomorphism.

Theorem 3.3 - Every semi-homeomorphism (B) is ps-homeomorphism but not conversely.

Proof - Let f be a semi-homeomorphism (B) from a topological space (X, τ) to (Y, σ). Since every semi-continuous map is pre semi continuous and every semi-open map is pre semi open, then f is ps-homeomorphism

The converse of the above theorem is not true as it can be seen the following example.

Let X=Y= {a,b,c}, τ = {ϕ,X,{a},{b,c}}and

□ = {□, Y,{a},{b},{a,b}}. Then
Let f**:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a semi-continuous map, f is not semi-homeomorphism (B).
However f is ps-homeomorphism.

Theorem 3.4 **-** Every pre-homeomorphism is ps homeomorphism but not conversely.
Proof - Let f be a pre-homeomorphism from a topological space (X, □) to (Y, □). Since every pre-continuous map is pre semi continuous and every pre-open map is pre semi open, then f is ps-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b,c}}and
□ = {□,Y,{a},{b},{a,b}}. Then
Let f**:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a pre open map, f is not pre-homeomorphism. However f is ps-homeomorphism.

Theorem 3.5 **-** Every gα-homeomorphism is ps-homeomorphism but not conversely.
Proof - Let f be a gα-homeomorphism from a topological space (X, □) to (Y, □). Since every gα-continuous map is pre semi continuous and every gα-open map is pre semi open, then f is ps-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{a,c}}. Then
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a gα-continuous map, f is not gα-homeomorphism. However f is ps-homeomorphism.

Theorem 3.6 **-** Every semi pre homeomorphism is ps-homeomorphism but not conversely.

Proof - Let f be a semi pre homeomorphism from a topological space (X, □) to (Y, □). Since every semi pre continuous map is pre semi continuous and every semi pre open map is pre semi open, then f is ps-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b,c}}and
□ = {□,Y,{a},{a,c}}. Then
Let f **:** (X, □) □ (Y, □) defined by f(a) = c, f(b) = a and f(c) = b.
Since f is not a semi pre open map, f is not semi pre homeomorphism. However f is ps-homeomorphism.

Theorem 3.7 **-** Every sg-homeomorphism is ps-homeomorphism but not conversely.

Proof - Let f be a sg-homeomorphism from a topological space (X, □) to (Y, □). Since every sg-continuous map is pre semi continuous and every sg-open map is pre semi open, then f is ps-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b,c)}and
□ = {□, Y, {a,b}}. Then
Let f**:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a sg-open map, f is not sg-homeomorphism, However f is ps-homeomorphism.

We prove that the following theorem:

Theorem 3.8 –Every ĝ-closed set is pre semi closed set but not conversely.
Proof – Let A is a ĝ-closed set and let U be an g-open set such that A□U. Since A is ĝ-closed cl(A)□ □ □A□ □U. But always spcl(A)□ □ □cl(A). Therefore spcl(A)□ □U. Hence A is pre semi closed.

The converse of the above theorem is not true as it can be seen the following example.
Consider the topological space X = {a,b,c} and □ □ □{□,X,{a},{b},{a,b}}
ĝ-closed of (X,□) are {□,X,{c},{a,c},{b,c}}and pre semi closed of (X,□) are {□,X,{a},{b},{c},{a,c},{b,c}}
Here the set {b} is pre semi closed but not ĝ closed in X.
Hence pre semi closed is not ĝ-closed.

This implies: ĝ-closed → pre semi closed.

Theorem 3.9 **-** Every ĝ-homeomorphism is ps-homeomorphism but not conversely.

Proof - Let f be a ĝ-homeomorphism from a topological space (X, □) to (Y, □). Since every ĝ-continuous map is pre semi continuous and every ĝ-open map is pre semi open, then f is ps-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b,c)}and
□ = {□,Y,{a},{b},{a,b}}. Then
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a ĝ-open map, f is not ĝ-homeomorphism. However f is ps-homeomorphism.

Theorem 3.10 **–** ps-homeomorphism is independent from g-homeomorphism and gs-homeomorphism.

This is proved by the following examples

Example 3.11
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{a,c}}. Then
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not g-continuous, gs- continuous, f is not g-homeomorphism also gs-homeomorphism. However f is ps-homeomorphism.

Example 3.12
Let X=Y= {a,b,c}, □ □ □{□,X,{a}}and
□ = {□,Y,{a},{b},{a,b}}. Then
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not pre-semi continuous, f is not ps-homeomorphism. However f is g-homeomorphism and gs-homeomorphism.

Proposition 3.13 – For any bijection
f**:** (X, □) □(Y, □) the following statements are equivalent.

i) Its inverse map f^{-1}: (Y, □) □ □(X, □) is pre semi continuous.
ii) f is pre semi open map.
iii) f is pre semi closed map.

Proof - To Prove- (i)⟹(ii) Let V be an open set of (X, □). By assumption $(f^{-1})^{-1}$ (V) = f(V) is pre semi open in (Y, □) and so f is pre semi open.

To Prove- (ii)⟹(iii) Let V be a closed set of (X, □). Then V^c is open in (X, □). By assumption, f (V^c) is pre semi open in (Y, □). ie., f(V^c) = $(f(V))^c$ is pre semi open in (Y, □) and therefore f(V) is per semi closed in (Y, □). Hence f is pre semi closed.

To Prove- (iii)⟹(i) Let V be a closed set in (X, □). By assumption f(V) is pre semi closed in (Y, □).
But f (V) = $(f^{-1})^{-1}$ (V) and therefore f^{-1} is pre semi continuous on (Y, □).

Proposition 3.14 **–** Let f **:** (X, □) □ (Y, □) be a bijective and pre semi continuous. Then the following statements are equivalent.

i) f is a pre semi open map.
ii) f is a ps-homeomorphism.
iii) f is a pre semi closed map.

Proof - To Prove- (i)⟹(ii) By the hypothesis and assumption f is a ps-homeomorphism.

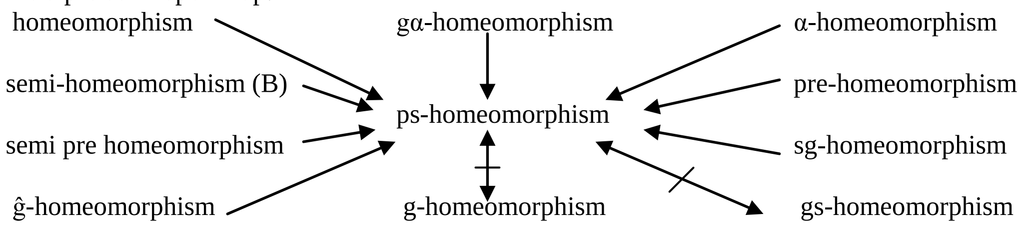
To Prove- (ii)⟹(iii) Let V be a closed set of (X, □). Then V^c is open in (X, □). By assumption, (f is a ps-homeomorphism, it is pre semi open) f(V^c) is pre semi open in (Y,□). ie., f(V^c) = $(f(V))^c$ is pre semi open in (Y, □) and therefore f(V) is per semi closed in (Y, □). Hence f is pre semi closed.

To Prove- (iii)⟹(i) Let V be a open set in (X, □). Then V^c is closed in (X, □).By assumption f(V^c) is pre semi closed in (Y, □). ie., f(V^c) = $(f(V))^c$ is pre semi closed in (Y,□) and therefore f(V) is per semi open in (Y, □). Hence f is pre semi open.

Result 3.15 The following example shows that the composition of two ps-homeomorphisms is not ps-homeomorphism.

Example 3.16 Let X = Y = Z = {a,b,c},
□ □ □{□,X,{a},{b,c}},
□ = {□, Y,{a},{b},{a,b}} and η = {□,Z,{a},{a,c}}.
Let f**:** (X, □) □ (Y, □) be defined as f(a) = a, f(b) = b, f(c) = c and h **:** (Y, □) □ (Z, η) be defined as h(a) = a, h(b) = c, h(c) = b. Then f and h are ps-homomorphism.
But **h**∘**f**: (X, □) □ (Z, η) is not a ps-homomorphism.

Remark 3.17 From the above results, examples and theorems, we have the following diagram:



Where A⟶ B (resp.A⟷B) represents that A implies B (resp. A and B are independent)

4. psc-Homeomorphisms

In this section we introduce the class of maps which are included in the class of ps-homeomorphisms and includes the class of homeomorphisms. Moreover, this class of maps is closed under the composition of maps.

Definition 4.1 - A bijection f **:**(X, □) □ (Y, □) is said to be psc-homeomorphisms if f is pre semi irresolute (briefly ps-irresolute) and its inverse f^{-1} is also pre semi irresolute. We say that spaces (X, □) and (Y, □) are psc-homeomorphic if their exists a psc –homeomorphism from (X, □) onto (Y, □)

Theorem 4.2 - Every psc-homeomorphism is ps homeomorphism.
Proof –Let f **:**(X, □) □ (Y, □) be psc-homeomorphism. Since every ps-irresolute map is pre semi continuous, by the hypothesis f is ps-homeomorphism.

Converse is not true from the following example:
Example 4.3 Let X=Y={a,b,c},
□ □ □{□,X,{a},{b,c}}and
□ = {□,Y,{a},{b},{a,b}}.
Let f **:** (X, □) □ (Y, □) be defined as f(a) = a,
f(b) = b, f(c) = c. Then f is not a psc-homomorphism. However f is a ps-homomorphism.

Remark 4.4 The following two examples show that the concepts of psc-homeomorphism and gc homeomorphisms are independent of each other.

Example 4.5
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{a,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = c, f(b) = a and f(c) = b.
Since f^{-1} is not ps- irresolute, f is not psc-homeomorphism. However f is gc-homeomorphism.

Example 4.6
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{a,c}}and
□ = {□,Y,{a},{a,b},{b},{b,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = b, f(b) = c and f(c) = a.
Since f is not gc- irresolute, f is not gc-homeomorphism. However f is psc-homeomorphism.

Remark 4.7 The following two examples show that the concepts of psc-homeomorphism and sgc homeomorphisms are independent of each other.

Example 4.8
Let X=Y= {a,b,c}, □ □ □{□,X,{a}}and
□ = {□,Y,{a},{a,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not ps-irresolute, f is not psc-homeomorphism. However f is sgc-homeomorphism.

Example 4.9
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{b},{a,b}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not sg- irresolute, f is not sgc-homeomorphism. However f is psc-homeomorphism.

Proposition 4.10 If f **:** (X, □) □ (Y, □) and g **:** (Y, □) □ (Z, η) are psc-homeomorphism, then their composition g∘f : (X, □) □ (Z, η) is also psc-homeomorphism.
Proof- Let U be ps open in (Z, η).
Now $(g\circ f)^{-1}(U)= f^{-1}(g^{-1}(U))= f^{-1}(V)$,where $V = g^{-1}(U)$. By hypothesis, V is ps open in (Y,□) and so again by hypothesis, $f^{-1}(V)$ is ps open in (X,□). Therefore g∘f is ps-irresolute.
Also for a ps open set B in (X, □), we have (g ∘ f)(B) = g(f(B))= g(H),where H = f(B). By hypothesis f (B) is ps open in (Y,□) and so again by hypothesis, g(f((B)) is ps open in (Z, η). ie., (g∘f)(B) is ps open in (Z, η) and therefore $(g\circ f)^{-1}$ is ps –irresolute. Hence g∘f is psc-homeomorphism

Remark 4.11 From the above results, examples and theorems, we have the following diagram:

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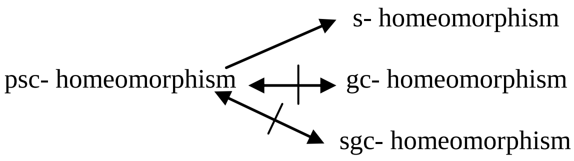


where A →B (resp. A ↔ B) represents that A implies B (resp. A and B are independent)

5. Generalized Semi Pre-Homeomorphisms

In this section we introduce the concepts of generalized semi pre open map (briefly gsp-open map), generalized semi pre irresolute (briefly gsp-irresolute) and generalized semi pre homeomorphisms (briefly gsp-homeomorphism); we discuss some of their properties.

Definition 5.1 - A function f **:** (X, □) □ (Y, □) is called gsp-open if f (V) is gsp-open in (Y, □) for every open set V of (X, □).

Definition 5.2 - A function f **:** (X, □) □ (Y, □) is called gsp-irresolute if $f^{-1}(V)$ is gsp-closed in (X, τ) for every gsp-closed set V of (Y, σ).

Definition 5.3 - A bijection f **:**(X, □) □ (Y, □) is called a gsp-homeomorphism if f is both gsp-continuous and gsp-open map.

Theorem 5.1 **-** Every homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a homeomorphism from a topological space (X, □) to (Y, □). Since every continuous map is gsp-continuous and every open map is gsp-open, we conclude that f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a}}and
□ = {□,Y,{a,b}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not open map as well as continuous, f is not homeomorphism. However f is gsp-homeomorphism.

Theorem 5.2 **-** Every α-homeomorphism is ps-homeomorphism but not conversely.

Proof - Let f be a α-homeomorphism from a topological space (X, □) to (Y, □). Since every α-continuous map is gsp-continuous and every α-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a, b}}and
□ = {□,Y,{a},{a, c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not α-continuous map, f is not α-homeomorphism. However f is gsp-homeomorphism.

Theorem 5.3 **-** Every semi-homeomorphism (B) is gsp-homeomorphism but not conversely.

Proof - Let f be a semi-homeomorphism (B) from a topological space (X, □) to (Y, □). Since every semi-continuous map is gsp-continuous and every semi-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b,c}}and
□ = {□,Y,{a},{b},{a,b}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a semi-continuous map, f is not semi-homeomorphism (B). However f is gsp-homeomorphism.

Theorem 5.4 **-** Every pre-homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a pre-homeomorphism from a topological space (X, □) to (Y, □). Since every pre-continuous map is gsp-continuous and every pre-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b,c}}and
□ = {□,Y,{a},{b},{a,b}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a pre open map, f is not pre-homeomorphism. However f is gsp-homeomorphism.

Theorem 5.5 **-** Every gα-homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a gα-homeomorphism from a topological space (X, □) to (Y, □). Since every gα-continuous map is gsp-continuous and every gα-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{a,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a gα-continuous map, f is not gα-homeomorphism.
However f is gsp-homeomorphism.

Theorem 5.6 **-** Every semi pre homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a semi pre homeomorphism from a topological space (X, □) to (Y, □). Since every semi pre continuous map is gsp-continuous and every semi pre open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b},{a,b}}and
□ = {□,Y,{a}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a semi pre open map, f is not semi pre homeomorphism. However f is gsp-homeomorphism.

Theorem 5.7 **-** Every ĝ-homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a ĝ-homeomorphism from a topological space (X, □) to (Y, □). Since every ĝ-continuous map is gsp-continuous and every ĝ-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b,c)}and
□ = {□,Y,{a},{b},{a,b}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a ĝ-open map, f is not ĝ-homeomorphism. However f is gsp-homeomorphism.

***Theorem 5.8* -** Every sg-homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a sg-homeomorphism from a topological space (X, □) to (Y, □). Since every sg-continuous map is gsp-continuous and every sg-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b,c)}and
□ = {□,Y,{a,b}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not a sg-open map, f is not sg-homeomorphism. However f is gsp-homeomorphism.

***Theorem 5.9* -** Every g-homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a g-homeomorphism from a topological space (X, □) to (Y, □). Since every g-continuous map is gsp-continuous and every g-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{a,c}}

Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not g-continuous as well as g- open, f is not g-homeomorphism.
However f is gsp-homeomorphism.

***Theorem 5.10* -** Every αg-homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a αg-homeomorphism from a topological space (X, □) to (Y, □). Since every αg-continuous map is gsp-continuous and every αg-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{a,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not αg-continuous, f is not αg-homeomorphism. However f is gsp-homeomorphism.

***Theorem 5.11* -** Every gs-homeomorphism is gsp-homeomorphism but not conversely.
Proof - Let f be a gs-homeomorphism from a topological space (X, □) to (Y, □). Since every gs-continuous map is gsp-continuous and every gs-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{a,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not gs-continuous, f is not gs-homeomorphism. However f is gsp-homeomorphism.

***Theorem 5.12* -** Every ps-homeomorphism is gsp-homeomorphism but not conversely.

Proof - Let f be a ps-homeomorphism from a topological space (X, □) to (Y, □). Since every ps-continuous map is gsp-continuous and every ps-open map is gsp-open, then f is gsp-homeomorphism.

The converse of the above theorem is not true as it can be seen the following example.
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{b},{a,b}}and
□ = {□,Y,{a}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not ps-open, f is not ps-homeomorphism. However f is gsp-homeomorphism.

***Proposition 5.13* –** For any bijection
f **:** (X, □) □ (Y, □) the following statements are equivalent.

i) Its inverse map f^{-1}: (Y, □) □ □(X, □) is gsp-continuous.
ii) f is gsp-open map.
iii) f is gsp-closed map.

Proof - To Prove- (i)$\Longrightarrow$(ii) Let V be an open set of (X, □). By assumption $(f^{-1})^{-1}$ (V)= f(V) is gsp-open in (Y, □) and so f is gsp-open.

To Prove- (ii)$\Longrightarrow$(iii) Let V be a closed set of (X, □). Then V^c is open in (X, □). By assumption, f (V^c) is gsp-open in (Y, □). ie., f(V^c) = $(f(V))^c$ is gsp-open in (Y, □) and therefore f(V) is gsp-closed in (Y, □). Hence f is gsp-closed.

To Prove- (iii)$\Longrightarrow$(i) Let V be a closed set in (X, □). By assumption f(V) is gsp-closed in (Y, □).
But f (V) = $(f^{-1})^{-1}$ (V) and therefore f^{-1} is gsp-continuous on (Y, □).

Proposition 5.14 **–** Let f **:** (X, □) □ (Y, □) be a bijective and gsp-continuous. Then the following statements are equivalent.

i) f is a gsp-open map.
ii) f is a gsp-homeomorphism.
iii) f is a gsp-closed map.

Proof - To Prove- (i)⟹(ii) By the hypothesis and assumption f is a gsp-homeomorphism.

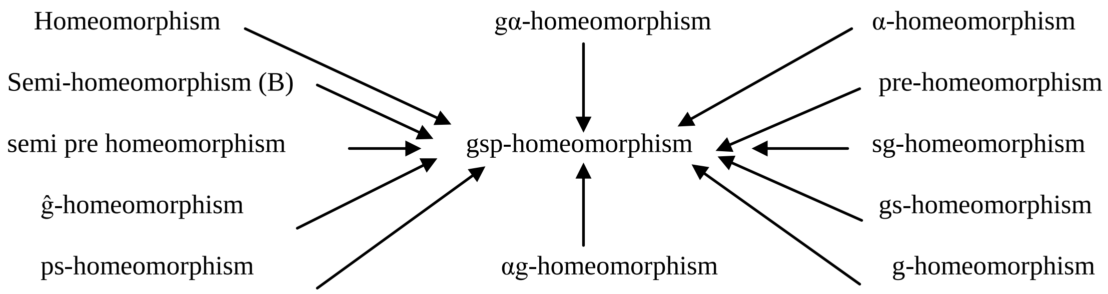
To Prove- (ii)⟹(iii) Let V be a closed set of (X, □). Then V^c is open in (X, □). By assumption, (f is a gsp-homeomorphism, it is gsp-open) $f(V^c)$ is gsp-open in (Y,□). ie., $f(V^c) = (f(V))^c$ is gsp-open in (Y, □) and therefore f(V) is gsp-closed in (Y, □). Hence f is gsp-closed.

To Prove- (iii)⟹(i) Let V be a open set in (X, □). Then V^c is closed in (X, □).By assumption $f(V^c)$ is gsp-closed in (Y, □). ie., $f(V^c) = (f(V))^c$ is gsp-closed in (Y,□) and therefore f(V) is gsp-open in (Y, □). Hence f is gsp-open

Result 5.15 The following example shows that the composition of two gsp-homeomorphisms is not gsp-homeomorphism.

Example 5.16 Let X =Y= Z ={a,b,c},
□ □ □{□,X,{a},{b,c}},
□ = {□,Y,{a},{b},{a,b}} and η = {□,Z,{a},{a,c}}.
Let f **:** (X, □) □ (Y, □) be defined as f(a) = a, f(b) = b, f(c) = c and h**:**(Y, □) □(Z, η) be defined as h(a) = a, h(b) = c, h(c) = b. Then f and h are gsp-homomorphism.
But **h** ∘ **f** : (X, □) □ (Z, η) is not a gsp-homomorphism.

Remark 5.17 From the above results, examples and theorems, we have the following diagram:



Where A⟶B represents that A implies B.

6. gspc-Homeomorphisms

In this section we introduce the class of maps which are included in the class of gsp-homeomorphisms and includes the class of homeomorphisms. Moreover, this class of maps is closed under the composition of maps.

Definition 6.1 - A bijection f **:**(X, □) □ (Y, □) is said to be gspc-homeomorphisms if f is gsp-irresolute and its inverse f^{-1} is also gsp-irresolute. We say that spaces (X, □) and (Y,□) are gspc-homeomorphic if their exists a gspc-homeomorphism from (X, □) onto (Y, □)

Theorem 6.2 **-** Every gspc-homeomorphism is gsp-homeomorphism.
Proof –Let f **:**(X, □) □ (Y, □) be gspc-homeomorphism. Since every gsp-irresolute map is gsp-continuous, by the hypothesis f is gsp-homeomorphism.

Converse is not true from the following example:
Example 6.3 Let X =Y ={a,b,c},
□ □ □{□,X,{a},{b,c}}and
□ = {□,Y,{a},{b},{a,b}}.
Let f **:** (X, □) □ (Y, □) be defined as f(a) = a, f(b) = b, f(c) = c. Then f is not a gspc-homomorphism. However f is a gsp-homomorphism.

Remark 6.4 The following two examples show that the concepts of gspc-homeomorphism and gc homeomorphisms are independent of each other.

Example 6.5
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{a,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = c, f(b) = a and f(c) = b.
Since f^{-1} is not gsp- irresolute, f is not gspc-homeomorphism. However f is gc-homeomorphism.

Example 6.6
Let X=Y= {a,b,c}, □ □ □{□,X,{a},{a,c}}and

□ = {□,Y,{a},{a,b},{b},{b,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = b, f(b) = c and f(c) = a.
Since f is not gc- irresolute, f is not gc-homeomorphism.
However f is gspc-homeomorphism.

Remark 6.7 The following two examples show that the concepts of gspc-homeomorphism and sgc homeomorphisms are independent of each other.

Example 6.8
Let X=Y= {a,b,c}, □ □ □{□,X,{a}}and
□ = {□,Y,{a},{a,c}}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f^{-1} is not gsp-irresolute, f is not gspc-homeomorphism. However f is sgc-homeomorphism.

Example 6.9
Let X=Y= {a,b,c}, □ □ □{□,X,{a,b}}and
□ = {□,Y,{a},{b},{a,b}.
Let f **:** (X, □) □ (Y, □) defined by f(a) = a, f(b) = b and f(c) = c.
Since f is not sg- irresolute, f is not sgc-homeomorphism. However f is gspc-homeomorphism.

Proposition 6.10 If f **:** (X, □) □ (Y, □) and g **:** (Y, □) □ (Z, η) are gspc-homeomorphism, then their composition $g \circ f$: (X, □) □ (Z, η) is also gspc-homeomorphism.

Proof- Let U be gsp-open in (Z, η).
Now $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) = f^{-1}(V)$, where $V = g^{-1}(U)$. By hypothesis, V is gsp-open in (Y,□) and so again by hypothesis, $f^{-1}(V)$ is gsp- open in (X,□). Therefore $g \circ f$ is gspc-irresolute.

Also for a gsp-open set B in (X, □), we have
$(g \circ f)(B) = g(f(B)) = g(H)$, where H = f(B). By hypothesis f (B) is gsp-open in (Y,□) and so again by hypothesis, g(f((B)) is gsp-open in (Z, η). ie., $(g \circ f)(B)$ is gsp-open in (Z, η) and therefore $(g \circ f)^{-1}$ is gsp-irresolute.

Hence $g \circ f$ is gspc-homeomorphism.

Remark 6.11 From the above results, examples and theorems, we have the following diagram:

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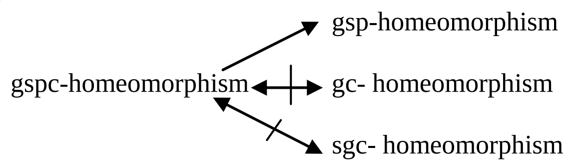

Where A ⟶ B (resp. A ⟷ B) represents that A implies B (resp. A and B are independent)

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