# Newton Raphson Method using Fuzzy Concept

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***Abstract-****Numerical methods are techniques which give approximate solutions to hard problems. Fuzzy numbers are foundation of fuzzy sets and fuzzy mathematics that extends the domain of numbers from those of real numbers to fuzzy numbers. Researchers in the past investigated a number of methods of numerical analysis with the help of Fuzzy theory. Recently, various methods have been developed for solving linear programming problems with fuzzy number. Many research works were done on fuzzy numbers and on its applications in various fields. But very few developments have been seen in the area of numerical methods using fuzzy triangular numbers. In this paper the fuzzification of Newton Raphson method to find the solution of cubic equation has been discussed. Results have been obtained in the form of triangular numbers along with the membership function.*

*Keywords: Newton Raphson Method, Fuzzy membership function, Triangular fuzzy number, α-cut etc.*

***Introduction***

Numerical methods are techniques which give approximate solutions to hard of problems. Many research works were done on fuzzy numbers and on its applications in various fields. But very few developments have been seen in the area of numerical methods using fuzzy triangular numbers. In this study effort is made to fuzzify the Newton Raphson method using fuzzy triangular numbers. An attempt has been also made to discuss an example using Fuzzified Newton Raphson Method to find the solution of a cubic equation.

***Objective of the Study***

The main objective of the study is to Fuzzify Newton Raphson Method.

***Methodology***

Fuzzification of the Newton Raphson method will be done using triangular fuzzy number.

***Fuzzification of Newton-Raphson Method***

Let us consider F(X) =0

Let the function F(X) changes its sign over an interval X=A and X=B.

Let $A=[A_1, A_2, A_3]$ and $B=[B_1, B_2, B_3]$ . Then there is a root of F(X)=0 lying between A and B. Now fuzzy membership function of A and B are respectively,

$$\mu_A = \begin{cases} \frac{X-A_1}{A_2-A_1}, A_1 \le X \le A_2 \\ \frac{X-A_3}{A_2-A_3}, A_2 \le X \le A_3 \\ 0 , otherwise \end{cases}$$

$$\mu_B = \begin{cases} \frac{X-B_1}{A_2-A_1}, B_1 \le X \le B_2 \\ \frac{X-B_3}{B_2-B_3}, B_2 \le X \le B_3 \\ 0 , otherwise \end{cases}$$

with respect to $\alpha$-cuts as

$$[A]^\alpha = [A_1 + \alpha(A_2 - A_1), A_3 + \alpha(A_2 - A_3)]$$

$$[B]^\alpha = [B_1 + \alpha(B_2 - B_1), B_3 + \alpha(B_2 - B_3)]$$

As a first approximation, the root of F(X)=0 is $X_0 = A \; or \; B$

Let $X_0 = A = [A_1, A_2, A_3]$

$$\mu_{X_0}(X) = \begin{cases} \frac{X-A_1}{A_2-A_1}, A_1 \le X \le A_2 \\ \frac{X-A_3}{A_2-A_3}, A_2 \le X \le A_3 \\ 0 , otherwise \end{cases}$$

$$[X_0]^\alpha = [A_1 + \alpha(A_2 - A_1), A_3 + \alpha(A_2 - A_3)]$$

According to Newton Raphson formula

$X_1 = X_0 - \frac{F(X_0)}{F'(X_0)} = [X_1', X_1'', X_1''']$

with membership function

$$\mu_{X_1}(X) = \begin{cases} \frac{X - X_1'}{X_1'' - X_1'}, X_1' \le X \le X_1'' \\ \frac{X - X_1'''}{X_1'' - X_1'''}, X_1'' \le X \le X_1''' \\ 0, otherwise \end{cases}$$

and $[X_1]^\alpha = [X_1' + \alpha(X_1'' - X_1'), X_1''' + \alpha(X_1'' - X_1''')]$

similarly

$X_2 = X_1 - \frac{F(X_1)}{F'(X_1)} = [X_2', X_2'', X_2''']$

with membership function

$$\mu_{X_2}(X) = \begin{cases} \frac{X - X_2'}{X_2'' - X_2'}, X_2' \le X \le X_2'' \\ \frac{X - X_2'''}{X_2'' - X_2'''}, X_2'' \le X \le X_2''' \\ 0, otherwise \end{cases}$$

and $[X_2]^\alpha = [X_2' + \alpha(X_2'' - X_2'), X_2''' + \alpha(X_2'' - X_2''')]$

Again

$X_3 = X_2 - \frac{F(X_2)}{F'(X_2)} = [X_3', X_3'', X_3''']$

with membership function

$$\mu_{X_3}(X) = \begin{cases} \frac{X - X_3'}{X_3'' - X_3'}, X_3' \le X \le X_3'' \\ \frac{X - X_3'''}{X_3'' - X_3'''}, X_3'' \le X \le X_3''' \\ 0, otherwise \end{cases}$$

and $[X_3]^\alpha = [X_3' + \alpha(X_3'' - X_3'), X_3''' + \alpha(X_3'' - X_3''')]$

Similarly

$X_4 = X_3 - \frac{F(X_3)}{F'(X_3)} = [X_4', X_4'', X_4''']$

with membership function

$$\mu_{X_4}(X) = \begin{cases} \frac{X - X_4'}{X_4'' - X_4'}, X_4' \le X \le X_4'' \\ \frac{X - X_1'''}{X_4'' - X_4'''}, X_4'' \le X \le X_4''' \\ 0, otherwise \end{cases}$$

and $[X_4]^\alpha = [X_4' + \alpha(X_4'' - X_4'), X_4''' + \alpha(X_4'' - X_4''')]$

and so on.

***Numerical Example***

Let us consider an algebraic equation

$$F(X) = X^3 - 6X + 4$$

Let $A = [-.01, 0, .01]$, $B = [.99, 1, 1.01]$

Since

$F(A) = F([-.01, 0, .01]) = A^3 - 6A + 4$

=[3.93,4,4.07]

=+ve

$F(B) = F([.99, 1, 1.01]) = B^3 - 6B + 4$

=[-1.09,-1,-.89]

=-ve

Since $F(A)$ and $F(B)$ are of opposite sign, a root lies between [-.01,0,.01] and [.99,1,1.01].

Let us take $X_0 = [-.99, 1, 1.01]$

Here $F'(X) = 3X^2 - 6$

So $X_1 = X_0 - \frac{F(X_0)}{F'(X_0)}$=[-2.02,.66,4.76]

The membership function of $X_1$ is

$$\mu_{X_1}(X) = \begin{cases} \frac{X - (-2.02)}{.66 - (-2.02)}, -2.02 \le X \le .66 \\ \frac{X - 4.76}{.66 - 4.76}, \quad .66 \le X \le 4.76 \\ 0, otherwise \end{cases}$$

w.r.t. α cut $[X_1]^\alpha = [-2.02 + \alpha(.66 + 2.02), 4.76 + \alpha(.66 - 4.76)]$

Again

$$X_2 = X_1 - \frac{F(X_1)}{F'(X_1)} = [-4.04, .73, 7.62]$$

The membership function of $X_2$ is

$$\mu_{X_2}(X) = \begin{cases} \frac{X-(-4.04)}{.73-(-4.04)}, -4.04 \leq X \leq .73 \\ \frac{X-7.62}{.73-7.62}, \quad .73 \leq X \leq 7.62 \\ 0, otherwise \end{cases}$$

w.r.t. α cut $[X_2]^\alpha$=$[-4.04+\alpha(.73+4.04), 7.62+\alpha(.73-7.62)]$

Similarly

$$X_3 = X_2 - \frac{F(X_2)}{F'(X_2)}=[-4.04,.73, 7.62]$$

The membership function of $X_3$ is

$$\mu_{X_3}(X) = \begin{cases} \frac{X-(-4.04)}{.73-(-4.04)}, -4.04 \leq X \leq .73 \\ \frac{X-7.62}{.73-7.62}, \quad .73 \leq X \leq 7.62 \\ 0, otherwise \end{cases}$$

w.r.t. α cut $[X_3]^\alpha$=$[-4.04+\alpha(.73+4.04), 7.62+\alpha(.73-7.62)]$

Again

$$X_4 = X_3 - \frac{F(X_3)}{F'(X_3)}=[-6.84,.73,12.41]$$

The membership function of $X_4$ is

$$\mu_{X_4}(X) = \begin{cases} \frac{X-(-6.84)}{.73-(-6.84)}, -6.84 \leq X \leq .73 \\ \frac{X-12.41}{.73-12.41}, \quad .73 \leq X \leq 12.41 \\ 0, otherwise \end{cases}$$

w.r.t. α cut $[X_4]^\alpha$ =$[-6.84+\alpha(.73+6.84), 12.41+\alpha(.73-12.41)]$

and so on.

***Conclusion***

In this study an attempt has been made to fuzzify the Newton-Raphson method using fuzzy triangular number to find the root of a cubic equation. After few numbers of iterations results have been observed in the form of triangular fuzzy number along with its membership function.

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