Mean, Odd Mean And Even Mean Labeling For the Extended Duplicate Graph of Kite Graph

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Abstract: *In this paper, we prove that the extended duplicate graph of kite graph admits mean, even mean and odd mean labeling.*
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Keywords: *Duplicate graph, Mean labeling, Odd mean labeling, Even mean labeling, Kite graph.*

1.INTRODUCTION

Graph theory is an area in discrete mathematics which studies graphs. Throughout this paper, by a graph, we mean a finite, undirected simple graph G = (V,E) with p vertices and q edges. Several researchers refer to Rosa's work [1]. For a detailed survey of graph labeling we refer to Gallian's work [2]. Somasundaram and Ponraj [3,4] have introduced the notion of mean labeling of graphs. In [5], K.Manickam and M.Marudai introduced odd mean labeling of a graph. The concept of even mean labeling was introduced and studied by B.Gayathri and R.Gopi [6]. The concept of duplicate graph was introduced by E.Sampathkumar and he proved many results [7]. K.Thirusangu, B.Selvam and P.P.Ulaganathan have proved that the Extended duplicate graph of twig graph is cordial and total cordial [8].

2. PRELIMINARIES

In this section, we give the basic definitions relevant to this paper. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels (usually the set of integers).

Definition 2.1 Kite Graph : The kite graph is obtained by attaching a path of length 'm' with a cycle of length 'n' and it is denoted as $KI_{n,m}$. Kite graphs is also known as the Dragon Graphs or Canoe Paddle Graphs or Tadpole graphs.

KITE GRAPH

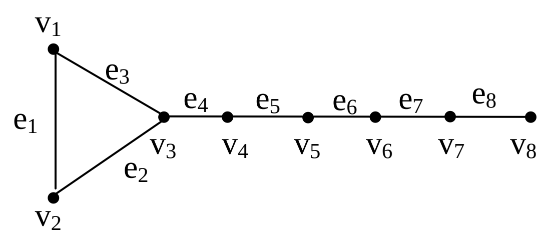

Fig: $KI_{3,5}$

Definition 2.2 Duplicate Graph: Let G (V,E) be a simple graph and the duplicate graph of G is DG = (V_1, E_1), where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as the edge *ab* is in E if and only if both ab' and $a'b$ are edges in E_1.

Definition 2.3 Extended duplicate graph of Kite graph: Let DG = (V_1, E_1) be a duplicate graph of the kite graph G(V,E). Extended duplicate graph of kite graph is obtained by adding the edge $v_2 v_2'$ to the duplicate graph. It is denoted by EDG $(KI_{3,m})$, $m \geq 1$. Clearly it has 2m+6 vertices and 2m+7 edges.

Definition 2.4 Mean labeling: A graph G with p vertices and q edges is called a mean graph if there is an injective function $f : v \to \{0,1,2,\ldots,q\}$ such that each edge uv is labeled with $\frac{(f(u)+f(v))}{2}$ if f(u)+f(v) is even and $\frac{(f(u)+f(v)+1)}{2}$ if f(u)+f(v) is odd, then the resulting edge labels are distinct.

Definition 2.5 Odd mean labeling: A function f is called an odd mean labeling of a graph G with p vertices and q edges. If f is an injection from the vertices of G to the set {1,3,5,…,2q-1} such that each edge uv is assigned the label $\frac{(f(u)+f(v))}{2}$, then the resulting edge labels are distinct. A graph which admits an odd mean labeling is said to be odd mean graph.

Definition 2.6 Even mean labeling: A function f is called an even mean labeling of a graph G with p vertices and q edges. If f is an injection from the vertices of G to the set {2,4,6,…,2q} such that each edge uv is assigned the label $\frac{(f(u)+f(v))}{2}$, then the resulting edge labels are distinct. A graph which admits an even mean labeling is said to be even mean graph.

EXTENDED DUPLICATE GRAPH OF KITE GRAPH

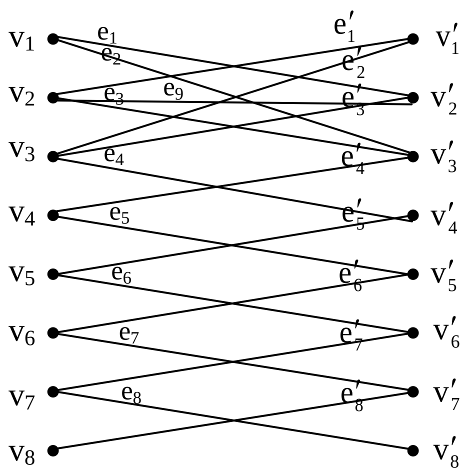

Fig: EDG ($KI_{3,5}$)

3. MAIN RESULTS

3.1 Mean labeling

In this section, we present an algorithm and prove the existence of mean labeling for the extended duplicate graph of kite graph $KI_{3,m}$, $m \geq 1$.

Algorithm:3.1

Procedure (Mean labeling for EDG ($KI_{3,m}$) , $m \geq 1$)

$V \leftarrow \{v_1, v_2, \dots, v_{m+3}, v_1', v_2', \dots, v_{m+3}'\}$

$E \leftarrow \{e_1, e_2, \dots, e_{m+4}, e_1', e_2', \dots, e_{m+3}'\}$

$v_1 \leftarrow 2m+7$, $v_2 \leftarrow 1$, $v'_1 \leftarrow 0$, $v'_2 \leftarrow 2m+6$

$for\ i = 1\ to \left\lfloor \frac{m+1}{2} \right\rfloor do$

$v_{2i+2} \leftarrow 2m+6-2i$

$v'_{2i+2} \leftarrow 2i$

end for

$for\ i = 1\ to \left\lfloor \frac{m+2}{2} \right\rfloor do$

$v_{2i+1} \leftarrow 2i+1$

$v'_{2i+1} \leftarrow 2m+7-2i$

end for

end procedure

Theorem 3.1

The graph EDG($KI_{3,m}$) for $m \geq 1$ admits mean labeling.

Proof

Let $KI_{3,m}$, $m \geq 1$ be the kite graph and EDG($KI_{3,m}$) , $m \geq 1$ be the extended duplicate graph of kite graph.

Let the vertex set and the edge set of the kite graph be defined as follows:

$V \leftarrow \{v_1, v_2, \dots, v_{m+3}, v_1', v_2', \dots, v_{m+3}'\}$

$E \leftarrow \{e_1, e_2, \dots, e_{m+4}, e_1', e_2', \dots, e_{m+3}'\}$

To label the vertices, define a function $f : v \rightarrow \{0,1,2,\dots,2m+7\}$ as given in algorithm 3.1 as follows.:

$f(v_1) = 2m+7$, $f(v'_1) = 0$, $f(v_2) = 1$, $f(v'_2) = 2m+6$

$$for\ 1 \le i \le \left\lfloor \frac{m+1}{2} \right\rfloor,$$

$f(v_{2i+2}) = 2m+6-2i$, $f(v'_{2i+2}) = 2i$.

$$for\ 1 \le i \le \left\lfloor \frac{m+2}{2} \right\rfloor,$$

$f(v_{2i+1}) = 2i+1$; $f(v'_{2i+1}) = 2m+7-2i$.

Thus the entire 2m+6 vertices are labeled by 0,1,2,…,2m+7 which are all distinct.

Now to compute the edge labeling, we define the induced function $f^*: E \rightarrow N$ such that

$$f^*(v_iv_j) = \begin{cases} \dfrac{f(v_i)+f(v_j)}{2} & if\ f(v_i)+f(v_j)\ is\ even \\ \dfrac{f(v_i)+f(v_j)+1}{2} & if\ f(v_i)+f(v_j)\ is\ odd \end{cases}$$

The edge functions are as follows:

if m≥ 1

$f^*(e_1) = m+7$, $f^*(e_2) = 2m+6$, $f^*(e_3) = m+3$, $f^*(e_4) = 3$

$f^*(e'_1) = 1$, $f^*(e'_2) = 2$, $f^*(e'_3) = m+5$, $f^*(e'_4) = 2m+5$.

if m≥2

when m is even,

$$for\ 1 \le i \le \left\lfloor \frac{m+1}{2} \right\rfloor,$$

$f^*(e_{2i+3}) = 2m+6-2i$, $f^*(e'_{2i+3}) = 2i+2$.

$$for\ 1 \le i \le \left\lfloor \frac{m+1}{3} \right\rfloor,$$

$f^*(e_{2i+4}) = 2i+3$, $f^*(e'_{2i+4}) = 2m+5-2i$.

when m is odd,

$$for\ 1 \le i \le \left\lfloor \frac{m+1}{3} \right\rfloor,\quad f^*(e_{2i+3}) = 2m+6-2i,$$

$f^*(e'_{2i+3}) = 2i+2$, $f^*(e_{2i+4}) = 2i+3$, $f^*(e'_{2i+4}) = 2m+5-2i$.

Thus the entire 2m+7 edges are labeled 1,2,3,4,..,2m+7, which are all distinct.

Hence the extended duplicate graph of kite graph $KI_{3,m}$, m≥1 admits mean labeling.

Example 1: MEAN LABELING IN EXTENDED DUPLICATE GRAPH FOR KITE GRAPH

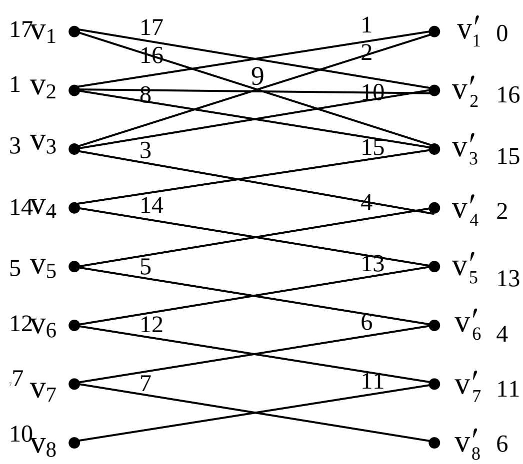

Fig: EDG ($KI_{3,5}$)

3.2 Odd mean labeling

In this section, we present an algorithm and prove the existence of odd mean labeling for the extended duplicate graph of kite graph $KI_{3,m}$, m ≥ 1.

Algorithm:3.2

Procedure [Odd mean labeling for EDG ($KI_{3,m}$) , m ≥ 1]

$V \leftarrow \{v_1, v_2, \dots, v_{m+3}, v'_1, v'_2, \dots, v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \dots, e_{m+4}, e'_1, e'_2, \dots, e'_{m+3}\}$

if m ≥ 1

$$for\ i = 0\ to \left\lfloor \frac{m+1}{2} \right\rfloor do$$

$v_{2i+2} \leftarrow 4m+11-4i$

end for

$$for\ i = 1 to \left\lfloor \frac{m+4}{2} \right\rfloor do$$

$v_{2i-1} \leftarrow 4i-3$

end for

$$for\ i = 0\ to \left\lfloor \frac{m+2}{2} \right\rfloor do$$

$v'_{2i+1} \leftarrow$ 4m+13-4i

$$for\ i = 1\ to \left\lfloor \frac{m+3}{2} \right\rfloor do$$

$v'_{2i} \leftarrow$ 4i-1

end for

end procedure

Theorem 3.2

The graph EDG($KI_{3,m}$) for m≥1 admits odd mean labeling.

Proof

Let $KI_{3,m}$ m≥1 be the kite graph and EDG($KI_{3,m}$), m≥1 be the extended duplicate graph of kite graph.

Let the vertex set and the edge set of the kite graph be defined as follows.

$$V \leftarrow \{v_1, v_2, \dots, v_{m+3}, v'_1, v'_2, \dots, v'_{m+3}\}$$

$$E \leftarrow \{e_1, e_2, \dots, e_{m+4}, e'_1, e'_2, \dots, e'_{m+3}\}$$

To label the vertices, define a function $f : v \to \{1,2,\dots,2m+7\}$ as given in algorithm 3.2 as follows:

If m≥1,

$for\ 0 \le i \le \left\lfloor \frac{m+1}{2} \right\rfloor$, $f(v_{2i+2})$=4m+11-4i ;

$for\ 1 \le i \le \left\lfloor \frac{m+4}{2} \right\rfloor$, $f(v_{2i-1})$=4i-3

$for\ 0 \le i \le \left\lfloor \frac{m+2}{2} \right\rfloor$, $f(v'_{2i+2})$ = 2i

$for\ 1 \le i \le \left\lfloor \frac{m+3}{2} \right\rfloor$, $f(v'_{2i})$ = 4i-1.

Thus the entire 2m+6 vertices are labeled by 1,3,5,…,4m+13, which are all distinct.

Now to compute the edge labeling, we define the induced function $f^* : E \to N$ such that

$$f^*(v_i v_j) = \frac{f(v_i) + f(v_j)}{2};\ v_i\, v_j \in V$$

The edge functions are as follows:

$f^*(e_1)$ = 3 , $f^*(e_2)$ = 2m+5 , $f^*(e_{m+4})$ = 2m+7,
$f^*(e'_1)$ = 4m+12 , $f^*(e'_2)$ = 2m+9,

$for\ 1 \le i \le \left\lfloor \frac{m+2}{2} \right\rfloor$, $f^*(e_{2i+1})$ = 4m+14-4i ,
$f^*(e'_{2i+1})$ = 4i

$for\ 0 \le i \le \left\lfloor \frac{m+1}{2} \right\rfloor$, $f^*(e_{2i+2})$ = 4i+2 ,
$f^*(e'_{2i+2})$ = 4m+12-4i.

Thus the entire 2m+7 edges are labeled, which are all distinct.

Hence the extended duplicate graph of kite graph $KI_{3,m}$, m≥1 admits odd mean labeling.

Example 2: ODD MEAN LABELING IN EXTENDED DUPLICATE GRAPH FOR KITE GRAPH

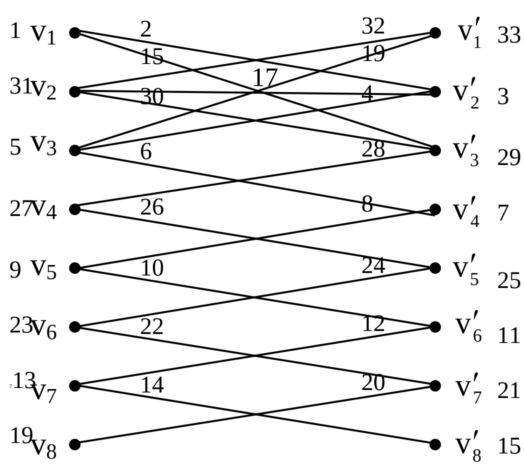

Fig: EDG ($KI_{3,5}$)

3.3 Even mean labeling

In this section, we present an algorithm and prove the existence of even mean labeling for the extended duplicate graph of kite graph $KI_{3,m}$, $m \geq 1$.

Algorithm:3.3

Procedure [Even mean labeling for EDG ($KI_{3,m}$) , m ≥ 1]

$V \leftarrow \{v_1, v_2, \ldots, v_{m+3}, v'_1, v'_2, \ldots, v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{m+4}, e'_1, e'_2, \ldots, e'_{m+3}\}$

if $m \geq 1$

$for\ i = 0\ to \left\lfloor \frac{m+1}{2} \right\rfloor do$

$v_{2i+2} \leftarrow 4m+12-4i$

end for

$for\ i = 1\ to \left\lfloor \frac{m+4}{2} \right\rfloor do$

$v_{2i-1} \leftarrow 4i-2$

end for

$for\ i = 0\ to \left\lfloor \frac{m+2}{2} \right\rfloor do$

$v'_{2i+1} \leftarrow 4m+14-4i$

end for

$for\ i = 1\ to \left\lfloor \frac{m+3}{2} \right\rfloor do$

$v'_{2i} \leftarrow 4i$

end for

end procedure

Theorem 3.3

The graph EDG($KI_{3,m}$) for $m \geq 1$ admits even mean labeling.

Proof: Let $KI_{3,m}$ $m \geq 1$ be the kite graph and EDG($KI_{3,m}$), $m \geq 1$ be the extended duplicate graph of kite graph. Let the vertex set and the edge set of the kite graph be defined as follows:

$V \leftarrow \{v_1, v_2, \ldots, v_{m+3}, v'_1, v'_2, \ldots, v'_{m+3}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{m+4}, e'_1, e'_2, \ldots, e'_{m+3}\}$

To label the vertices, define a function $f : v \rightarrow \{1, 2, \ldots, 2m+7\}$ as given in algorithm 3.3 as follows:

If $m \geq 1$, $for\ 0 \leq i \leq \left\lfloor \frac{m+1}{2} \right\rfloor$, $f(v_{2i+2}) = 4m+12-4i$,

$for\ 1 \leq i \leq \left\lfloor \frac{m+4}{2} \right\rfloor$, $f(v_{2i-1}) = 4i-2$,

$for\ 0 \leq i \leq \left\lfloor \frac{m+2}{2} \right\rfloor$, $f(v'_{2i+1}) = 4m+14-4i$,

$for\ 1 \leq i \leq \left\lfloor \frac{m+3}{2} \right\rfloor$, $f(v'_{2i}) = 4i$.

Thus the entire 2m+6 vertices are labeled by 2,4,6,…,4m+14, which are all distinct.

Now to compute the edge labeling, we define the induced function $f^* : E \rightarrow N$ such that

$$f^*(v_i v_j) = \frac{f(v_i) + f(v_j)}{2};\ v_i v_j \in V$$

The edge functions are as follows:

$f^*(e_1) = 3$, $f^*(e_2) = 2m+6$, $f^*(e_{m+4}) = 2m+8$,
$f^*(e'_1) = 4m+13$, $f^*(e'_2) = 2m+10$,

$for\ 1 \leq i \leq \left\lfloor \frac{m+2}{2} \right\rfloor$,

$f^*(e_{2i+1}) = 4m+15-4i$, $f^*(e'_{2i+1}) = 4i+1$;

$for\ 0 \leq i \leq \left\lfloor \frac{m+1}{2} \right\rfloor$,

$f^*(e_{2i+2}) = 4i+3$, $f^*(e'_{2i+2}) = 4m+13-4i$.

Thus the entire 2m+7 edges are labeled, which are all distinct. Hence the extended duplicate graph of kite graph $KI_{3,m}$, $m \geq 1$ admits even mean labeling.

Example 3: EVEN MEAN LABELING IN EXTENDED DUPLICATE GRAPH FOR KITE GRAPH

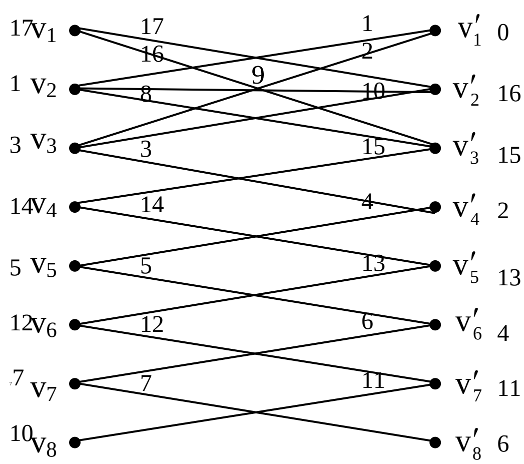

Fig: EDG ($KI_{3,5}$)

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4. CONCLUSION In this paper, we presented algorithms and proved the extended duplicate of kite graph $KI_{3,m}$, $m\geq 1$ admits mean, odd mean and even meanlabeling

5. REFERENCES

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