Existence and Non-Existence of some V_4 Cordial graphs

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Abstract:
Let $< A, *>$ be any abelian group. A graph G = (V(G), E(G)) is said to be A-cordial[6,9] if there is a mapping f: V(G) →A which satisfies the following two conditions with each edge
*e= uv is labeled as f(u)*f(v),*
(i)$|v_f(a) - v_f(b)| \leq 1, \forall$ a,b∈A
(ii)$|e_f(a) - e_f(b)| \leq 1, \forall$ a,b∈A
where$v_f(a)$= the number of vertices with label a
$v_f(b)$= the number of vertices with label b
$e_f(a)$= the number of edges with label a
$e_f(b)$= the number of edges with label b
*We note that if A = <V_4 , *> is a multiplicative group. Then the labeling is known as V_4 Cordial Labeling. A graph is called a V_4 Cordial graph if it admits a V_4 Cordial Labeling.In this paper C_n(when n $\not\equiv$ 4(mod 8)), K_n(n<7) are V_4 Cordial graphs and Globe Gl(n)(when n ≡ 2(mod 4)), C_n(when n ≡ 4(mod 8)) and K_n (n≥7) are not V_4 Cordial graphs.*
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Keywords and Phrases: Cordial labeling, **V_4**Cordial Labeling and **V_4**Cordial Graph.

I. Introduction

By a graph, itmeans a finite undirected graph without loops or multiple edges. For graph theoretic terminology, we referred Harary[4]. For labeling of graphs, we referred Gallian[1].

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.

A graph *G* is said to be labeled if the *n* vertices are distinguished from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2].Already we proved that the graph Globe Gl(n)(when n ≡ 0,1,3(mod 4)) are V_4 Cordial graph in [12].In this paperC_n (when n $\not\equiv$ 4(mod 8)), K_n (n<7) are V_4 Cordial graphs and Globe Gl(n)(when n ≡ 2(mod 4)), C_n (when n ≡ 4(mod 8)) and K_n (n≥7) are not V_4 Cordial graphs.

II. Preliminaries

Definition 2.1:
Let G = (V,E) be a simple graph. Let f:V(G)→{0,1} and for each edge uv , assign the label $|f(u) - f(v)|$. f is called a **cordial labeling** if the number of vertices labeled 0 and the the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the the number of edges labeled 1 differ by atmost 1. A graph is called **Cordial** if it has a cordial labeling.

Definition 2.2:
Let $< A, *>$ be any abelian group. A graph G = (V(G), E(G)) is said to be A-cordial[6,9] if there is a mapping f: V(G) →A which satisfies the following two conditions with each edge
e= uv is labeled as f(u)*f(v).
(i)$|v_f(a) - v_f(b)| \leq 1, \forall$ a,b∈A
(ii)$|e_f(a) - e_f(b)| \leq 1, \forall$ a,b∈A
where$v_f(a)$= the number of vertices with label a.
$v_f(b)$= the number of vertices with label b.
$e_f(a)$= the number of edges with label a.
$e_f(b)$= the number of edges with label b.
We note that if A = <V_4 , *> is a multiplicative group. Then the labeling is known as **V_4 Cordial Labeling.** A graph is called a **V_4 Cordial graph** if it admits a V_4 Cordial Labeling.

Definition 2.3:
A closed trail whose origin and internal vertices are distinct is called a cycle

Definition 2.4:
Globe is a graph obtained from two isolated vertex which are joined by n paths of length 2. It is denoted by (Gl(n)).

Definition 2.5:
A simple graph in which each pair of distinct vertices is joined by an edge is called a

complete graph. The complete graph on n vertices is denoted by K_n .

II .Main Results

Theorem:3.1

C_n is a V_4 Cordial graph , when n $\not\equiv$ 4(mod 8).

Proof:

Let V(C_n) ={u_i: $1 \le i \le n$}.

Let E (C_n) = {$(u_i u_{i+1})$: $1 \le i \le n-1$} $\cup$ {$u_n u_1$}.

Case(i):

Define f : V(C_n) → V_4 by

$$f(u_i) = \begin{cases} i & if\ i \equiv 0,3(mod\ 8) \\ 1 & if\ i \equiv 1,6(mod\ 8) \\ -1 & if\ i \equiv 2,5(mod\ 8) \\ -i & if\ i \equiv 4,7(mod\ 8) \end{cases},$$

$1 \le i \le n$

Subcase(i): when n ≡ 0(mod 8)

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} i & if\ i \equiv 0(mod\ 4) \\ -1 & if\ i \equiv 1(mod\ 4) \\ -i & if\ i \equiv 2(mod\ 4) \\ 1 & if\ i \equiv 3(mod\ 4) \end{cases},$$

$1 \le i \le n-1$

f(u_n)* f(u_1)= i

Vertex Conditions:

Here,$v_f(1) = v_f(i) = v_f(-i) = v_f(-1) = \frac{n}{4}$

Hence,$|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4.

Edge Conditions:

Here, $e_f(1) = e_f(i) = e_f(-1) = e_f(-i) = \frac{n}{4}$

Hence, $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4.

Hence, C_n is a V_4Cordial graph.

For example, the V_4Cordial Labeling of $C_8$is shown in the Figure 3.1.1.

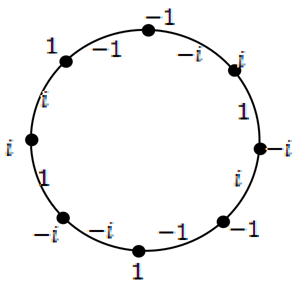

Figure 3.1.1

Subcase(ii): when n ≡ 1(mod 8)

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} i & if\ i \equiv 0(mod\ 4) \\ -1 & if\ i \equiv 1(mod\ 4) \\ -i & if\ i \equiv 2(mod\ 4) \\ 1 & if\ i \equiv 3(mod\ 4) \end{cases},$$

$1 \le i \le n-1$

f(u_n)* f(u_1)= 1

Vertex Conditions:

Here, $v_f(1) = \frac{n-1}{4} + 1$ and $v_f(i) = v_f(-i) = v_f(-1) = \frac{n-1}{4}$

Hence,$|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4.

Edge Conditions:

Here, $e_f(1) = \frac{n-1}{4} + 1$ and $e_f(-i) = e_f(-1) = e_f(i) = \frac{n-1}{4}$

Hence, $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4.

Hence, $C_n$is a V_4Cordial graph.

For example, the V_4Cordial Labeling of $C_9$is shown in the Figure 3.1.2.

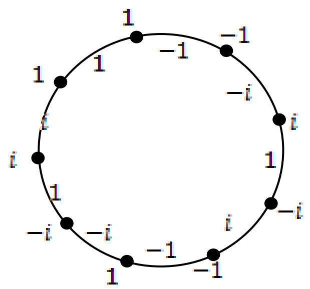

Figure 3.1.2

Subcase(iii): when n ≡ 3(mod 8)

The induced edge labelings are

$$f(u_i) * f(u_{i+1}) = \begin{cases} i & if\ i \equiv 0(mod\ 4) \\ -1 & if\ i \equiv 1(mod\ 4) \\ -i & if\ i \equiv 2(mod\ 4) \\ 1 & if\ i \equiv 3(mod\ 4) \end{cases},$$

$1 \le i \le n-1$

f(u_n)* f(u_1)= i

Vertex Conditions:

Here, $v_f(1) = v_f(i) = v_f(-1) = \frac{n+1}{4}$ and $v_f(-i) = \frac{n-3}{4}$

Hence,$|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4.

Edge Conditions:

Here, $e_f(1) = \frac{n-3}{4}$ and $e_f(-i) = e_f(-1) = e_f(i) = \frac{n+1}{4}$

Hence, $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4.

Hence, $C_n$is a V_4Cordial graph.

For example, the V_4Cordial Labeling of $C_{11}$is shown in the Figure 3.1.3.

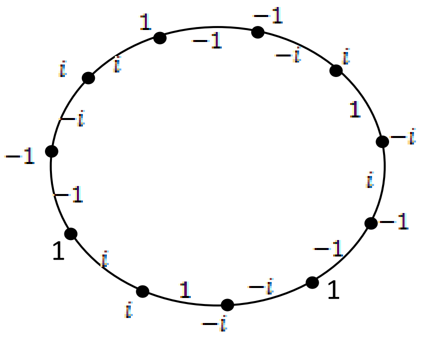

Figure 3.1.3

Subcase(iv): when n ≡ 5(mod 8)

The induced edge labelings are

$$f(u_i)*f(u_{i+1}) = \begin{cases} i & if\ i \equiv 0(mod\ 4) \\ -1 & if\ i \equiv 1(mod\ 4) \\ -i & if\ i \equiv 2(mod\ 4) \\ 1 & if\ i \equiv 3(mod\ 4) \end{cases},$$

$1 \le i \le n-1$

$f(u_n)*f(u_1) = -1$

Vertex Conditions:

Here, $v_f(1) = v_f(i) = v_f(-i) = \frac{n-1}{4}$ and $v_f(-1) = \frac{n-1}{4}+1$

Hence, $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4.

Edge Conditions:

Here, $e_f(1) = e_f(i) = e_f(-i) = \frac{n-1}{4}$ and $e_f(-1) = \frac{n-1}{4}+1$

Hence, $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4.

Hence, C_n is a V_4Cordial graph.

For example, the V_4Cordial Labeling of C_5 is shown in the Figure 3.1.4.

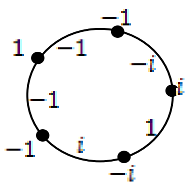

Figure 3.1.4

Subcase(v): when n ≡ 6(mod 8)

The induced edge labelings are

$$f(u_i)*f(u_{i+1}) = \begin{cases} i & if\ i \equiv 0(mod\ 4) \\ -1 & if\ i \equiv 1(mod\ 4) \\ -i & if\ i \equiv 2(mod\ 4) \\ 1 & if\ i \equiv 3(mod\ 4) \end{cases},$$

$1 \le i \le n-1$

$f(u_n)*f(u_1) = 1$

Vertex Conditions:

Here, $v_f(1) = v_f(-1) = \frac{n-2}{4}+1$ and $v_f(-i) = v_f(i) = \frac{n-2}{4}$

Hence, $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4.

Edge Conditions:

Here, $e_f(1) = e_f(-1) = \frac{n-2}{4}+1$ and $e_f(i) = e_f(-i) = \frac{n-2}{4}$

Hence, $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4.

Hence, C_n is a V_4Cordial graph.

For example, the V_4Cordial Labeling of C_6 is shown in the Figure 3.1.5.

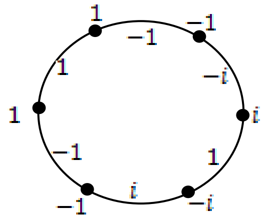

Figure 3.1.5

Case(ii): when n ≡ 2(mod 8)

Define f : V(C_n) → V_4 by

$$f(u_i) = \begin{cases} i & if\ i \equiv 0,3(mod\ 8) \\ 1 & if\ i \equiv 1,6(mod\ 8) \\ -1 & if\ i \equiv 2,5(mod\ 8) \\ -i & if\ i \equiv 4,7(mod\ 8) \end{cases},$$

$1 \le i \le n-2$

$f(u_{n-1}) = i$, $f(u_n) = 1$

The induced edge labelings are

$$f(u_i)*f(u_{i+1}) = \begin{cases} i & if\ i \equiv 0(mod\ 4) \\ -1 & if\ i \equiv 1(mod\ 4) \\ -i & if\ i \equiv 2(mod\ 4) \\ 1 & if\ i \equiv 3(mod\ 4) \end{cases},$$

$1 \le i \le n-3$

$f(u_{n-2})*f(u_{n-1}) = -1$, $f(u_{n-1})*f(u_n) = i$, $f(u_n)*f(u_1) = 1$

Vertex Conditions:

Here, $v_f(1) = v_f(-1) = \frac{n+2}{4}$ and $v_f(i) = v_f(-i) = \frac{n+2}{4} - 1$

Hence, $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4.

Edge Conditions:

Here, $e_f(1) = e_f(-1) = \frac{n+2}{4}$ and $e_f(-i) = e_f(i) = \frac{n+2}{4} - 1$

Hence, $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4.

Hence, C_n is a V_4Cordial graph.

For example, the V_4Cordial Labeling of C_{10} is shown in the Figure 3.1.6.

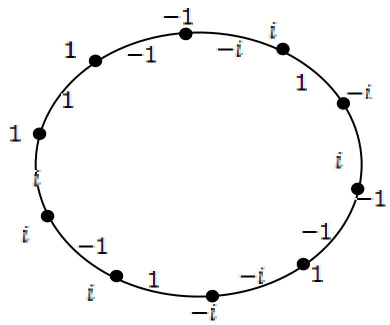

Figure 3.1.6

Case(iii): when n ≡ 7(mod 8)

Define f : V(C_n) → V_4 by

$$f(u_i) = \begin{cases} i & if\ i \equiv 0,3(mod\ 8) \\ 1 & if\ i \equiv 1,6(mod\ 8) \\ -1 & if\ i \equiv 2,5(mod\ 8) \\ -i & if\ i \equiv 4,7(mod\ 8) \end{cases} ,$$

$1 \le i \le n-3$

$f(u_{n-1}) = -i$ and $f(u_n) = 1$

The induced edge labelings are

$$f(u_i)*\ f(u_{i+1}) = \begin{cases} i & if\ i \equiv 0(mod\ 4) \\ -1 & if\ i \equiv 1(mod\ 4) \\ -i & if\ i \equiv 2(mod\ 4) \\ 1 & if\ i \equiv 3(mod\ 4) \end{cases} ,$$

$1 \le i \le n-4$

$f(u_{n-2})*\ f(u_{n-1}) = i$, $f(u_{n-1})*\ f(u_n) = -i$,

$f(u_n)*\ f(u_1) = 1$

Vertex Conditions:

Here, $v_f(1) = v_f(-1) = v_f(-i) = \frac{n+1}{4}$ and $v_f(i) = \frac{n+1}{4} - 1$

Hence, $|v_f(a) - v_f(b)| \le 1$, ∀ a,b∈ V_4.

Edge Conditions:

Here, $e_f(1) = e_f(-i) = e_f(i) = \frac{n+1}{4}$ and $e_f(-1) = \frac{n+1}{4} - 1$

Hence, $|e_f(a) - e_f(b)| \le 1$, ∀ a,b∈ V_4.

Hence, $C_n$is a V_4Cordial graph.

For example, the V_4Cordial Labeling of $C_7$is shown in the Figure 3.1.7.

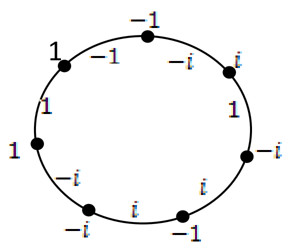

Figure 3.1.7

Theorem: 3.2

$C_n$is not a V_4 Cordial graph , when n ≡ 4(mod 8).

Proof:

For all the 4k vertices of v_i assign the label values as 1,−1,i and − i in any order . So that each labela ∈ V_4 occurs k times . In this case C_n(n ≡ 4(mod 8)) satisfies the vertex condition of V_4Cordial Labeling. It is verified that in any case it does not satisfy the edge condition of V_4Cordial Labeling . Therefore, C_n(n ≡ 4(mod 8)) is not a V_4 Cordial graph.

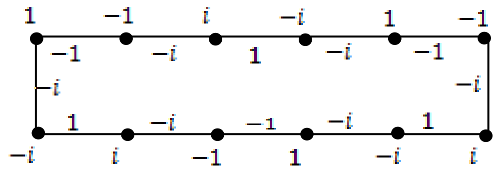

Figure 3.2.1

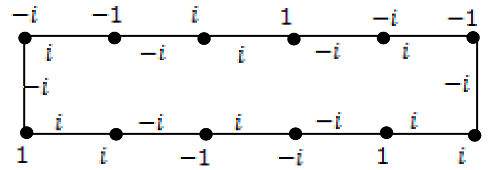

Figure 3.2.2

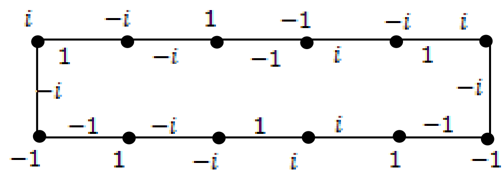

Figure 3.2.3

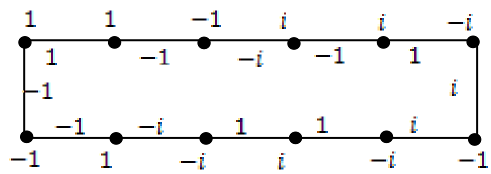

Figure 3.2.4

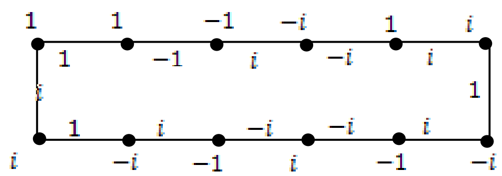

Figure 3.2.5

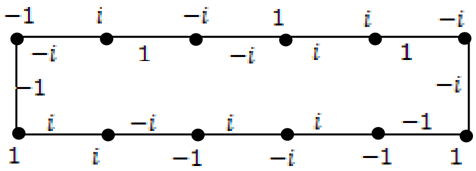

Figure 3.2.6

For example, the V_4Cordial Labeling of $C_{12}$is shown in Figures 3.2.1-3.2.6.

Here, $v_f(1)$ =$v_f(i)$ =$v_f(-1)$ =$v_f(-i)$ =3
Here, $v_f(1)$ =$v_f(i)$ =$v_f(-1)$ =$v_f(-i)$ =3
&$e_f(1)$ =3,$e_f(i)$ =0, $e_f(-1)$ =3,$e_f(-i)$ =6
&$e_f(1)$ =0,$e_f(i)$=6, $e_f(-1)$=0,$e_f(-i)$ =6
Here, $v_f(1)$ =$v_f(i)$ =$v_f(-1)$ =$v_f(-i)$ =3
Here, $v_f(1)$ =$v_f(i)$ =$v_f(-1)$ =$v_f(-i)$ =3
&$e_f(1)$ =3,$e_f(i)$ =2, $e_f(-1)$ =3,$e_f(-i)$ =4
&$e_f(1)$ =4,$e_f(i)$ =2, $e_f(-1)$ =4,$e_f(-i)$ =2
Here, $v_f(1)$ =$v_f(i)$ =$v_f(-1)$ =$v_f(-i)$ =3
Here, $v_f(1)$ =$v_f(i)$ =$v_f(-1)$ =$v_f(-i)$ =3
&$e_f(1)$ =3,$e_f(i)$ =5, $e_f(-1)$ =1,$e_f(-i)$ =3
&$e_f(1)$ =2,$e_f(i)$ =4, $e_f(-1)$ =2,$e_f(-i)$ =4

Theorem :3.3

Globe Gl(n) is not a V_4 cordial graph, (when n ≡ 2 (mod 4)).

*	1	−1	i	−i
1	1	−1	i	−i
−1	−1	1	−i	i
i	i	−i	−1	1
−i	−i	i	1	−1

Proof:

Let V(G) = $\{u, v, v_i : 1 \le i \le n\}$.
Let E(G) = $\{uv_i : 1 \le i \le n\} \cup \{vv_i : 1 \le i \le n\}$.
Case(i) : Suppose the label values of u and v are different.
For the first 4k vertices of v_i assign values from $\cup_{j=1}^{k} S_j$, S_j={1,−1, i, −i} for $1 \le j \le k$ in any order. From the table, it is observed that the edge values 1,−1,i and −i occur equal number of times for the edges induced by 4k vertices together with u and v . Further, it also satisfy the vertex condition of V_4Cordial Labeling . Hence for any choice of labeling of G, if there is any difference in vertex labeling as well as edge labeling occurs it depends on the choice of vertex labelings of two vertices of v_i, say v_{4k+1} ,v_{4k+2} and u, v and the corresponding edge values among them.

Given that f(u) ≠ f(v) . If two of f(v_{4k+1}) ,f(v_{4k+2}),f(u) and f(v) are equal, then the vertex condition is not satisfied. Therefore the vertex labelings of u, v,v_{4k+1} andv_{4k+2} are different. Now consider f(u) = 1, f(v) = −1, f(v_{4k+1}) = i and f(v_{4k+2}) = −i . The other possible different vertex labelings of u, v,v_{4k+1} andv_{4k+2} can be similarly discussed.

From this table, we observed that $v_f(1)$=$v_f(i) = v_f(-i) = v_f(-1)$=$\frac{n+2}{4}$.
So, For all a,b ∈ V_4 , the number of vertices with label *a* and the number of vertices with label *b* differ by atmost 1. Without affecting vertex conditions of V_4Cordial Labeling. We check the edge condition for all possible cases.

1.

*	i	-i
1	i	-i
-1	-i	i

Here ,$e_f(1)$ =$e_f(-1)\frac{n-2}{2}$ and $e_f(-i)$ = $e_f(i)$ = $\frac{n+2}{2}$.

It is observed that, $|e_f(1) - e_f(b)| > 1$, where b∈{i, −i} and $|e_f(-1) - e_f(b)| > 1$, where b∈{ i, −i}.

*	1	−1	i	−i	f(v_{4k+1}) =i	f(v_{4k+2}) =−i
f(u)=1	1	−1	i	−i	i	−i
f(v)= −1	−1	1	−i	i	−i	i

2.

*	-1	-i
1	-1	-i
i	-i	1

It follows that, $|e_f(a) - e_f(b)| > 1$, where a,b∈ V_4− {1, −1}.

3.

*	-1	i
1	-1	i
-i	i	1

It is seen that, $|e_f(a) - e_f(b)| > 1$, where a,b$\in$ $V_4 - \{1, -1\}$.

4.

*	1	-i
-1	-1	i
i	i	1

It is observed that, $|e_f(a) - e_f(b)| > 1$, where a,b$\in$ $V_4 - \{1, -1\}$.

5.

*	1	i
-1	-1	-i
-i	-i	1

It follows that, $|e_f(a) - e_f(b)| > 1$, where a,b$\in$ $V_4 - \{1, -1\}$.

6.

*	1	-1
i	i	-i
-i	-i	i

It is seen that, $|e_f(1) - e_f(b)| > 1$, where b$\in$ $\{i, -i\}$ and $|e_f(-1) - e_f(b)| > 1$, where b$\in$ $\{i, -i\}$.

From the above 6 cases, we observed that it does not satisfy the edge condition of V_4Cordial Labeling. Hence Globe Gl(n)(when n $\equiv$ 2 (mod 4)) is not a V_4 cordial graph.

Case(ii): Suppose f(u) = f(v) .

First we consider vertex condition. Let f(u) = f(v) = i (say). For the other choices the result follows in similar way. It remains 4n+2 vertices. Label 4(n$-$1) vertices with 1,-1,i and $-i$. So that eachlabel occur n$-$1 times. The vertex condition depends on the choices of the 6 remaining vertices of v_i . As we assigned the label i for u and v , we label 1,-1 and $-i$ for 6 vertices so that each label occur two times. There is no other choice of labeling of G, otherwise vertex condition will be violated.

For edge condition the edges formal form 4(n$-$1)v_i's together with u and v have equal representation for each labels of V_4. So it is enough to consider edges induced by u and v together with six left over vertices of v_i . Consider,

*	1	-1	$-i$	1	-1	$-i$
i	i	$-i$	1	i	$-i$	1
i	i	$-i$	1	i	$-i$	1

If f(u) = f(v) = i , as discussed above the only vertex labeling of the 6 vertices are shown above. Hence, $|e_f(-1) - e_f(a)| > 1$, for all a$\in$ $V_4 - \{-1\}$. Hence, if f(u) = f(v) = a , a$\in$ V_4 , then the edge condition is violated for any labeling of satisfying vertex condition.

Hence Globe Gl(n)(when n $\equiv$ 2 (mod 4)) is not a V_4 cordial graph.

Theorem: 3.4

K_n is a V_4 Cordial graph **,** when n $<$ 7.

Proof:

(i) when n= 2

vertex		v_1	v_2
		1	-1
v_1	1	**1**	-1
v_2	-1	-1	**1**

*	1	i
1	**1**	i
i	i	**-1**

*	1	$-i$
1	**1**	$-i$
$-i$	$-i$	**-1**

*	-1	i
-1	**1**	-1
i	-1	**1**

*	-1	$-i$
-1	**1**	i
$-i$	i	**-1**

*	i	$-i$
i	**-1**	1
$-i$	1	**-1**

(ii) when n=3

Vertex		v_1	v_2	v_3
		1	-1	i
v_1	1	**1**	-1	i
v_2	-1	-1	**1**	$-i$
v_3	i	i	$-i$	**-1**

*	i	$-i$	-1
i	**-1**	1	$-i$
$-i$	1	**-1**	i
-1	$-i$	i	**1**

*	1	-1	$-i$
1	**1**	-1	$-i$
-1	-1	**1**	i
$-i$	$-i$	i	**-1**

*	1	i	$-i$
1	**1**	i	$-i$
i	i	**-1**	1
$-i$	$-i$	1	**-1**

(iii) when n=4

Vertex		v_1	v_2	v_3	v_4
		1	-1	i	$-i$
v_1	1	**1**	-1	i	$-i$
v_2	-1	-1	**1**	$-i$	i
v_3	i	i	$-i$	**-1**	1
v_4	$-i$	$-i$	i	1	**-1**

(iv) when n=5

vertex		v_1	v_2	v_3	v_4	v_5
		1	-1	i	$-i$	1
v_1	1	**1**	-1	i	$-i$	1
v_2	-1	-1	**1**	$-i$	i	-1
v_3	i	i	$-i$	**-1**	1	i
v_4	$-i$	$-i$	i	1	**-1**	$-i$
v_5	1	1	-1	i	$-i$	**1**

*	1	-1	i	$-i$	-1
1	**1**	-1	i	$-i$	-1
-1	-1	**1**	$-i$	i	1
i	i	$-i$	**-1**	1	$-i$
$-i$	$-i$	i	1	**-1**	i
-1	-1	1	$-i$	i	**1**

*	1	-1	i	$-i$	i
1	**1**	-1	i	$-i$	i
-1	-1	**1**	$-i$	i	$-i$
i	i	$-i$	**-1**	1	-1
$-i$	$-i$	i	1	**-1**	1
i	i	$-i$	-1	1	**-1**

*	1	-1	i	$-i$	$-i$
1	**1**	-1	i	$-i$	$-i$
-1	-1	**1**	$-i$	i	i
i	i	$-i$	**-1**	1	1
$-i$	$-i$	i	1	**-1**	-1
$-i$	$-i$	i	1	-1	**-1**

(v) when n=6

Vertex		v_1	v_2	v_3	v_4	v_5	v_6
		1	-1	i	$-i$	1	-1
v_1	1	**1**	-1	i	$-i$	1	-1
v_2	-1	-1	**1**	$-i$	i	-1	1
v_3	i	i	$-i$	**-1**	1	i	$-i$
v_4	$-i$	$-i$	i	1	**-1**	$-i$	i
v_5	1	1	-1	i	$-i$	**1**	-1
v_6	-1	-1	1	$-i$	i	-1	**1**

*	1	-1	i	$-i$	1	i
1	**1**	-1	i	$-i$	1	i
-1	-1	**1**	$-i$	i	-1	$-i$
i	i	$-i$	**-1**	1	i	-1
$-i$	$-i$	i	1	**-1**	$-i$	1
1	1	-1	i	$-i$	**1**	i
i	i	$-i$	-1	1	i	**-1**

*	1	-1	i	$-i$	1	$-i$
1	**1**	-1	i	$-i$	1	$-i$
-1	-1	**1**	$-i$	i	-1	i
i	i	$-i$	**-1**	1	i	1
$-i$	$-i$	i	1	**-1**	$-i$	-1
1	1	-1	i	$-i$	**1**	$-i$
$-i$	$-i$	i	1	-1	$-i$	**-1**

*	1	-1	i	$-i$	-1	i
1	**1**	-1	i	$-i$	-1	i
-1	-1	**1**	$-i$	i	1	$-i$
i	i	$-i$	**-1**	1	$-i$	-1
$-i$	$-i$	i	1	**-1**	i	1
-1	-1	1	$-i$	i	**1**	$-i$
i	i	$-i$	-1	1	$-i$	**-1**

*	1	-1	i	$-i$	-1	$-i$
1	**1**	-1	i	$-i$	-1	$-i$
-1	-1	**1**	$-i$	i	1	i
i	i	$-i$	**-1**	1	$-i$	1
$-i$	$-i$	i	1	**-1**	i	-1
-1	-1	1	$-i$	i	**1**	i
$-i$	$-i$	i	1	-1	$-i$	**-1**

*	1	-1	i	$-i$	i	$-i$
1	**1**	-1	i	$-i$	i	$-i$
-1	-1	**1**	$-i$	i	$-i$	i
i	i	$-i$	**-1**	1	-1	1
$-i$	$-i$	i	1	**-1**	1	-1
i	i	$-i$	-1	1	**-1**	1
$-i$	$-i$	i	1	-1	1	**-1**

From all the table,it is observed that, the vertex condition and edge condition of V_4Cordial Labeling is satisfied. Hence $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(a) - e_f(b)| \leq 1, \forall$ a,b$\in$ V_4 . HenceK_n(n $\leq$6) is a V_4 Cordial graph.

Theorem: 3.5

$K_n$is not a V_4 Cordial graph , when n $\geq$ 7.

Proof: when $n = 7$

vertex		v_1	v_2	v_3	v_4	v_5	v_6	v_7
		1	-1	i	$-i$	1	-1	$-i$
v_1	1	**1**	-1	i	$-i$	1	-1	$-i$
v_2	-1	-1	**1**	$-i$	i	-1	1	i
v_3	i	i	$-i$	**-1**	1	i	$-i$	1
v_4	$-i$	$-i$	i	1	**-1**	$-i$	i	-1
v_5	1	1	-1	i	$-i$	**1**	-1	$-i$
v_6	-1	-1	1	$-i$	i	-1	**1**	i
v_7	$-i$	$-i$	i	1	-1	$-i$	i	**-1**

*	1	-1	i	$-i$	i	$-i$	-1
1	**1**	-1	i	$-i$	i	$-i$	-1
-1	-1	**1**	$-i$	i	$-i$	i	1
i	i	$-i$	**-1**	1	-1	1	$-i$
$-i$	$-i$	i	1	**-1**	1	-1	i
i	i	$-i$	-1	1	**-1**	1	$-i$
$-i$	$-i$	i	1	-1	1	**-1**	i
-1	-1	1	$-i$	i	$-i$	i	**1**

*	1	-1	i	$-i$	1	i	$-i$
1	**1**	-1	i	$-i$	1	i	$-i$
-1	-1	**1**	$-i$	i	-1	$-i$	i
i	i	$-i$	**-1**	1	i	-1	1
$-i$	$-i$	i	1	**-1**	$-i$	1	-1
1	1	-1	i	$-i$	**1**	i	$-i$
i	i	$-i$	-1	1	i	**-1**	1
$-i$	$-i$	i	1	-1	$-i$	1	**-1**

*	1	-1	i	$-i$	1	-1	i
1	**1**	-1	i	$-i$	1	-1	i
-1	-1	**1**	$-i$	i	-1	1	$-i$
i	i	$-i$	**-1**	1	i	$-i$	-1
$-i$	$-i$	i	1	**-1**	$-i$	i	1
1	1	-1	i	$-i$	**1**	-1	i
-1	-1	1	$-i$	i	-1	**1**	$-i$
i	i	$-i$	-1	1	i	$-i$	**-1**

In this case, the vertex condition of V_4Cordial Labeling is satisfied. It is observed that, $|v_f(a) - v_f(b)| \leq 1$ $\forall$ a,b$\in$ V_4. But it does not satisfy the edge condition of V_4Cordial Labeling. Hence,$|e_f(a) - e_f(b)| >1$. We can extend $K_n(n > 8)$it also does not satisfy the edge condition of V_4Cordial Labeling. HenceK_n(n $\geq$ 7) is not a V_4 Cordial graph.

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