Fuzzy Soft Almost Continuous Mappings in the Sense of Singal and Hussain

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***Abstract*:** *In this paper the concepts of fuzzy soft almost continuous mappings in the sense of Singal and Hussain have been introduced and their basic properties are studied.*

Keywords - *Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft mapping, Fuzzy soft almost continuity.*

1. INTRODUCTION

Molodtsov[7] initiated the concept of soft sets as a new Mathematical tool for dealing with uncertainties. Majiet. al.[6] introduced the notion of Fuzzy Soft Set and discussed some operations on fuzzy soft sets. These results were further improved by Ahmad and Kharal[1]. In 2011,Tanayet. al.[13] introduced the notion of a fuzzy soft topology.Kharal and Ahmad [1]have been introduced the concepts of fuzzy soft mappings. BanashreeBora[3] studied the notion of fuzzy soft continuous mappings. In the year 1968, Singal and A.R Singal [11]discussed the concept of almost continuous mappings. Thakur and Alpa Singh Rajput[12]discussed the concept of Soft Almost Continuous Mappings. In the year 2008, Ahmad and Athar [2] introduced the concepts of Fuzzy Almost Continuous Functions in the sense of Singal and Hussain.The purpose of this paper is to introduce the notion of fuzzy soft almost continuity in both the senses and characterize their properties.Throughout this paper X and Y denote the initial sets. E and K denotethe parameter spaces.

2. PRELIMINARIES

Definition 2.1 A pair (F,E) is called a soft set [7] over X where F:E→ 2^X is a function and 2^X is the power set of X

Definition 2.2

A fuzzy set [15] of on X is a mapping f: X→I^X where I=[0,1].

Definition 2.3

A pair $\tilde{\lambda} = (\lambda, E)$ is called a fuzzy soft set [13] over (X,E) where λ:E→ I^X is a mapping , I^X is the collection of all fuzzy subsets of X. FS(X,E) denotes the collection of all fuzzy soft sets over (X,E). We denote $\tilde{\lambda}$ by $\tilde{\lambda}$={(e,λ(e)):e∈E}where λ(e) is a fuzzy subset of X for each e in E.

Definition 2.4 [13]

For any two fuzzy soft sets$\tilde{\lambda}$ and $\tilde{\mu}$ over a common universe X and a common parameter space E, $\tilde{\lambda}$is a fuzzy soft subset of $\tilde{\mu}$ if λ(e)≤μ(e) for all e∈E.If $\tilde{\lambda}$ is a fuzzy soft subset of $\tilde{\mu}$ then we write $\tilde{\lambda} \subseteq \tilde{\mu}$ and $\tilde{\mu}$ contains $\tilde{\lambda}$.

Two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over (X,E) are soft equal if $\tilde{\lambda} \subseteq \tilde{\mu}$ and $\tilde{\mu} \subseteq \tilde{\lambda}$. That is $\tilde{\lambda} = \tilde{\mu}$ if and only if λ(e) = μ(e) for all e∈ E.We use the following notations:

$\bar{0}(x) = 0$, for all x in X and $\bar{1}(x)$ = 1, for all x in X.

Definition 2.5 [13]

A fuzzy soft set $\tilde{\varphi}_X$ over (X,E) is said to be a null

fuzzy soft set if for all e∈ E, $\varphi_X(e) = \bar{0}$ and $\tilde{\varphi}_X = (\varphi_X, E)$.

Definition 2.6 [13]

A fuzzy soft set $\tilde{1}_X$ over (X,E) is said to be absolute fuzzy soft set if for all e∈ E, $1_X(e) = \bar{1}$ and

$\tilde{1}_X = (1_X, E)$.

Definition 2.7 [14]

The union of two fuzzy soft sets$\tilde{\lambda}$ and $\tilde{\mu}$ over (X,E) is defined as $\tilde{\lambda} \tilde{\cup} \tilde{\mu}$ = $(\lambda \tilde{\cup} \mu, E)$ where $(\lambda \tilde{\cup} \mu)(e)$ = λ(e)∪ μ(e) = the union of fuzzy sets λ(e) and μ(e) for all $e \in E$.

Definition 2.8 [14]

The intersection of two fuzzy soft sets $\tilde{\lambda}$ and $\tilde{\mu}$ over (X,E) is defined as $\tilde{\lambda} \tilde{\cap} \tilde{\mu} = (\lambda \tilde{\cap} \mu, E)$ where $(\lambda \tilde{\cap} \mu)(e)$ = λ(e)∩ μ(e) = the intersection of fuzzy sets λ(e) and μ(e) for all $e \in E$.

The arbitrary union and arbitrary intersection of fuzzy soft sets over (X,E) are defined as

$\tilde{\cup}\{\tilde{\lambda}_\alpha : \alpha \in \Delta\} = (\tilde{\cup}\{\lambda_\alpha : \alpha \in \Delta\}$, E) and $\tilde{\cap}\{\tilde{\lambda}_\alpha : \alpha \in \Delta\} = (\tilde{\cap}\{\lambda_\alpha : \alpha \in \Delta\}$, E) where

$(\tilde{\cup}\{\lambda_\alpha : \alpha \in \Delta\})(e) = \tilde{\cup}\{\lambda_\alpha(e) : \alpha \in \Delta\}$ =the union of fuzzy sets λ_α(e),$\alpha \in \Delta$ and

$(\tilde{\cap}\{\lambda_\alpha : \alpha \in \Delta\})(e) = \tilde{\cap}\{\lambda_\alpha(e) : \alpha \in \Delta\}$ =the intersection of fuzzy sets λ_α(e),$\alpha \in \Delta$, for all e∈E.

Definition 2.9 [14]

The complement of a fuzzy soft set (λ,E) over (X,E), denoted by $(\lambda,E)^C$is defined as $(\lambda,E)^C = (\lambda^C,E)$ where λ^C:E → I^X is a mapping given by $\lambda^C(e) = 1-\lambda(e)$ for every e inE.

Definition 2.10 [14]

A fuzzy soft topology $\tilde{\tau}$ on (X,E) is a family of fuzzy soft sets over (X,E) satisfying the following axioms.

i. $\tilde{\varphi}_X, \tilde{1}_X$ belong to $\tilde{\tau}$,
ii. Arbitrary union of fuzzy soft sets in $\tilde{\tau}$,belongs to $\tilde{\tau}$,
iii. The intersection of any two fuzzy soft sets in $\tilde{\tau}$,belongs to $\tilde{\tau}$.

Members of $\tilde{\tau}$are called fuzzy soft open sets in(X,$\tilde{\tau}$,E). A fuzzy soft set $\tilde{\lambda}$ over (X,E) is fuzzy soft closed in (X,$\tilde{\tau}$,E) if $(\tilde{\lambda})^C \in \tilde{\tau}$. The fuzzy soft interior of $\tilde{\lambda}$ in (X,$\tilde{\tau}$,E) is the union of all fuzzy soft open sets $\tilde{\mu} \subseteq \tilde{\lambda}$ denoted by $\widetilde{fs}$ int($\tilde{\lambda}$) = $\tilde{\cup}\{\tilde{\mu} : \tilde{\mu} \subseteq \tilde{\lambda}, \tilde{\mu} \in \tilde{\tau}\}$. The fuzzy soft closure of $\tilde{\lambda}$ in (X,$\tilde{\tau}$,E) is the intersection of all fuzzy soft closed sets $\tilde{\eta}$, $\tilde{\lambda} \subseteq \tilde{\eta}$ denoted by $\widetilde{fs}$ cl($\tilde{\lambda}$) = $\tilde{\cap}\{\tilde{\eta} : \tilde{\lambda} \subseteq \tilde{\eta}, (\tilde{\eta})^C \in \tilde{\tau}\}$.

Definition 2.11 [2]

Let (X,$\tilde{\tau}$,E). be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then $\tilde{\lambda}$ is fuzzy soft regular open if $\tilde{\lambda} = \widetilde{fsint}(\widetilde{fscl}(\tilde{\lambda}))$ and fuzzy soft regular closed if $\widetilde{fsCl}\widetilde{fsInt}(\tilde{\lambda}) = \tilde{\lambda}$.

Definition 2.12 [2]

Let (X,$\tilde{\tau}$,E). be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then $\tilde{\lambda}$ is fuzzy soft semi-open if $\tilde{\lambda} \subseteq \widetilde{fsCl}(\widetilde{fsInt}(\tilde{\lambda}))$ and fuzzy soft semi closed if $\widetilde{fsInt}(\widetilde{fsCl}(\tilde{\lambda})) \subseteq \tilde{\lambda}$.

Definition 2.13 [2]

Let (X,$\tilde{\tau}$,E). be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then $\tilde{\lambda}$ is fuzzy soft pre-open if $\tilde{\lambda} \subseteq \widetilde{fsint}(\widetilde{fscl}(\tilde{\lambda}))$ and fuzzy soft pre-closed if $\widetilde{fscl}(\widetilde{fsInt}(\tilde{\lambda})) \subseteq \tilde{\lambda}$.

Definition 2.14 [2]

Let (X,$\tilde{\tau}$,E) be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then $\tilde{\lambda}$ is fuzzy soft α-open if $\tilde{\lambda} \subseteq \widetilde{fsInt}(\widetilde{fsCl}(\widetilde{fsInt}(\tilde{\lambda})))$ and fuzzy soft α-closed if $\tilde{\lambda} \supseteq \widetilde{fsCl}(\widetilde{fsInt}(\widetilde{fsCl}(\tilde{\lambda})))$.

The classes of all fuzzy soft α-open, fuzzy soft pre-open, fuzzy soft semi-open and fuzzy soft regular open sets ,fuzzy soft semi-pre-open sets over (X,E) are denoted as $\widetilde{FS}\alpha(X)$,$\widetilde{FSSO}(X)$,$\widetilde{FSPO}(X)$ and $\widetilde{FSRO}(X)$, $\widetilde{FSSP}(X)$ respectively.

The fuzzy soft pre-interior, fuzzy soft pre-closure, fuzzy soft semi-interior,fuzzy soft semi-closure and fuzzy soft α-interior,fuzzy soft α-closure ,fuzzy soft semi-pre-interior, fuzzy soft semi-pre-closure of X are denoted by $\widetilde{fsPCl}(\tilde{\lambda})$, $\widetilde{fsPInt}(\tilde{\lambda})$, $\widetilde{fsSInt}(\tilde{\lambda})$, $\widetilde{fsSCl}(\tilde{\lambda})$,$\widetilde{fs\alpha Cl}(\tilde{\lambda})$,$\widetilde{fs\alpha Int}(\tilde{\lambda})$, $\widetilde{fsSPInt}(\tilde{\lambda})$,$\widetilde{fsSPCl}(\tilde{\lambda})$ respectively

Definition 2.15 [2]

Let (X,$\tilde{\tau}$,E). be a fuzzy soft topological space and let $\tilde{\lambda}$ be a fuzzy soft set over (X,E). Then its fuzzy soft pre-closure and fuzzy soft pre-interior are defined as:

$\widetilde{fsPCl}(\tilde{\lambda}) = \cap \{ \tilde{\mu} \mid \tilde{\mu} \supseteq \tilde{\lambda}, \tilde{\mu} \in \widetilde{FSPC}(X)\}$.

$\widetilde{fsPInt}(\tilde{\lambda}) = \cup \{ \tilde{\eta} \mid \tilde{\eta} \subseteq \tilde{\lambda}, \tilde{\eta} \in \widetilde{FSPO}(X)\}$.

The definitions for $\widetilde{fsSCl}$, $\widetilde{fsSInt}$,$\widetilde{fs\alpha Cl}$ and $\widetilde{fs\alpha Int}$ are similar.

The following extension principle is used to define the mapping between the classes of fuzzy soft sets.

Definition 2.16 [14]

Let X and Y be any two non-empty sets. Let g :X→Y be a mapping. Let λ be a fuzzy subset of X and $\tilde{\mu}$ be a fuzzy subset of Y. Then g(λ) is a fuzzy subset of Yand for y inY

g(λ)(y)=
$$\begin{cases} Sup\{\lambda(f(x)) : x \in g^{-1}(y)\}, & g^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

g^{-1}(μ) is a fuzzy subset of X, defined by g^{-1}(μ)(x)= μ(f(x)) for all x∈X.

Definition 2.17 [13]

Let FS(X,E) and FS(Y,K) be classes of fuzzy soft sets over (X,E) and (Y,K) respectively.
ρ: X→Y andψ : E → K be any two mappings. Then a fuzzy soft mapping g=(ρ, ψ) : FS(X,E)→ FS(Y,K) is defined as follows:
For a fuzzy soft set $\tilde{\lambda}$ in FS(X,E), g($\tilde{\lambda}$) is a fuzzy soft set in FS(Y,K) obtained as follows:

$$g(\tilde{\lambda})(k)=\begin{cases} \bigcup_{e \in \psi^{-1}(k)} \rho(\lambda(e)) , & \psi^{-1}(k) \neq \phi \\ \bar{0} & , \text{ Otherwise} \end{cases}$$

For every y in Y, where ρ(λ(e))(y)=
$$\begin{cases} Sup\{\lambda(e)(x) : x \in \rho^{-1}(y)\} , & \rho^{-1}(y) \neq \phi \\ 0 & , \text{ Otherwise} \end{cases}$$

Thatis g($\tilde{\lambda}$)(k) **(y)** =
$$\begin{cases} \underset{e \in \psi^{-1}(k)}{Sup}\left\{ \underset{x \in \rho^{-1}(y)}{Sup} \lambda(e)(x) \right\}, \rho^{-1}(y) \neq \phi, \psi^{-1}(k) \neq \phi \\ o \quad , \text{ Otherwise} \end{cases}$$

g ($\tilde{\lambda}$) is the image of the fuzzy soft set $\tilde{\lambda}$ under the fuzzy mapping g=(ρ,ψ).
For a fuzzy soft set $\tilde{\mu}$ in FS(Y,K) , $g^{-1}(\tilde{\mu})$ is a fuzzy soft set in FS(X,E) obtained as follows:
$g^{-1}(\tilde{\mu})(e)(x) = \rho^{-1}(\tilde{\mu}(\psi(e)))(x)$ for every x in X
$g^{-1}(\tilde{\mu})$ is the inverse image of the fuzzy soft set $\tilde{\mu}$.

Lemma 2.18 [4]

Let (X,$\tilde{\tau}$,E). and (Y,$\tilde{\sigma}$,K) be fuzzy soft topological spaces. Let ρ: X→ Y and ψ : E → K be the two mappings and g = (ρ,ψ) : FS(X,E)→ FS(Y,K) be a fuzzy soft mapping. Let $\tilde{\lambda}, \tilde{\lambda}_1, (\tilde{\lambda})_i$ ∈ FS(X,E) and $\tilde{\mu}, \tilde{\mu}_1, (\tilde{\mu})_i$ ∈ FS(Y,K), where $i \in J$ is an index set.

1. If $\tilde{\lambda}_1 \subseteq \tilde{\lambda}_2$,then $g(\tilde{\lambda}_1) \subseteq g(\tilde{\lambda}_2)$.
2. If $\tilde{\mu}_1 \subseteq \tilde{\mu}_2$,then $g^{-1}(\tilde{\mu}_1) \subseteq g^{-1}(\tilde{\mu}_2)$.
3. $\tilde{\lambda} \subseteq g^{-1}(g(\tilde{\lambda}))$, the equality holds if g is injective.
4. $g(g^{-1}(\tilde{\mu})) \subseteq \tilde{\mu}$, the equality holds if g is surjective.
5. $g^{-1}((\tilde{\mu})^C) = [g^{-1}(\tilde{\mu})]^C$.
6. $[g(\tilde{\lambda})]^C \subseteq g((\tilde{\lambda})^C)$.
7. $g^{-1}(\tilde{1}_K) = \tilde{1}_E, g^{-1}(\tilde{0}_K) = \tilde{0}_E$.
8. $g(\tilde{1}_E) = \tilde{1}_Kifg$ is surjective.
9. $g(\tilde{0}_E) = \tilde{0}_K$.

Lemma 2.19 [4]

Let (X,$\tilde{\tau}$,E). and (Y,$\tilde{\sigma}$,K) be the two fuzzy soft topological spaces. Let ρ: X→ Y and ψ : E → K bethe two mappings and g = (ρ,ψ) : FS(X,E)→ FS(Y,K) be a fuzzy soft mapping. Let $\tilde{\lambda}, \tilde{\lambda}_1, (\tilde{\lambda})_i$ ∈FS(X,E) and $\tilde{\mu}, \tilde{\mu}_1, (\tilde{\mu})_i \in$ FS(Y,K), where J is an index set.

1. $g(\cup_{i \in J} \tilde{\lambda}_i) = \cup_{i \in J} g(\tilde{\lambda}_i)$.
2. $g(\cap_{i \in J} \tilde{\lambda}_i) \subseteq \cap_{i \in J} g(\tilde{\lambda}_i)$, the equality holds if g is injective.
3. $g^{-1}(\cup_{i \in J} \tilde{\mu}_i) = \cup_{i \in J} g^{-1}(\tilde{\mu}_i)$.
4. $g^{-1}(\cap_{i \in J} \tilde{\mu}_i) = \cap_{i \in J} g^{-1}(\tilde{\mu}_i)$.

Definition 2.20 [14]

Fix x∈ X, 0< $\alpha < 1$ .Then the fuzzy subset x^α of X is called fuzzy point if
$$x^\alpha(y) = \begin{cases} \alpha & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$$

Definition 2.21 [14]

Fix x∈X, 0< $\alpha < 1$, e∈E.The fuzzy soft set x_e^α over (X,E) is called fuzzy soft point if

$$x_e^\alpha(e_1) = \begin{cases} x^\alpha & \text{for } e_1 = e \\ \bar{0} & \text{otherwise} \end{cases}$$

$$x_e^\alpha(e_1)(y) = \begin{cases} \alpha & \text{for } e_1 = e, y = x \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.22[3]

Let (X,$\tilde{\tau}$,E). and (Y,$\tilde{\sigma}$,K) be the fuzzy soft topological spaces. Let ρ: X→ Y and ψ : E → K be the two mappings and g = (ρ,ψ) : FS(X,E)→ FS(Y,K) be a fuzzy soft mapping. Then g=(ρ,ψ) is said to be fuzzy soft continuous if the inverse image of every fuzzy soft open set in (Y,$\tilde{\sigma}$,K) is fuzzy soft open in (X,$\tilde{\tau}$,E). That is $g^{-1}(\tilde{\mu}) \in \tilde{\tau}$,for all $\tilde{\mu} \in \tilde{\sigma}$.

3. Fuzzy Soft Almost Continuous Mappings

3.1 Fuzzy Soft Almost Continuity in the sense of Singal Definition 3.1.1

A fuzzy soft mapping g :(X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$) is said to be fuzzy soft almost continuous(S) if $g^{-1}(\tilde{\mu})$ is a fuzzy soft open in (X,$\tilde{\tau}$,E) for each fuzzy soft regular open set $\tilde{\mu}$ in (Y,$\tilde{\sigma}, K$).

Example 3.1.2

Let X = { a, b, c },Y={x, y, z},

E = {e_1, e_2, e_3}, K={$\alpha_1,\alpha_2,\alpha_3$}.

Let $\tilde{\lambda} = \left\{\left(e_1,\left\{\frac{a}{0.2},\frac{b}{0.2},\frac{c}{0.2}\right\}\right),\left(e_2,\left\{\frac{a}{0.2},\frac{b}{0.2},\frac{c}{0.2}\right\}\right),\left(e_3,\left\{\frac{a}{0.1},\frac{b}{0.1},\frac{c}{0.1}\right\}\right)\right\}$

$\tilde{\mu} = \left\{(\alpha_1,\{\frac{x}{0.1},\frac{y}{0.2},\frac{z}{0.2}\}),(\alpha_2,\{\frac{x}{0.1},\frac{y}{0.2},\frac{z}{0.2}\}),(\alpha_3,\{\frac{x}{0.1},\frac{y}{0.1},\frac{z}{0.1}\})\right\}$

.

Let $\tilde{\tau}$ = {$\tilde{\Phi}_X$, $\tilde{1}_X$, $\tilde{\lambda}$} and $\tilde{\sigma}$ ={$\tilde{\Phi}_Y$,$\tilde{1}_Y$,$\tilde{\mu}$}.

Then $(X,\tilde{\tau},E)$ and $(Y,\tilde{\sigma},K)$ are fuzzy soft topological spaces.

It can be verified that $\widetilde{fsint}\widetilde{fsCl}\tilde{1}_Y = \tilde{1}_Y$.

$\widetilde{FSRO}(Y) = \{\tilde{\Phi}_Y,\tilde{1}_Y\}$.

For any fuzzy soft mapping

$g = (\rho,\psi): (X,\tilde{\tau},E) \longrightarrow (Y,\tilde{\sigma},K)$,

By using lemma 2.18(7), $g^{-1}(\tilde{\Phi}_Y) = \tilde{\Phi}_X$ and $g^{-1}(\tilde{1}_Y) = \tilde{1}_X$.

This shows that g is fuzzy soft almost continuous.

Example 3.1.3

If g is fuzzy soft almost continuous but not fuzzy soft continuous.

Let X = { a, b, c },Y={x, y, z},

E = {e_1, e_2, e_3}, K={$\alpha_1,\alpha_2,\alpha_3$}.

Let $\tilde{\lambda} = \left\{\left(e_1,\left\{\frac{a}{0.6},\frac{b}{0.8},\frac{c}{0.9}\right\}\right),\left(e_2,\left\{\frac{a}{0.2},\frac{b}{0.1},\frac{c}{0.8}\right\}\right),\left(e_3,\left\{\frac{a}{0.3},\frac{b}{0.7},\frac{c}{0.9}\right\}\right)\right\}$

$\tilde{\mu} = \left\{(\alpha_1,\{\frac{x}{0.6},\frac{y}{0.4},\frac{z}{0.2}\}),(\alpha_2,\{\frac{x}{0.7},\frac{y}{0.5},\frac{z}{0.4}\}),(\alpha_3,\{\frac{x}{0.9},\frac{y}{0.4},\frac{z}{0.6}\})\right\}$

Let $\tilde{\tau}$ = {$\tilde{\Phi}_X$, $\tilde{1}_X$, $\tilde{\lambda}$} and $\tilde{\sigma}$ ={$\tilde{\Phi}_Y$,$\tilde{1}_Y$,$\tilde{\mu}$}.

Then $(X,\tilde{\tau},E)$ and $(Y,\tilde{\sigma},K)$ are fuzzy soft topological spaces.

It can be verified that $\widetilde{fsint}\widetilde{fsCl}\tilde{1}_Y = \tilde{1}_Y$.

$\widetilde{FSRO}(Y) = \{\tilde{\Phi}_Y,\tilde{1}_Y\}$.

Define $\rho(a) = z$, $\rho(b) = x, \rho(c) = y$ and $\psi(e_1) = \alpha_1$, $\psi(e_2) = \alpha_1$, $\psi(e_3) = \alpha_3$.

Let $g = (\rho,\psi): (X,\tilde{\tau},E) \longrightarrow (Y,\tilde{\sigma},K)$ be a fuzzy soft mapping.

By using lemma 2.18(7), $g^{-1}(\tilde{\Phi}_Y) = \tilde{\Phi}_X$ and $g^{-1}(\tilde{1}_Y) = \tilde{1}_X$.

$g^{-1}(\tilde{\mu})(e_1)(a) = \tilde{\mu}(\psi(e_1))\rho(a)$=$\tilde{\mu}(\alpha_1)(z) = \{\frac{x}{0.6},\frac{y}{0.4},\frac{z}{0.2}\}$)(z) = 0.2,

$g^{-1}(\tilde{\mu})(e_1)(b) = \tilde{\mu}(\psi(e_1))\rho(b)$=$\tilde{\mu}(\alpha_1)(x) = \{\frac{x}{0.6},\frac{y}{0.4},\frac{z}{0.2}\}$) (x)=0.6,

$g^{-1}(\tilde{\mu})(e_1)(c) = \tilde{\mu}(\psi(e_1))\rho(c)$=$\tilde{\mu}(\alpha_1)(y) = \{\frac{x}{0.6},\frac{y}{0.4},\frac{z}{0.2}\}$)(y)=0.4,

$g^{-1}(\tilde{\mu})(e_1)(X) = \left\{e_1,\left\{\frac{a}{0.2},\frac{b}{0.2},\frac{c}{0.2}\right\}\right\}$.

$g^{-1}(\tilde{\mu})(e_2)(a) = \tilde{\mu}(\psi(e_2))\rho(a)$=$\tilde{\mu}(\alpha_1)(z) = \left\{\frac{x}{0.6},\frac{y}{0.4},\frac{z}{0.2}\right\}(z) = 0.2$,

$g^{-1}(\tilde{\mu})(e_2)(b) = \tilde{\mu}(\psi(e_2))\rho(b)$=$\tilde{\mu}(\alpha_1)(x) = \{\frac{x}{0.6},\frac{y}{0.4},\frac{z}{0.2}\}$(x) =0.6,

$g^{-1}(\tilde{\mu})(e_2)(c) = \tilde{\mu}(\psi(e_2))\rho(c)$=$\tilde{\mu}(\alpha_1)(y) = \{\frac{x}{0.6},\frac{y}{0.4},\frac{z}{0.2}\}$ (y)=0.4,

$g^{-1}(\tilde{\mu})(e_2)(X)$= $\left\{e_2,\left\{\frac{a}{0.2},\frac{b}{0.2},\frac{c}{0.2}\right\}\right\}$.

$g^{-1}(\tilde{\mu})(e_3)(a) = \tilde{\mu}(\psi(e_3))\rho(a)$=$\tilde{\mu}(\alpha_3)(z) = \{\frac{x}{0.9},\frac{y}{0.4},\frac{z}{0.6}\}$ (z)=0.6,

$g^{-1}(\tilde{\mu})(e_3)(b) = \tilde{\mu}(\psi(e_3))\rho(b)$=$\tilde{\mu}(\alpha_3)(x) = \{\frac{x}{0.9},\frac{y}{0.4},\frac{z}{0.6}\}$ (x)=0.9,

$g^{-1}(\tilde{\mu})(e_3)(c) = \tilde{\mu}(\psi(e_3))\rho(c)$=$\tilde{\mu}(\alpha_3)(y) = \{\frac{x}{0.9},\frac{y}{0.4},\frac{z}{0.6}\}$ (y)=0.4,

$g^{-1}(\tilde{\mu})(e_3)(X)$= $\left\{e_2,\left\{\frac{a}{0.6},\frac{b}{0.9},\frac{c}{0.4}\right\}\right\}$

Thus

$g^{-1}(\tilde{\mu}) = \left\{\left(\boldsymbol{e_1},\left\{\frac{\boldsymbol{a}}{\mathbf{0.2}},\frac{\boldsymbol{b}}{\mathbf{0.6}},\frac{\boldsymbol{c}}{\mathbf{0.4}}\right\}\right),\left(\boldsymbol{e_2},\left\{\frac{\boldsymbol{a}}{\mathbf{0.2}},\frac{\boldsymbol{b}}{\mathbf{0.6}},\frac{\boldsymbol{c}}{\mathbf{0.4}}\right\}\right),\left(\boldsymbol{e_3},\left\{\frac{\boldsymbol{a}}{\mathbf{0.6}},\frac{\boldsymbol{b}}{\mathbf{0.9}},\frac{\boldsymbol{c}}{\mathbf{0.4}}\right\}\right)\right\} \notin \tilde{\tau}$

Therefore g is fuzzy soft almost continuous but not fuzzy soft continuous.

Theorem 3.1.4

For a fuzzy soft mappingg = (ρ,ψ) :(X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}, K$), the following are equivalent

(i) g is fuzzy soft almost continuous (S).

(ii) The inverse image of every fuzzy soft regular closed set in (Y,$\tilde{\sigma}, K$) is fuzzy soft closed in (X,$\tilde{\tau}$,E).

Proof:

Suppose (i) holds. Let $\tilde{\mu}$ be a fuzzy soft regular closed in (Y,$\tilde{\sigma}, K$). Then $(\tilde{\mu})^C$ is fuzzy soft regular open in (Y,$\tilde{\sigma}, K$).Using Definition 3.1.1, $g^{-1}((\tilde{\mu})^C)$is fuzzy soft open. Since$g^{-1}((\tilde{\mu})^C)$= $[g^{-1}(\tilde{\mu})]^C$, $g^{-1}(\tilde{\mu})$ is fuzzy soft closed. This proves (i) ⟹ (ii).

Conversely we assume that (ii) holds. Let $\tilde{\mu}$ be fuzzy soft regular open in (Y,$\tilde{\sigma}, K$). Therefore $(\tilde{\mu})^C$ is fuzzy soft regular closed set in (Y,$\tilde{\sigma}, K$). Then by applying (ii), $[g^{-1}(\tilde{\mu})]^C$ is fuzzy soft closed in (X,$\tilde{\tau}$,E). That implies $g^{-1}(\tilde{\mu})$ is fuzzy soft open in (X,$\tilde{\tau}$,E). This proves (ii) ⟹ (i).

Theorem 3.1.5 Let g :(X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma},K$) be a fuzzy soft almost continuous(S) mapping. Then for each fuzzy soft open set $\tilde{\mu}$ in (Y,$\tilde{\sigma},K$),$\widetilde{fsCl}g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsCl}(\tilde{\mu})$.

Proof

Let $\tilde{\mu}$ be a fuzzy soft open set in (Y,$\tilde{\sigma},K$). Since $\tilde{\mu} \mathrel{\tilde{\subseteq}} \widetilde{fsCl}(\tilde{\mu})$, $g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsCl}(\tilde{\mu}))$. Since $\tilde{\mu}$ is fuzzy soft open, $\widetilde{fsCl}(\tilde{\mu})$ is fuzzy soft regular closed. Since g is fuzzy soft almost continuous(S), $g^{-1}(\widetilde{fsCl}(\tilde{\mu}))$ is fuzzy soft closed. That implies
$\widetilde{fsCl}g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} \widetilde{fsCl}(g^{-1}(\widetilde{fsCl}(\tilde{\mu})) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsCl}(\tilde{\mu})$.

Theorem 3.1.6

For a fuzzy soft mapping **g** = (ρ,ψ) :(X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma},K$), the following are equivalent

(i) g is fuzzy soft almost continuous(S).
(ii) $\widetilde{fsCl}g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsCl}(\tilde{\mu}))$, for every $\tilde{\mu} \in \widetilde{FSSPO}(Y)$.
(iii) $\widetilde{fsCl}g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsCl}(\tilde{\mu}))$, for every $\tilde{\mu} \in \widetilde{FSSO}(Y)$.
(iv) $g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} \widetilde{fsInt}\, g^{-1}(\widetilde{fsInt}\,\widetilde{fsCl}(\mu))$, for every $\tilde{\mu} \in \widetilde{FSPO}(Y)$.

Proof:

Suppose (i) holds. Let $\tilde{\mu}$ be a fuzzy soft semi- pre-open set in(Y,$\tilde{\sigma},K$). Since fuzzy soft closure of a fuzzy soft semi-pre-open set is fuzzy soft regularly closed,$\widetilde{fsCl}(\tilde{\mu})$ is fuzzy soft regularly closed in (Y,$\tilde{\sigma},K$). By using Theorem 3.1.4, $g^{-1}(\widetilde{fsCl}(\tilde{\mu}))$ is fuzzy soft closed in (X,$\tilde{\tau}$,E). Since$\tilde{\mu} \mathrel{\tilde{\subseteq}} \widetilde{fsCl}(\tilde{\mu})$, $g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsCl}(\tilde{\mu}))$. Since $g^{-1}(\widetilde{fsCl}(\tilde{\mu}))$ is fuzzy soft closed,
$\widetilde{fsCl}g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} \widetilde{fsCl}(g^{-1}(\widetilde{fsCl}(\tilde{\mu}))=g^{-1}(\widetilde{fsCl}(\tilde{\mu}))$.
This proves (i) ⟹ (ii).

Since each fuzzy soft semi open set is fuzzy soft semi pre-open set, (ii) ⟹ (iii) is obvious.
Suppose (iii) holds. Let $\tilde{\mu}$ be a fuzzy soft regular closed set in (Y,$\tilde{\sigma},K$). Then $\tilde{\mu} = \widetilde{fsCl}\widetilde{fsInt}(\tilde{\mu})$ and hence$\tilde{\mu} \in \widetilde{FSSO}(Y)$. Therefore, we have $\widetilde{fsCl}g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsCl}(\tilde{\mu}))= g^{-1}(\tilde{\mu})$. Hence $g^{-1}(\tilde{\mu})$ is fuzzy soft closed and by using Theorem 3.1.4, g is fuzzy soft almost continuous. This proves (iii) ⟹ (i).

Suppose (i) holds. Let $\tilde{\mu}$ be a fuzzy soft semi pre-open set in (Y,$\tilde{\sigma},K$). Then $\tilde{\mu} \mathrel{\tilde{\subseteq}} \widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu})$ and $\widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu})$ is fuzzy soft regularly open. Since g is fuzzy soft almost continuous, by using Theorem 3.1.4, $g^{-1}(\widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu}))$ is fuzzy soft open in (X,$\tilde{\tau}$,E) and hence $g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu})) = \widetilde{fsInt}(g^{-1}(\widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu}))$. This proves (i) ⟹ (iv).

Suppose (iv) holds. Let $\tilde{\mu}$ be a fuzzy soft regular open set in (Y,$\tilde{\sigma},K$). Then $\tilde{\mu} \in \widetilde{FSPO}(Y)$ and hence $g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} \widetilde{fsInt}(g^{-1}(\widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu}))$ = $\widetilde{fsInt}(g^{-1}(\tilde{\mu}))$. Therefore $g^{-1}(\tilde{\mu})$ is fuzzy soft open in (X,$\tilde{\tau}$,E) and g is fuzzy soft almost continuous.

Theorem 3.1.7

Let $\tilde{\lambda}$ be a fuzzy soft set over (X,E) and let (X,$\tilde{\tau}$,E) be fuzzy soft topological spaces. Then

(i) $\widetilde{fs}$Pcl$\tilde{\lambda}$ = $\widetilde{fs}$Cl$\tilde{\lambda}$ for every fuzzy soft semi-open set $\tilde{\lambda}$.
(ii) $\widetilde{fs}$Scl$\tilde{\lambda}$ = $\widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\lambda})$for every fuzzy soft pre-open set $\tilde{\lambda}$.
(iii) $\widetilde{fs\alpha}$cl$\tilde{\lambda}$ = $\widetilde{fs}$Cl$\tilde{\lambda}$ for every fuzzy soft semi-pre-open set $\tilde{\lambda}$.

Proof:

Let $\tilde{\lambda}$ be a fuzzy soft semi open set. Since every fuzzy soft closed set is fuzzy soft pre closed,
$\widetilde{fs}$Pcl$\tilde{\lambda} \mathrel{\tilde{\subseteq}} \widetilde{fs}Cl\tilde{\lambda}$ is always true.
To prove the reverse inclusion, let $x_e^\alpha \in \widetilde{fs}Cl\tilde{\lambda}$ be a fuzzy soft point. Let $\tilde{\mu}$ be a fuzzy soft pre open set containing x_e^α. Then $x_e^\alpha \in \tilde{\mu} \mathrel{\tilde{\subseteq}} \widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu})$. Since $x_e^\alpha \in \widetilde{fs}Cl\tilde{\lambda}$,
$\widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu}) \cap \tilde{\lambda} \neq \varphi$. Since $\tilde{\lambda}$ is fuzzy soft semi open,

$$\tilde{\lambda} \cap \widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} \widetilde{fsCl}\widetilde{fsInt}(\tilde{\lambda})\widetilde{fsInt}\,\widetilde{fsCl}(\tilde{\mu})$$
$$\mathrel{\tilde{\subseteq}} \widetilde{fsCl}\widetilde{fsInt}(\tilde{\lambda}) \cap \widetilde{fsCl}(\tilde{\mu})$$
$$\mathrel{\tilde{\subseteq}} \widetilde{fsCl}(\widetilde{fsInt}\tilde{\lambda} \cap \tilde{\mu})$$
$$\mathrel{\tilde{\subseteq}} \widetilde{fsCl}(\tilde{\lambda} \cap \tilde{\mu}).$$

This implies $\tilde{\lambda} \cap \tilde{\mu} \neq \varphi$. That implies $x_e^\alpha \in \widetilde{fs}Pcl\tilde{\lambda}$. That is $\widetilde{fs}$Cl$\tilde{\lambda} \subseteq \widetilde{fs}Pcl\tilde{\lambda}$. This proves (i).
(ii) and (iii) can be analogously proved.

Theorem 3.1.8

For a fuzzy soft mapping **g** = (ρ,ψ) :(X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma},K$), the following are equivalent

(i) g is fuzzy soft almost continuous(S).
(ii) $\widetilde{fsCl}g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fs}\,\alpha Cl(\tilde{\mu}))$, for every $\tilde{\mu} \in \widetilde{FSSPO}(Y)$.
(iii) $\widetilde{fsCl}g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} g^{-1}(\widetilde{fsPcl}(\tilde{\mu}))$, for every $\tilde{\mu} \in \widetilde{FSSO}(Y)$.
(iv) $g^{-1}(\tilde{\mu}) \mathrel{\tilde{\subseteq}} \widetilde{fsInt}\, g^{-1}(\widetilde{fsScl}(\tilde{\mu}))$, for every $\tilde{\mu} \in \widetilde{FSPO}(Y)$.

Proof:

Follows from Theorem 3.1.6 and Theorem 3.1.7.

Definition 3.1.9

A fuzzy soft mapping g = (ρ,ψ) : (X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}$,K) is said to be fuzzy soft almost open if for each fuzzy soft regular open set $\tilde{\lambda}$ in X , g($\tilde{\lambda}$) is fuzzy soft open in Y.

Definition 3.1.10

A surjective mapping g=(ρ,ψ) : (X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}$,K) is said to be fuzzy soft almost quasi-compact if g^{-1} ($\tilde{\mu}$) is fuzzy soft regular open in (X,$\tilde{\tau}$,E) implies $\tilde{\mu}$ is fuzzy soft open in (Y,$\tilde{\sigma}$,K).

Theorem 3.1.11

A bijective mappingg=(ρ,ψ) : (X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}$,K) is fuzzy soft almost quasi compact if and only if

(i) The image of every fuzzy soft regular open under g is fuzzy soft open.
(ii) The image of every fuzzy soft regular closed under g is fuzzy soft closed.

Proof:

Letg=(ρ,ψ) : (X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}$,K) be bijective mapping. Suppose g is fuzzy soft almost quasi compact Let $\tilde{\lambda}$ be a fuzzy soft set over (X,E) and$\tilde{\lambda}$ be fuzzy soft regular open in (X,$\tilde{\tau}$,E) . Since g is fuzzy soft almost quasi compact, $g^{-1}(g(\tilde{\lambda})) = \tilde{\lambda}$ is fuzzy soft regular open. Then using Definition 3.1.9, $g(\tilde{\lambda})$ is fuzzy soft open. This proves (i).

Now $\tilde{\lambda}$ is fuzzy soft regular closed in (X,$\tilde{\tau}$,E). Then $(\tilde{\lambda})^C$ is fuzzy soft regular open. That implies

$g^{-1}(g(\tilde{\lambda})^C) = (\tilde{\lambda})^C$ is fuzzy soft regular open. Then using Definition 3.1.9, $g((\tilde{\lambda})^C)$ is fuzzy soft open, $(g(\tilde{\lambda}))^C$ is fuzzy soft open, $g(\tilde{\lambda})$ is fuzzy soft closed. This proves (ii).

Conversely, let $g(\tilde{\lambda})$ be fuzzy soft open (resp. closed) for every $\tilde{\lambda}$ is fuzzy soft regular open (regular closed) set. g^{-1} ($\tilde{\lambda}$) is fuzzy soft regular open, g(g^{-1} ($\tilde{\lambda}$) is fuzzy soft open (by our assumption), $\tilde{\lambda}$ is fuzzy soft open. Therefore g is fuzzy soft almost quasi compact.

Definition 3.1.12A fuzzy soft mapping g=(ρ,ψ) : (X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}$,K) is said to be fuzzy soft almost closed if for each fuzzy soft regular closed $\tilde{\lambda}$ in X , g($\tilde{\lambda}$) is fuzzy soft closed in Y.

Theorem 3.1.13

Let **g** = (ρ,ψ) :(X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}$,K) be fuzzy soft almost open and fuzzy soft almost continuous(S) if and only if $\widetilde{fs}\ Cl\ g^{-1}(\tilde{\mu}) = g^{-1}(\widetilde{fs}\ Cl\ \tilde{\mu})$for every $\tilde{\mu} \in \widetilde{FS}\ SO\ (Y)$.

Proof:

Let $\tilde{\mu} \in \widetilde{FS}\ SO\ (Y)$. Since f is fuzzy soft almost continuous(S), by Theorem 3.1.6 (iii), $\widetilde{fs}\ Cl\ g^{-1}(\tilde{\mu}) \tilde{\subseteq} g^{-1}(\widetilde{fs}\ Cl\ (\tilde{\mu}))$. Since g is fuzzy soft almost open , we have $g^{-1}(\widetilde{fs}\ Cl\ (\tilde{\mu}))$ = $g^{-1}(\widetilde{fs}\ Cl\ \widetilde{fs}\ Int\ (\tilde{\mu})) \tilde{\subseteq} \widetilde{fs}\ Cl\ g^{-1}(\widetilde{fs}\ Int\ \tilde{\mu}) = \widetilde{fs}\ Cl\ g^{-1}(\tilde{\mu})$.

Therefore, we obtain $\widetilde{fs}\ Cl\ g^{-1}(\tilde{\mu})$= $g^{-1}(\widetilde{fs}\ Cl\ (\tilde{\mu}))$, for every $\tilde{\mu} \in \widetilde{FS}\ SO\ (Y)$.

Conversely we assume that

$\widetilde{fs}\ Cl\ g^{-1}(\tilde{\mu}) = g^{-1}(\widetilde{fs}\ Cl\ \tilde{\mu})$for every $\tilde{\mu} \in \widetilde{FS}\ SO\ (Y)$.

Let $\tilde{\mu}$ be fuzzy soft regular closed in (Y,$\tilde{\sigma}$,K). Then $\tilde{\mu} = \widetilde{fs}\ Cl\ \widetilde{fs}\ Int\ (\tilde{\mu})$ and hence $\tilde{\mu} \in \widetilde{FS}\ SO\ (Y)$. By the hypothesis, $\widetilde{fs}\ Cl\ g^{-1}(\tilde{\mu})$ = $g^{-1}(\tilde{\mu})$ and hence $g^{-1}(\tilde{\mu})$ is fuzzy soft closed in (X,$\tilde{\tau}$,E). Therefore by Theorem 3.1.4, g is fuzzy soft almost continuous(S).

Theorem 3.1.14

For a bijective fuzzy soft mapping **g** = (ρ,ψ) :(X,$\tilde{\tau}$,E) → (Y,$\tilde{\sigma}$,K), the following are equivalent
g is fuzzy soft almost open.
g is fuzzy soft almost closed.
g is fuzzy soft almost quasi-compact.
g^{-1} is fuzzy soft almost continuous(S).

Proof:

Suppose g is fuzzy soft almost open. Let $\tilde{\lambda}$ be a fuzzy soft regular closed setin (X,$\tilde{\tau}$,E). Then $(\tilde{\lambda})^C$ is fuzzy soft regular open in (X,$\tilde{\tau}$,E). Since g is fuzzy soft almost open, $g((\tilde{\lambda})^C)$= $[g(\tilde{\lambda})]^C$, g($\tilde{\lambda}$) is fuzzy soft closed in (Y,$\tilde{\sigma}$,K). That is g is fuzzy soft almost closed. This proves (i) ⟹ (ii).
Suppose g is fuzzy soft almost closed. Let $\tilde{\mu}$ be a fuzzy soft over (Y,K) such that $g^{-1}(\tilde{\mu})$ is fuzzy soft regular closed. Since g is fuzzy soft almost closed, g(g^{-1} ($\tilde{\mu}$) is fuzzy soft closed.That is $\tilde{\mu}$ is fuzzy soft closed. Therefore by Theorem 3.1.11, g is fuzzy soft almost quasi compact. This proves (ii) ⟹ (iii).
Suppose g is fuzzy soft almost quasi compact. Let $\tilde{\lambda}$ be fuzzy soft regular open set in (X,$\tilde{\tau}$,E).
Then $\tilde{\lambda}$ = g^{-1} (g($\tilde{\lambda}$)) is fuzzy soft regular open. That implies g($\tilde{\lambda}$) is fuzzy soft open in (Y,$\tilde{\sigma}$,K).

That is $(g^{-1})^{-1}(\tilde{\lambda})$ = g($\tilde{\lambda}$) is fuzzy soft open in (Y,$\tilde{\sigma}$, K). Therefore $g^{-1}$is fuzzy soft almost continuous(S). This proves (iii) ⟹ (iv).

Suppose $g^{-1}$is fuzzy soft almost continuous(S). Let $\tilde{\lambda}$ be fuzzy soft regular open set in (X,$\tilde{\tau}$,E).

Since $g^{-1}$is fuzzy soft almost continuous(S), $((g^{-1})^{-1})(\tilde{\lambda})$ is fuzzy soft open set in (Y,$\tilde{\sigma}$, K). That is g($\tilde{\lambda}$) is fuzzy soft open in (Y,$\tilde{\sigma}$, K). Therefore g is fuzzy soft almost open.

This proves (iv) ⟹ (i).

Definition 3.1.15

A surjective mapping g=(ρ,ψ) **:** (X,$\tilde{\tau}$,E)→ (Y,$\tilde{\sigma}$, K) is said to be fuzzy soft quasi-compact if $g^{-1}(\tilde{\mu})$ is fuzzy soft open in (X,$\tilde{\tau}$,E) implies $\tilde{\mu}$ is fuzzy soft open in (Y,$\tilde{\sigma}$, K).

Theorem 3.1.16

Let h = (ρ,ψ) **:** (X,$\tilde{\tau}$,E)→(Y,$\tilde{\sigma}$, K) and g = (α,β)**:** (Y,$\tilde{\sigma}$,K)→ (Z,$\tilde{\eta}$, L) be two fuzzy soft mappings. Suppose h is surjective and fuzzy soft almost continuous(S). Then

(i) If g∘h is fuzzy soft open then g is fuzzy soft almost open.

(ii) If g∘h is fuzzy soft closed then g is fuzzy soft almost closed .

(iii) If g∘h is fuzzy soft quasi-compact then g is fuzzy soft almost quasi-compact.

Proof:

Supposeg∘h is fuzzy soft open. Let $\tilde{\mu}$ be fuzzy soft regular open set in (Y,$\tilde{\sigma}$, K). Since h is fuzzy soft almost continuous(S), $h^{-1}(\tilde{\mu})$ is fuzzy soft open set in (X,$\tilde{\tau}$,E). Since g∘h is fuzzy soft open, (g∘h)($h^{-1}(\tilde{\mu})$) is fuzzy soft open in (Z,$\tilde{\eta}$, L). That is g($\tilde{\mu}$) is fuzzy soft open in (Z,$\tilde{\eta}$, L). Therefore g is fuzzy soft almost open. This proves (i).

Supposeg∘h is fuzzy soft closed. Let $\tilde{\mu}$ be fuzzy soft regular closed set in (Y,$\tilde{\sigma}$, K). Since h is fuzzy soft almost continuous(S), $h^{-1}(\tilde{\mu})$ is fuzzy soft closed set in (X,$\tilde{\tau}$,E). Since g∘h is fuzzy soft closed,(g∘h)($h^{-1}(\tilde{\mu})$) is fuzzy soft closed in (Z,$\tilde{\eta}$, L). That is g($\tilde{\mu}$) is fuzzy soft closed in (Z,$\tilde{\eta}$, L). Therefore g is fuzzy soft almost closed. This proves (ii).

Suppose g∘h is fuzzy soft quasi-compact. Let $\tilde{\mu}$ be fuzzy soft set in (Z,$\tilde{\eta}$, L) such that $g^{-1}(\tilde{\mu})$ is fuzzy soft regular open set in (Y,$\tilde{\sigma}$, K). Since h is fuzzy soft almost continuous(S), $h^{-1}(g^{-1}(\tilde{\mu}))$ is fuzzy soft open set in (X,$\tilde{\tau}$,E). That is ,$(g∘h)^{-1}(\tilde{\mu})$ is fuzzy soft open set in (X,$\tilde{\tau}$,E). since g∘h is fuzzy soft quasi-compact, $\tilde{\mu}$ is fuzzy soft open in (Z,$\tilde{\eta}$, L). Therefore g is fuzzy soft almost quasi compact.

3.2. Fuzzy Soft Almost Continuity in the sense of Hussain's sense

Definition 3.2.1

A fuzzy soft mapping g **:** (X,$\tilde{\tau}$,E)→(Y,$\tilde{\sigma}$, K) is said to be a fuzzy soft almost continuous(H) at $x_e^{\alpha} \in X$, if for each fuzzy soft open set $\tilde{\mu}$in (Y,$\tilde{\sigma}$, K) with g(x_e^{α}) ∈ $\tilde{\mu}$, $fs\ Cl\ g^{-1}(\tilde{\mu})$ is a fuzzy soft neighbourhood of x_e^{α}. If g is fuzzy soft almost continuous(H) at each fuzzy soft point of X, then g is called fuzzy soft almost continuous(H).

Definition 3.2.2

A fuzzy soft set $\tilde{\lambda}$ is said to be fuzzy soft dense in another fuzzy soft set $\tilde{\mu}$ both being fuzzy soft sets in an fuzzy soft topological space (X,$\tilde{\tau}$,E), if $fs\ Cl\ \tilde{\lambda} = \tilde{\mu}$.

Definition 3.2.3

A fuzzy soft mapping **g :** (X,$\tilde{\tau}$,E)→(Y,$\tilde{\sigma}$, K)is said to be fuzzy soft weakly continuous if for each point x_e^{α} ∈X and each open set $\tilde{\mu}$ in Y containing g(x_e^{α}), there existis an open set $\tilde{\lambda}$ in X containing x_e^{α} such that g($\tilde{\lambda}$)⊂ fs cl ($\tilde{\lambda}$).

Definition 3.2.4

A fuzzy soft mapping g **:**(X,$\tilde{\tau}$,E)→(Y,$\tilde{\sigma}$, K) is said to be fuzzy soft almost weakly continuous if $g^{-1}(\tilde{\mu}) \sqsubseteq fs\ Int\ fs\ Cl\ g^{-1}(fs\ Cl\ \tilde{\mu})$, for every fuzzy soft open set $\tilde{\mu}$ in (Y,$\tilde{\sigma}$, K).

Theorem 3.2.5

For a fuzzy soft mappingg **:**(X,$\tilde{\tau}$,E)→(Y,$\tilde{\sigma}$, K), the following are equivalent:

i. g is fuzzy soft almost weakly continuous.
ii. $fs\ Cl\ fs\ Int\ g^{-1}(\tilde{\mu}) \sqsubseteq g^{-1}(fs\ Cl\ (\tilde{\mu}))$, for every fuzzy soft open set $\tilde{\mu}$in (Y,$\tilde{\sigma}$, K).
iii. For each $x_e^{\alpha} \in X$ and each fuzzy soft-open set $\tilde{\lambda}$ such that $g(x_e^{\alpha}) \in \tilde{\lambda}$, $fs\ Cl\ g^{-1}(fs\ Cl\ \tilde{\lambda})$ is a fuzzy soft neighbourhood of x_e^{α}.

Proof:

Let $\tilde{\mu}$ be a fuzzy soft open set in $(Y,\tilde{\sigma},K)$. Then $(\widetilde{fs}\ Cl\ \tilde{\mu})^C$ is fuzzy soft open in $(Y,\tilde{\sigma},K)$ and we have

$$(\widetilde{fs}\ Int\ \ \widetilde{fs}\ Cl\ g^{-1}(\widetilde{fs}\ Cl\ \tilde{\mu}))^C = \widetilde{fs}\ Cl\widetilde{fs}\ Int\ \ g^{-1}((\widetilde{fs}\ Cl\ \tilde{\mu})^C)$$

$$\tilde{\subseteq}\ g^{-1}(\widetilde{fs}\ Cl\ (\widetilde{fs}\ Cl\ (\tilde{\mu})^C))$$

$$= g^{-1}((\widetilde{fs}\ Int\ \ \widetilde{fs}\ Cl\ \tilde{\mu})^C)$$

$$\tilde{\subseteq}\ g^{-1}(\tilde{\mu}^C)$$

$$= (g^{-1}(\tilde{\mu}))^C.$$

Therefore we obtain, $\widetilde{fs}\ Cl\ \ \widetilde{fs}\ Int\ \ g^{-1}(\tilde{\mu})\ \tilde{\subseteq}\ g^{-1}(\widetilde{fs}\ Cl\ (\tilde{\mu}))$.

This proves (i) ⟹(ii).

Let $x_e^\alpha \in X$ and $\tilde{\mu}$ be a fuzzy soft open set such that $g(x_e^\alpha) \in \tilde{\mu}$. Since $(\widetilde{fs}\ Cl\ \tilde{\mu})^C$is fuzzy soft open in $(Y,\tilde{\sigma},K)$,we have

$$\tilde{\subseteq}\ g^{-1}(\widetilde{fs}\ Cl\ (\widetilde{fs}\ Cl\ (\tilde{\mu})^C))$$

$$= g^{-1}((\widetilde{fs}\ Int\ \ \widetilde{fs}\ Cl\ \tilde{\mu})^C)$$

$$\tilde{\subseteq}\ g^{-1}(\tilde{\mu}^C)$$

$$= (g^{-1}(\tilde{\mu}))^C.$$

Therefore we obtain $x_e^\alpha \in g^{-1}(\tilde{\mu})\ \tilde{\subseteq}\ \widetilde{fs}\ Int\ \ \widetilde{fs}\ Cl\ g^{-1}(\widetilde{fs}\ Cl\ \tilde{\mu})$ and hence $\widetilde{fs}\ Cl\ g^{-1}(\widetilde{fs}\ Cl\ \tilde{\mu})$is a fuzzy soft neighbourhood of x. This proves (ii) ⟹(iii).

Let $\tilde{\mu}$ be a fuzzy soft open set in $(Y,\tilde{\sigma},K)$ and $x_e^\alpha \in g^{-1}(\tilde{\mu})$. Then $g(x_e^\alpha) \in \tilde{\mu}$ and $\widetilde{fs}Clg^{-1}(\widetilde{fs}Cl\tilde{\mu})$is a fuzzy soft neighbourhood of x_e^α. Therfore $x_e^\alpha \in \widetilde{fs}Int\widetilde{fs}Clg^{-1}(\widetilde{fs}Cl\tilde{\mu})$ and we obtain $g^{-1}(\tilde{\mu})\ \tilde{\subseteq}\ \widetilde{fs}Int\widetilde{fs}Clg^{-1}(\widetilde{fs}Cl\tilde{\mu})$. This proves (iii)⟹(i).

Theorem 3.2.6

Every fuzzy soft almost continuous(H) mapping g **:** $(X,\tilde{\tau},E) \to (Y,\tilde{\sigma},K)$ is fuzzy soft almost weakly continuous mapping.

Proof: By Theorem 3.2.5, (iii)⟹(i), every fuzzy soft almost

continuous(H) mapping is fuzzy soft almost weakly continuous mapping.

The next two propositions can be easily proved.

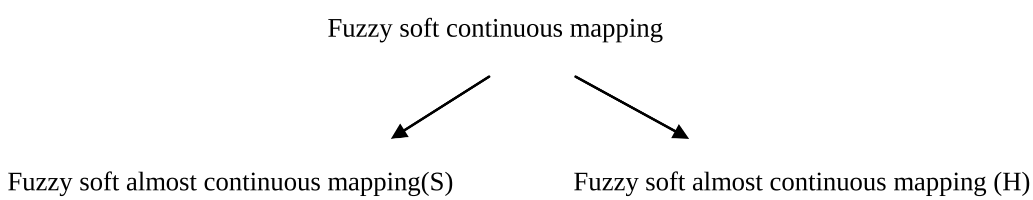
Proposition 3.2.7

Every fuzzy soft continuous mapping is fuzzy soft almost continuous (S).

Proposition 3.2.8

Every fuzzy soft continuous mapping is fuzzy soft almost continuous (H).

Remark 3.2.9
The above discussions give the following implication diagram.



Conclusion

Fuzzy Soft almost continuous mappingsin the sense of Singal and Hussian have been Characterized using recent concepts in the literature of fuzzy soft topology..

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