Eulerian integral associated with product of two multivariable I-functions, a class of polynomials and the multivariable Aleph-function I

$F.Y.\ AYANT$[1]

1 Teacher in High School , France

ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of two multivariable I-functions defined by Prasad [1] a generalized Lauricella function , a class of multivariable polynomials and multivariable Aleph-function with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [7] and Srivastava-Daoust polynomial [4].

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable H-function, generalized hypergeometric function, class of polynomials

2010 Mathematics Subject Classification. 33C60, 82C31

1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable I-functions defined by Prasad [1], the Aleph-function of several variables and a class of polynomials with general arguments. The Aleph-function of several variables is an extension of multivariable I-function defined by Sharma et al [3].

We define : $\aleph(z_1''',\cdots,z_v''') = \aleph^{0,N:M_1,N_1,\cdots,M_v,N_v}_{P_i,Q_i,\tau_i;R:P_{i(1)},Q_{i(1)},\tau_{i(1)};R^{(1)};\cdots;P_{i(r)},Q_{i(v)};\tau_{i(v)};R^{(v)}} \left(\begin{array}{c} z'''_1 \\ \cdot \\ \cdot \\ \cdot \\ z'''_v \end{array} \right|$

$[(\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(v)})_{1,\mathfrak{n}}]$, $[\tau_i'(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(v)})_{\mathfrak{n}+1,p_i}]$:
.... , $[\tau_i'(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(v)})_{m+1,q_i}]$:

$$\left. \begin{array}{c} [(\mathrm{c}_j^{(1)});\gamma_j^{(1)})_{1,n_1}],[\tau'_{i^{(1)}}(c_{ji^{(1)}}^{(1)};\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;\ [(\mathrm{c}_j^{(v)});\gamma_j^{(v)})_{1,n_v}],[\tau'_{i^{(v)}}(c_{ji^{(v)}}^{(v)};\gamma_{ji^{(v)}}^{(v)})_{n_v+1,p_i^{(v)}}] \\ [(\mathrm{d}_j^{(1)});\delta_j^{(1)})_{1,m_1}],[\tau'_{i^{(1)}}(d_{ji^{(1)}}^{(1)};\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;\ [(\mathrm{d}_j^{(v)});\delta_j^{(v)})_{1,m_v}],[\tau'_{i^{(v)}}(d_{ji^{(v)}}^{(v)};\delta_{ji^{(v)}}^{(v)})_{m_v+1,q_i^{(v)}}] \end{array} \right)$$

$$= \frac{1}{(2\pi\omega)^v}\int_{L_1}\cdots\int_{L_v}\psi_1(s_1,\cdots,s_v)\prod_{k=1}^{v}\xi_k(s_k)z_k^{'''s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi_1(s_1,\cdots,s_v) = \frac{\prod_{j=1}^{N}\Gamma(1-a_j+\sum_{k=1}^{v}\alpha_j^{(k)}s_k)}{\sum_{i=1}^{R}[\tau_i'\prod_{j=N+1}^{P_i}\Gamma(a_{ji}-\sum_{k=1}^{v}\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{Q_i}\Gamma(1-b_{ji}+\sum_{k=1}^{v}\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\xi_k(s_k) = \frac{\prod_{j=1}^{M_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{N_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau'_{i^{(k)}}\prod_{j=M_k+1}^{Q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=N_k+1}^{P_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

Suppose , as usual , that the parameters

$a_j, j=1,\cdots,P; b_j, j=1,\cdots,Q;$

$c_j^{(k)}, j=1,\cdots,N_k; c_{ji^{(k)}}^{(k)}, j=N_k+1,\cdots,P_{i^{(k)}};$

$d_j^{(k)}, j=1,\cdots,M_k; d_{ji^{(k)}}^{(k)}, j=M_k+1,\cdots,Q_{i^{(k)}};$

with $k=1\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)}$

are complex numbers , and the $\alpha's, \beta's, \gamma's$ and $\delta's$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)}=\sum_{j=1}^{N}\alpha_j^{(k)}+\tau_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(k)}+\sum_{j=1}^{N_k}\gamma_j^{(k)}+\tau'_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}-\tau_i\sum_{j=1}^{Q_i}\beta_{ji}^{(k)}-\sum_{j=1}^{M_k}\delta_j^{(k)}$$

$$-\tau'_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}\leqslant 0 \tag{1.4}$$

The reals numbers τ_i are positives for $i=1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)}=1$ to $R^{(k)}$

The contour L_k is in the s_k-p lane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ is a real number with loop , if necessary ,ensure that the poles of $\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)$ with $j=1$ to M_k are separated from those of $\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j^{(k)}s_k)$ with $j=1$ to n and $\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)$ with $j=1$ to N_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz'''_k|<\frac{1}{2}A_i^{(k)}\pi$, where

$$A_i^{(k)}=\sum_{j=1}^{N}\alpha_j^{(k)}-\tau'_i\sum_{j=N+1}^{P_i}\alpha_{ji}^{(k)}-\tau'_i\sum_{j=1}^{Q_i}\beta_{ji}^{(k)}+\sum_{j=1}^{N_k}\gamma_j^{(k)}-\tau'_{i^{(k)}}\sum_{j=N_k+1}^{P_{i^{(k)}}}\gamma_{ji^{(k)}}^{(k)}$$

$$+\sum_{j=1}^{M_k}\delta_j^{(k)}-\tau'_{i^{(k)}}\sum_{j=M_k+1}^{Q_{i^{(k)}}}\delta_{ji^{(k)}}^{(k)}>0,\quad \text{with } k=1,\cdots,r, i=1,\cdots,R\ , i^{(k)}=1,\cdots,R^{(k)} \tag{1.5}$$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$\aleph(z'''_1,\cdots,z'''_v)=0(\,|z'''_1|^{\alpha_1},\cdots,|z'''_r|^{\alpha_r}\,), max(\,|z'''_1|,\cdots,|z'''_v|\,)\rightarrow 0$

$\aleph(z'''_1,\cdots,z'''_v)=0(\,|z'''_1|^{\beta_1},\cdots,|z'''_v|^{\beta_r}\,), min(\,|z'''_1|,\cdots,|z'''_v|\,)\rightarrow \infty$

where $k=1,\cdots,r:\alpha_k=min[Re(d_j^{(k)}/\delta_j^{(k)})], j=1,\cdots,m_k$ and

$\beta_k=max[Re((c_j^{(k)}-1)/\gamma_j^{(k)})], j=1,\cdots,n_k$

Serie representation of Aleph-function of $u-$variables is given by

$$\aleph(z_1''',\cdots,z_v''') = \sum_{G_1,\cdots,G_v=0}^{\infty}\sum_{g_1=0}^{M_1}\cdots\sum_{g_v=0}^{M_v}\frac{(-)^{G_1+\cdots+G_v}}{\delta_{g_1}G_1!\cdots\delta_{g_v}G_v!}\psi_1(\eta_{G_1,g_1},\cdots,\eta_{G_v,g_v})$$

$$\times\ \xi_1(\eta_{G_1,g_1})\cdots\xi_v(\eta_{G_v,g_v})z_1^{-\eta_{G_1,g_1}}\cdots z_v^{-\eta_{G_v,g_v}} \tag{1.6}$$

Where $\psi(.,\cdots,.)$, $\theta_i(.)$,$i=1,\cdots,r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1} = \frac{d_{g_1}^{(1)}+G_1}{\delta_{g_1}^{(1)}},\cdots,\ \eta_{G_v,g_v} = \frac{d_{g_v}^{(v)}+G_v}{\delta_{g_v}^{(v)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[d_j^i+p_i]\neq\delta_j^{(i)}[d_{g_i}^i+G_i]$ (1.7)
for $j\neq m_i, m_i=1,\cdots\eta_{G_i,g_i};p_i,n_i=0,1,2,\cdots,;y_i\neq 0,i=1,\cdots,v$

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)})_{1,p_r}:(a_j^{(1)},\alpha_j^{(1)})_{1,p^{(1)}};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ \\ (\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)})_{1,q_r}:(b_j^{(1)},\beta_j^{(1)})_{1,q^{(1)}};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array}\right) \tag{1.8}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.9}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i|<\frac{1}{2}\Omega_i\pi$, where

$$\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}+\left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right)+\cdots+$$

$$\left(\sum_{k=1}^{n_s}\alpha_{sk}^{(i)}-\sum_{k=n_s+1}^{p_s}\alpha_{sk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right) \tag{1.10}$$

where $i=1,\cdots,r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1,\cdots,z_r)=0(\,|z_1|^{\alpha_1},\cdots,|z_r|^{\alpha_r}\,)\,,max(\,|z_1|,\cdots,|z_r|\,)\rightarrow 0$$

$$I(z_1,\cdots,z_r)\;=0(\,|z_1|^{\beta_1},\cdots,|z_r|^{\beta_r}\,)\,,min(\,|z_1|,\cdots,|z_r|\,)\rightarrow\infty$$

where $k=1,\cdots,r:\alpha'_k=min[Re(b_j^{(k)}/\beta_j^{(k)})],j=1,\cdots,m_k$ and

$$\beta'_k=max[Re((a_j^{(k)}-1)/\alpha_j^{(k)})],j=1,\cdots,n_k$$

Condider a second multivariable I-function defined by Prasad [1]

$$I(z'_1,\cdots,z'_s)=\;I^{0,n'_2;0,n'_3;\cdots;0,n'_r:m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)}}_{p'_2,q'_2,p'_3,q'_3;\cdots;p'_s,q'_s:p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)}}\left(\begin{array}{c|c} z'_1 & (a'_{2j};\alpha'^{(1)}_{2j},\alpha'^{(2)}_{2j})_{1,p'_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z'_s & (b'_{2j};\beta'^{(1)}_{2j},\beta'^{(2)}_{2j})_{1,q'_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c}(a'_{sj};\alpha'^{(1)}_{sj},\cdots,\alpha'^{(s)}_{sj})_{1,p'_s}:(a'^{(1)}_j,\alpha'^{(1)}_j)_{1,p'^{(1)}};\cdots;(a'^{(s)}_j,\alpha'^{(s)}_j)_{1,p'^{(s)}} \\ \\ (b'_{sj};\beta'^{(1)}_{sj},\cdots,\beta'^{(s)}_{sj})_{1,q'_s}:(b'^{(1)}_j,\beta'^{(1)}_j)_{1,q'^{(1)}};\cdots;(b'^{(s)}_j,\beta'^{(s)}_j)_{1,q'^{(s)}}\end{array}\right) \tag{1.11}$$

$$=\frac{1}{(2\pi\omega)^s}\int_{L_1}\cdots\int_{L_s}\psi(t_1,\cdots,t_s)\prod_{i=1}^{s}\xi_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_s \tag{1.12}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [1]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

where $|argz'_i|<\frac{1}{2}\Omega'_i\pi$,

$$\Omega'_i=\sum_{k=1}^{n'^{(i)}}\alpha'^{(i)}_k-\sum_{k=n'^{(i)}+1}^{p'^{(i)}}\alpha'^{(i)}_k+\sum_{k=1}^{m'^{(i)}}\beta'^{(i)}_k-\sum_{k=m^{(i)}+1}^{q'^{(i)}}\beta'^{(i)}_k+\left(\sum_{k=1}^{n'_2}\alpha'^{(i)}_{2k}-\sum_{k=n_2+1}^{p'_2}\alpha'^{(i)}_{2k}\right)$$

$$+\cdots+\left(\sum_{k=1}^{n'_s}\alpha'_{sk}{}^{(i)}-\sum_{k=n'_s+1}^{p'_s}\alpha'_{sk}{}^{(i)}\right)-\left(\sum_{k=1}^{q'_2}\beta'_{2k}{}^{(i)}+\sum_{k=1}^{q'_3}\beta'_{3k}{}^{(i)}+\cdots+\sum_{k=1}^{q'_s}\beta'_{sk}{}^{(i)}\right) \tag{1.13}$$

where $i=1,\cdots,s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z'_1,\cdots,z'_s)=0(\,|z'_1|^{\alpha'_1},\cdots,|z'_s|^{\alpha'_s}\,),max(\,|z'_1|,\cdots,|z'_s|\,)\to 0$$

$$I(z'_1,\cdots,z'_s)\;=0(\,|z'_1|^{\beta'_1}\;,\cdots,|z'_s|^{\beta'_s}\,),min(\,|z'_1|,\cdots,|z'_s|\,)\to\infty$$

where $k=1,\cdots,z:\alpha''_k=min[Re(b'^{(k)}_j/\beta'^{(k)}_j)],j=1,\cdots,m'_k$ and

$$\beta''_k=max[Re((a'^{(k)}_j-1)/\alpha'^{(k)}_j)],j=1,\cdots,n'_k$$

where $k=1,\cdots,z:\alpha''_k=min[Re(b'^{(k)}_j/\beta'^{(k)}_j)],j=1,\cdots,m'_k$ and

$$\beta''_k=max[Re((a'^{(k)}_j-1)/\alpha'^{(k)}_j)],j=1,\cdots,n'_k$$

Srivastava and Garg [5] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]=\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \tag{1.14}$$

The coefficients are $B[E;R_1,\ldots,R_v]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j),j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j),j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \tag{2.2}$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[4,page 454] given by :

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j=L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [2, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [4, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}},\zeta_j^{(i)}>0(i=1,\cdots,r)\,;\,\theta_i'=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}},\zeta_j'^{(i)}>0(i=1,\cdots,s)$$

$$\theta_i''=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j''^{(i)}},\zeta_j''^{(i)}>0(i=1,\cdots,u)$$

$$\theta_i'''=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}},\zeta_j'''^{(i)}>0(i=1,\cdots,v) \tag{3.1}$$

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};p_2',q_2';p_3',q_3';\cdots;p_{s-1}',q_{s-1}';0,0;\cdots;0,0;0,0;\cdots;0,0 \tag{3.2}$$

$$V=0,n_2;0,n_3;\cdots;0,n_{r-1};0,n_2';0,n_3';\cdots;0,n_{s-1}';0,0;\cdots;0,0;0,0;\cdots;0,0 \tag{3.3}$$

$$X=m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)};m'^{(1)},n'^{(1)};\cdots;m'^{(s)},n'^{(s)};1,0;\cdots;1,0;1,0;\cdots;1,0 \tag{3.4}$$

$$Y=p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)};p'^{(1)},q'^{(1)};\cdots;p'^{(s)},q'^{(s)};0,1;\cdots;0,1;0,1;\cdots;0,1 \tag{3.5}$$

$$A=(a_{2k};\alpha_{2k}^{(1)},\alpha_{2k}^{(2)});\cdots;(a_{(r-1)k};\alpha_{(r-1)k}^{(1)},\alpha_{(r-1)k}^{(2)},\cdots,\alpha_{(r-1)k}^{(r-1)})\,;(a_{2k}';\alpha_{2k}'^{(1)},\alpha_{2k}'^{(2)});\cdots;$$

$$(a_{(s-1)k}';\alpha_{(s-1)k}'^{(1)},\alpha_{(s-1)k}'^{(2)},\cdots,\alpha_{(s-1)k}'^{(s-1)}) \tag{3.6}$$

$$B=(b_{2k};\beta_{2k}^{(1)},\beta_{2k}^{(2)});\cdots;(b_{(r-1)k};\beta_{(r-1)k}^{(1)},\beta_{(r-1)k}^{(2)},\cdots,\beta_{(r-1)k}^{(r-1)})\,;(b_{2k}';\beta_{2k}'^{(1)},\beta_{2k}'^{(2)});\cdots;$$

$$(b_{(s-1)k}';\beta_{(s-1)k}'^{(1)},\beta_{(s-1)k}'^{(2)},\cdots,\beta_{(s-1)k}'^{(s-1)}) \tag{3.7}$$

$$\mathfrak{A}=(a_{rk};\alpha_{rk}^{(1)},\alpha_{rk}^{(2)},\cdots,\alpha_{rk}^{(r)},0,\cdots,0,0\cdots,0,0,\cdots,0) \tag{3.8}$$

$$\mathfrak{A}'=(a_{sk}';0,\cdots,0,\alpha_{sk}'^{(1)},\alpha_{sk}'^{(2)},\cdots,\alpha_{sk}'^{(s)},0,\cdots,0,0,\cdots,0) \tag{3.9}$$

$$\mathfrak{B}=(b_{rk};\beta_{rk}^{(1)},\beta_{rk}^{(2)},\cdots,\beta_{rk}^{(r)},0,\cdots,0,0,\cdots,0,0,\cdots,0) \tag{3.10}$$

$$\mathfrak{B}'=(b_{sk}';0,\cdots,0,\beta_{sk}'^{(1)},\beta_{sk}'^{(2)},\cdots,\beta_{sk}'^{(s)},0,\cdots,0,0,\cdots,0) \tag{3.11}$$

$$\mathfrak{A}_1=(a_k^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k^{(r)},\alpha_k^{(r)})_{1,p^{(r)}};(a_k'^{(1)},\alpha_k^{(1)})_{1,p^{(1)}};\cdots;(a_k'^{(s)},\alpha_k'^{(s)})_{1,p'^{(s)}};$$

$$(1,0);\cdots;(1,0);(1.0);\cdots;(1.0) \tag{3.12}$$

$$\mathfrak{B}_1 = (b_k^{(1)}, \beta_k^{(1)})_{1,q^{(1)}}; \cdots ; (b_k^{(r)}, \beta_k^{(r)})_{1,q^{(r)}}; (b_k'^{(1)}, \beta_k'^{(1)})_{1,q'^{(1)}}; \cdots ; (b_k'^{(s)}, \beta_k'^{(s)})_{1,q'^{(s)}};$$
$$(0,1); \cdots ; (0,1); (0,1); \cdots ; (0,1) \quad (3.13)$$

$$K_1 = (1 - \alpha - \sum_{i=1}^{u} R_i a_i - \sum_{i=1}^{v} \eta_{G_i,g_i} a_i'; \mu_1, \cdots, \mu_r, \mu_1', \cdots, \mu_s', h_1, \cdots, h_l, 1, \cdots, 1) \quad (3.14)$$

$$K_2 = (1 - \beta - \sum_{i=1}^{u} R_i b_i - \sum_{i=1}^{v} \eta_{G_i,g_i} b_i'; \rho_1, \cdots, \rho_r, \rho_1', \cdots, \rho_s', 0, \cdots, 0, 0 \cdots, 0) \quad (3.15)$$

$$K_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \zeta_j'''^{(i)}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)},$$
$$0, \cdots, \underset{\text{j}}{1}, \cdots, 0, 0 \cdots, 0]_{1,l} \quad (3.16)$$

$$K_j' = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \lambda_j'''^{(i)}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)},$$
$$0, \cdots, 0, 0 \cdots, \underset{\text{j}}{1}, \cdots, 0]_{1,k} \quad (3.17)$$

$$L_1 = (1 - \alpha - \beta - \sum_{i=1}^{u} R_i(a_i + b_i) - \sum_{i=1}^{v} (a_i' + b_i') \eta_{G_i,g_i}; \mu_1 + \rho_1, \cdots, \mu_r + \rho_r, \mu_1' + \rho_1', \cdots, \mu_r' + \rho_r',$$
$$h_1, \cdots, h_l, 1, \cdots, 1) \quad (3.18)$$

$$L_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{s} \zeta_j'''^{(i)} \eta_{G_i,g_i}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)}, 0, \cdots, 0, 0 \cdots, 0]_{1,l} \quad (3.19)$$

$$L_j' = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \lambda_j'''^{(i)} \eta_{G_i,g_i}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0]_{1,k} \quad (3.20)$$

$$P_1 = (b-a)^{\alpha+\beta-1} \left\{ \prod_{j=1}^{h} (af_j + g_j)^{\sigma_j} \right\} \quad (3.21)$$

$$B_{u,v} = (b-a)^{\sum_{i=1}^{v}(a_i'+b_i')\eta_{G_i,g_i} + \sum_{i=1}^{u}(a_i+b_i)R_i} \left\{ \prod_{j=1}^{h} (af_j + g_j)^{-\sum_{i=1}^{v} \lambda_i''' \eta_{g_i,h_i} - \sum_{i=1}^{u} \lambda_i'' R_i} \right\} G_v \quad (3.22)$$

where $G_v = \psi(\eta_{G_1,g_1}, \cdots, \eta_{G_v,g_v}) \times \ \xi_1(\eta_{G_1,g_1}) \cdots \xi_v(\eta_{G_v,g_v})$

$\psi_1, \xi_i, i = 1, \cdots, v$ are defined respectively by (1.2) and (1.3)

$$B_u = \frac{(-L)_{h_1R_1+\cdots+h_uR_u} B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!} \tag{3.23}$$

We have the general Eulerian integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c} \mathrm{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{array}\right)$$

$$\aleph\left(\begin{array}{c} \mathrm{z}_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{i=1}^{v} z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u} z''^{R_k} B_u B_{u,v}$$

$$I^{V;0,n_r+n'_s+l+k+2;X}_{U:p_r+p'_s+l+k+2,q_r+q'_s+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A}\ ;\ \mathrm{K}_1, K_2, K_j, K'_j, \mathfrak{A} : A' \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(s)}_j}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \tau_l(b-a)^{h_l} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \mathrm{B}\ ;\ \mathrm{L}_1, L_j, L'_j : D_1, \mathfrak{B} : B' \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (3.24)$$

We obtain the I-function of $r+s+k+l$ variables. The quantities $U, V, X, Y, A, B, K_1, K_2, K_j, K'_j, \mathfrak{A}, \mathfrak{A}', \mathfrak{A}_1, L_1, L_j, L'_j,$ $\mathfrak{B}, \mathfrak{B}', P_1, B_u, B_{u,v}$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a, b \in \mathbb{R}(a < b); \mu_i, \mu'_u, \rho_i, \rho'_u, \lambda_j^{(i)}, \lambda_j'^{(u)}, h_v \in \mathbb{R}^+$, $f_i, g_j, \tau_v, \sigma_j, \lambda_v \in \mathbb{C}$ $(i = 1, \cdots, r; j = 1, \cdots; k;$

$u = 1, \cdots, s; v = 1, \cdots, l), a_i, b_i, \lambda_j''^{(i)}, \zeta_j''^{(i)} \in \mathbb{R}^+, (i = 1, \cdots, u; j = 1, \cdots, k)$

$a'_i, b'_i, \lambda_j'''^{(i)}, \zeta_j'''^{(i)} \in \mathbb{R}^+, (i = 1, \cdots, v; j = 1, \cdots, k)$

(B) $a_{ij}, b_{ik}, \in \mathbb{C}\ (i = 1, \cdots, r; j = 1, \cdots, p_i; k = 1, \cdots, q_i); a_j^{(i)}, b_j^{(k)} \in \mathbb{C}$

$(i = 1, \cdots, r; j = 1, \cdots, p^{(i)}; k = 1, \cdots, q^{(i)})$

$a'_{ij}, b'_{ik}, \in \mathbb{C}\ (i = 1, \cdots, s; j = 1, \cdots, p'_i; k = 1, \cdots, q'_i); a'^{(i)}_j, b'^{(k)}_j, \in \mathbb{C}$

$(i = 1, \cdots, r; j = 1, \cdots, p'^{i)}; k = 1, \cdots, q'^{(i)})$

$\alpha_{ij}^{(k)}, \beta_{ij}^{(k)} \in \mathbb{R}^+\ ((i = 1, \cdots, r, j = 1, \cdots, p_i, k = 1, \cdots, r)\ ; \alpha_j^{(i)}, \beta_i^{(i)} \in \mathbb{R}^+\ (i = 1, \cdots, r; j = 1, \cdots, p_i)$

$\alpha'^{(k)}_{ij}, \beta'^{(k)}_{ij} \in \mathbb{R}^+\ ((i = 1, \cdots, s, j = 1, \cdots, p'_i, k = 1, \cdots, s)\ ; \alpha'^{(i)}_j, \beta'^{(i)}_i \in \mathbb{R}^+\ (i = 1, \cdots, s; j = 1, \cdots, p'_i)$

(C) $\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1$

(D) $Re\big[\alpha+\sum_{j=1}^{v}a'_j\min_{1\leqslant k\leqslant M_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{r}\mu_j\min_{1\leqslant k\leqslant m^{(i)}}\frac{b_k^{(j)}}{\beta_k^{(j)}}+\sum_{j=1}^{s}\mu'_i\min_{1\leqslant k\leqslant m'^{(i)}}\frac{b'^{(j)}_k}{\beta'^{(j)}_k}\big]>0$

$$Re\big[\beta+\sum_{j=1}^{v}b'_j\min_{1\leqslant k\leqslant M_i}\frac{d_k^{(j)}}{\delta_k^{(j)}}+\sum_{j=1}^{r}\rho_j\min_{1\leqslant k\leqslant m^{(i)}}\frac{b_k^{(j)}}{\beta_k^{(j)}}+\sum_{j=1}^{s}\rho'_j\min_{1\leqslant k\leqslant m'^{(i)}}\frac{b'^{(j)}_k}{\beta'^{(j)}_k}\big]>0$$

(E) $Re\left(\alpha+\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i+\sum_{i=1}^{u}R_ia_i+\sum_{i=1}^{r}\mu_is_i+\sum_{i=1}^{s}t_i\mu'_i\right)>0$

$$Re\left(\beta+\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i+\sum_{i=1}^{u}R_ib_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}_j+\sum_{i=1}^{u}R_i\lambda''^{(i)}_j+\sum_{i=1}^{r}s_i\zeta_j^{(i)}+\sum_{i=1}^{s}t_i\zeta'^{(i)}_j\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}+\sum_{i=1}^{u}R_i\lambda''^{(i)}_j+\sum_{i=1}^{r}s_i\lambda_j^{(i)}+\sum_{i=1}^{s}t_i\lambda'^{(i)}_j\right)>0(j=1,\cdots,k);$$

(F) $\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta_k^{(i)}+\left(\sum_{k=1}^{n_2}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}\alpha_{2k}^{(i)}\right)+\cdots+$

$$\left(\sum_{k=1}^{n_s}\alpha_{sk}^{(i)}-\sum_{k=n_s+1}^{p_s}\alpha_{sk}^{(i)}\right)-\left(\sum_{k=1}^{q_2}\beta_{2k}^{(i)}+\sum_{k=1}^{q_3}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{q_s}\beta_{sk}^{(i)}\right)-\mu_i-\rho_i$$

$$-\sum_{l=1}^{k}\lambda_j^{(i)}-\sum_{l=1}^{l}\zeta_j^{(i)}>0\quad(i=1,\cdots,r)$$

$$\Omega'_i=\sum_{k=1}^{n'^{(i)}}\alpha'^{(i)}_k-\sum_{k=n'^{(i)}+1}^{p'^{(i)}}\alpha'^{(i)}_k+\sum_{k=1}^{m'^{(i)}}\beta'^{(i)}_k-\sum_{k=m^{(i)}+1}^{q'^{(i)}}\beta'^{(i)}_k+\left(\sum_{k=1}^{n'_2}\alpha'^{(i)}_{2k}-\sum_{k=n_2+1}^{p'_2}\alpha'^{(i)}_{2k}\right)+$$

$$\cdots+\left(\sum_{k=1}^{n'_s}\alpha'^{(i)}_{sk}-\sum_{k=n'_s+1}^{p'_s}\alpha'^{(i)}_{sk}\right)-\left(\sum_{k=1}^{q'_2}\beta'^{(i)}_{2k}+\sum_{k=1}^{q'_3}\beta'^{(i)}_{3k}+\cdots+\sum_{k=1}^{q'_s}\beta'^{(i)}_{sk}\right)-\mu'_i-\rho'_i$$

$$-\sum_{l=1}^{k}\lambda'^{(i)}_j-\sum_{l=1}^{l}\zeta'^{(i)}_j>0\quad(i=1,\cdots,s)$$

(G) $\left|arg\left(z_i\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(i)}}\right)\right|<\frac{1}{2}\Omega_i\pi\quad(a\leqslant t\leqslant b;i=1,\cdots,r)$

$$\left|arg\left(z'_i\prod_{j=1}^{l}\left[1-\tau'_j(t-a)^{h'_i}\right]^{-\zeta'^{(i)}_j}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(i)}_j}\right)\right|<\frac{1}{2}\Omega'_i\pi\quad(a\leqslant t\leqslant b;i=1,\cdots,s)$$

(H) The multiple series occuring on the right-hand side of (3.24) is absolutely and uniformly convergent.

Proof

To prove (3.24), first, we express in serie the multivariable Aleph-function with the help of (1.6), a class of multivariable polynomials defined by Srivastava et al [5] $S_L^{h_1,\cdots,h_u}[.]$ in serie with the help of (1.14) and we interchange the order of summations and t-integral (which is permissible under the conditions stated). Expressing the I-functions of r-variables and s-variables defined by Prasad [1] in terms of Mellin-Barnes type contour integral with the help of (1.9) and (1.21) respectively and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. . Now collect the power of $\left[1-\tau_j(t-a)^{h_i}\right]$ with $(i=1,\cdots,r;j=1,\cdots,l)$ and collect the power of (f_jt+g_j) with $j=1,\cdots,k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integral in multivariable I-function defined by Prasad [1], we obtain the equation (3.24).

Remarks

If a) $\rho_1=\cdots,\rho_r=\rho'_1=\cdots,\rho'_s=0$; b) $\mu_1=\cdots,\mu_r=\mu'_1=\cdots,\mu'_s=0$, we obtain the similar formulas that (3.24) with the corresponding simplifications.

4. Particular cases

a) If $U=V=A=B=0$, **the multivariable I-function defined by Prasad reduces to multivariable H-function defined by Srivastava et al [7] and we obtain :**

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\left(\begin{array}{c}z''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda''^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ z''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda''^{(u)}_j}\end{array}\right)$$

$$\aleph\left(\begin{array}{c}z'''_1\theta'''_1(t-a)^{a'_1}(b-t)^{b'_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda'''^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ z'''_v\theta'''_v(t-a)^{a'_v}(b-t)^{b'_v}\prod_{j=1}^k(f_jt+g_j)^{-\lambda'''^{(v)}_j}\end{array}\right)$$

$$H\left(\begin{array}{c}z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda^{(r)}_j}\end{array}\right)$$

$$H\left(\begin{array}{c} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$= P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{i=1}^{v}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}B_uB_{u,v}$$

$$H_{p_r+p'_s+l+k+2,q_r+q'_s+l+k+1;Y}^{0,n_r+n'_s+l+k+2;X}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{K}_1,K_2,K_j,K'_j,\mathfrak{A}:A' \\ \cdots & . \\ \cdots & . \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & . \\ \tau_1(b-a)^{h_1} & . \\ \cdots & . \\ \cdots & . \\ \tau_l(b-a)^{h_l} & . \\ \frac{(b-a)f_1}{af_1+g_1} & . \\ \cdots & \mathrm{L}_1,L_j,L'_j:D_1,\mathfrak{B}:B' \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (4.1)$$

under the same notations and conditions that (3.24) with $U=V=A=B=0$

b) If $B(L;R_1,\cdots,R_u) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{R_1\psi'_j+\cdots+R_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ (4.2)

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [4]. We have the following integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$F^{1+\bar{A}:B';\cdots;B^{(u)}}_{\bar{C}:D';\cdots;D^{(u)}}\left(\begin{array}{c} \mathrm{z}''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda''^{(u)}_j} \end{array}\right.$$

$$\left.\begin{array}{c} [(\text{-L});\mathrm{R}_1,\cdots,R_u][(a);\theta',\cdots,\theta^{(u)}]:[(b');\phi'];\cdots;[(b^{(u)});\phi^{(u)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(u)}]:[(d');\delta'];\cdots;[(d^{(u)});\delta^{(u)}] \end{array}\right)$$

$$\aleph\left(\begin{array}{c} \mathrm{z}'''_1\theta'''_1(t-a)^{a'_1}(b-t)^{b'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'''^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'''_v\theta'''_v(t-a)^{a'_v}(b-t)^{b'_v}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'''^{(v)}_j} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda^{(r)}_j} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(1)}_j} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(s)}_j} \end{array}\right)\mathrm{d}t$$

$$=P_1\sum_{h_1=1}^{M_1}\cdots\sum_{h_v=1}^{M_v}\sum_{k_1=0}^{\infty}\cdots\sum_{k_v=0}^{\infty}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{i=1}^{v}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}B'_uB_{u,v}$$

$$I_{U:p_r+p'_s+l+k+2,q_r+q'_s+l+k+1;Y}^{V;0,n_r+n'_s+l+k+2;X}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(1)}}} & \mathrm{A} ; \mathrm{K}_1, K_2, K_j, K'_j, \mathfrak{A} : A' \\ \cdot\cdot\cdot & \cdot \\ \cdot\cdot\cdot & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^k(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^k(af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot \\ \cdot\cdot\cdot & \cdot \\ \cdot\cdot\cdot & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^k(af_j+g_j)^{\lambda'^{(s)}_j}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdot\cdot\cdot & \cdot \\ \cdot\cdot\cdot & \cdot \\ \tau_l(b-a)^{h_l} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdot\cdot\cdot & \mathrm{B} ; \mathrm{L}_1, L_j, L'_j : D_1, \mathfrak{B} : B' \\ \cdot\cdot\cdot & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad \textbf{(4.3)}$$

under the same conditions and notations that (3.24)

where $B'_u = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$, $B[E;R_1,\ldots,R_v]$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable I-functions defined by Prasad [1] and a class of multivariable polynomials defined by Srivastava et al [5].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable I-functions defined by Prasad [1],a expansion of multivariable Aleph-function and a class of multivariable polynomials defined by Srivastava et al [5] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES

[1] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.

[2] Saigo M. and Saxena R.K. Unified fractional integral formulas for the multivariable H-function I. J.Fractional Calculus 15 (1999), page 91-107.

[3] Sharma C.K.and Ahmad S.S.: On the multivariable I-function. Acta ciencia Indica Math , 1994 vol 20,no2, p 113-116.

[4] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[5] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function. Rev. Roumaine Phys. 32(1987), page 685-692.

[6] Srivastava H.M. and Karlsson P.W. Multiple Gaussian Hypergeometric series. Ellis.Horwood. Limited. New-York, Chichester. Brisbane. Toronto , 1985.

[7] H.M. Srivastava and R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

Personal adress : 411 Avenue Joseph Raynaud
Le parc Fleuri , Bat B
83140 , Six-Fours les plages
Tel : 06-83-12-49-68
Department : VAR
Country : FRANCE