Eulerian integral associated with product of two multivariable Kumari's I-functions and classes of polynomials

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ABSTRACT

The present paper is evaluated a new Eulerian integral associated with the product of two multivariable I-functions defined by Kumari [2] a generalized Lauricella function and a classes of multivariable polynomials with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [8] and Srivastava-Daoust polynomial [5].

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable H-function, generalized hypergeometric function, class of polynomials

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1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of two multivariable I-functions defined by Kumari et al [2] , and classes of polynomials with general arguments.

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) \;=\; I^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r} \left(\begin{array}{c} z_1 \\ . \\ . \\ . \\ z_r \end{array} \middle| \begin{array}{l} (a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p}: \\[2ex] (b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q}: \end{array} \right.$$

$$(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r}$$

$$\left. (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r} \right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1,\cdots,s_r) \prod_{i=1}^{r} \theta_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.2}$$

where $\phi(s_1,\cdots,s_r), \theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{n} \Gamma^{A_j}\left(1-aj+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=n+1}^{p} \Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right) \prod_{j=1}^{q} \Gamma^{B_j}\left(1-bj+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right) \prod_{j=1}^{m_i} \Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right) \prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\delta_j^{(i)}s_i\right)} \tag{1.4}$$

For more details, see Nambisan et al [2].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i = \sum_{j=1}^{p} A_j \alpha_j^{(i)} - \sum_{j=1}^{q} B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)} \delta_j^{(i)} \leqslant 0, i = 1, \cdots, r \tag{1.5}$$

The integral (2.1) converges absolutely if

$|arg(z_k)| < \frac{1}{2} \Delta_k \pi, k = 1, \cdots, r$ where

$$\Delta_k = - \sum_{j=n+1}^{p} A_j \alpha_j^{(k)} - \sum_{j=1}^{q} B_j \beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} > 0 \tag{1.6}$$

Consider the second multivariable I-function.

$$I(z'_1, \cdots, z'_s) \ = \ I_{p',q':p'_1,q'_1;\cdots;p'_s,q'_s}^{0,n':m'_1,n'_1;\cdots;m'_s,n'_s} \left(\begin{array}{c} z'_1 \\ . \\ . \\ . \\ z'_s \end{array} \left| \begin{array}{c} (a'_j; \alpha'^{(1)}_j, \cdots, \alpha'^{(s)}_j; A'_j)_{1,p'} : \\ \\ \\ (b'_j; \beta'^{(1)}_j, \cdots, \beta'^{(s)}_j; B'_j)_{1,q'} : \end{array} \right. \right.$$

$$(c'^{(1)}_j, \gamma'^{(1)}_j; C'^{(1)}_j)_{1,p'_1}; \cdots; (c'^{(s)}_j, \gamma'^{(s)}_j; C'^{(s)}_j)_{1,p'_s}$$

$$\left. (d'^{(1)}_j, \delta'^{(1)}_j; D'^{(1)}_j)_{1,q'_1}; \cdots; (d'^{(s)}_j, \delta'^{(s)}_j; D'^{(s)}_j)_{1,q'_s} \right) \tag{1.7}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L'_1} \cdots \int_{L'_s} \psi(t_1, \cdots, t_s) \prod_{i=1}^{s} \xi_i(t_i) z_i^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.8}$$

where $\psi(t_1, \cdots, t_s), \xi_i(s_i), i = 1, \cdots, s$ are given by :

$$\psi(t_1, \cdots, t_s) = \frac{\prod_{j=1}^{n'} \Gamma^{A'_j} \left(1 - a'_j + \sum_{i=1}^{s} \alpha'^{(i)}_j t_j\right)}{\prod_{j=n'+1}^{p'} \Gamma^{A'_j} \left(a'_j - \sum_{i=1}^{s} \alpha'^{(i)}_j t_j\right) \prod_{j=1}^{q'} \Gamma^{B'_j} \left(1 - b'_j + \sum_{i=1}^{s} \beta'^{(i)}_j t_j\right)} \tag{1.9}$$

$$\xi_i(s_i) = \frac{\prod_{j=1}^{n'_i} \Gamma^{C'^{(i)}_j} \left(1 - c'^{(i)}_j + \gamma'^{(i)}_j t_i\right) \prod_{j=1}^{m'_i} \Gamma^{D'^{(i)}_j} \left(d'^{(i)}_j - \delta'^{(i)}_j t_i\right)}{\prod_{j=n'_i+1}^{p'_i} \Gamma^{C'^{(i)}_j} \left(c'^{(i)}_j - \gamma'^{(i)}_j t_i\right) \prod_{j=m'_i+1}^{q'_i} \Gamma^{D'^{(i)}_j} \left(1 - d'^{(i)}_j + \delta'^{(i)}_j t_i\right)} \tag{1.10}$$

For more details, see Nambisan et al [2].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U'_i = \sum_{j=1}^{p'} A'_j \alpha'^{(i)}_j - \sum_{j=1}^{q'} B'_j \beta'^{(i)}_j + \sum_{j=1}^{p'_i} C'^{(i)}_j \gamma'^{(i)}_j - \sum_{j=1}^{q'_i} D'^{(i)}_j \delta'^{(i)}_j \leqslant 0, i = 1, \cdots, s \tag{1.11}$$

The integral (2.1) converges absolutely if

where $|arg(z'_k)| < \frac{1}{2} \Delta'_k \pi, k = 1, \cdots, s$

$$\Delta'_k = - \sum_{j=n'+1}^{p'} A'_j \alpha'^{(k)}_j - \sum_{j=1}^{q'} B'_j \beta'^{(k)}_j + \sum_{j=1}^{m'_k} D'^{(k)}_j \delta'^{(k)}_j - \sum_{j=m'_k+1}^{q'_k} D'^{(k)}_j \delta'^{(k)}_j + \sum_{j=1}^{n'_k} C'^{(k)}_j \gamma'^{(k)}_j - \sum_{j=n'_k+1}^{p'_k} C'^{(k)}_j \gamma'^{(k)}_j > 0 \quad (1.12)$$

The generalized polynomials of multivariables defined by Srivastava [4], is given in the following manner :

$$S^{\mathfrak{M}_1,\cdots,\mathfrak{M}_\mathfrak{v}}_{N_1,\cdots,N_v}[y_1,\cdots,y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_\mathfrak{v}]} \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \frac{(-N_v)_{\mathfrak{M}_\mathfrak{v} K_v}}{K_v!}$$

$$A[N_1,K_1;\cdots;N_v,K_v]y_1^{K_1}\cdots y_v^{K_v} \quad (1.13)$$

where $\mathfrak{M}_1,\cdots,\mathfrak{M}_\mathfrak{v}$ are arbitrary positive integers and the coefficients $A[N_1,K_1;\cdots;N_v,K_v]$ are arbitrary constants, real or complex.

Srivastava and Garg [6] introduced and defined a general class of multivariable polynomials as follows

$$S^{h_1,\cdots,h_u}_L[z_1,\cdots,z_u] = \sum_{R_1,\cdots,R_u=0}^{h_1 R_1+\cdots h_u R_u \leqslant L} (-L)_{h_1 R_1+\cdots+h_u R_u} B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \quad (1.14)$$

The coefficients are $B[E;R_1,\ldots,R_v]$ arbitrary constants, real or complex.

We will note $a_v = \dfrac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \cdots \dfrac{(-N_v)_{\mathfrak{M}_\mathfrak{v} K_v}}{K_v!} A[N_1,K_1;\cdots;N_v,K_v]$ and

$$b_u = \frac{(-E)_{F_1 L_1+\cdots+F_u L_u} B(E;L_1,\cdots,L_u)}{L_1!\cdots L_u!} \quad (1.15)$$

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [7 ,page 39 eq .30]

$$\frac{\prod_{j=1}^P \Gamma(A_j)}{\prod_{j=1}^Q \Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^P \Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^Q \Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (2.1)$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j), j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^l \left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^k (f_j t+g_j)^{\sigma_j}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^k (af_j+g_j)^{\sigma_j}$$

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1} \left(\begin{array}{c} (\alpha : h_1, \cdots, h_l, 1, \cdots, 1) : (\lambda_1 : 1), \cdots, (\lambda_l : 1); (-\sigma_1 : 1), \cdots, (-\sigma_k : 1) \\ \cdots \\ (\alpha + \beta : h_1, \cdots, h_l, 1, \cdots, 1) : -, \cdots, -; -, \cdots, - \end{array} \right.$$

$$\left. ; \tau_1(b-a)^{h_1}, \cdots, \tau_l(b-a)^{h_l}, -\frac{(b-a)f_1}{af_1+g_1}, \cdots, -\frac{(b-a)f_k}{af_k+g_k} \right) \tag{2.2}$$

where $a, b \in \mathbb{R}(a < b), \alpha, \beta, f_i, g_i, \sigma_i, \tau_j, h_j \in \mathbb{C}, \lambda_j \in \mathbb{R}^+(i = 1, \cdots, k; j = 1, \cdots, l)$

$$min(Re(\alpha), Re(\beta)) > 0, \max_{1 \leqslant j \leqslant l} \left\{ \left| \tau_j(b-a)^{h_j} \right| \right\} < 1, \max_{1 \leqslant j \leqslant k} \left\{ \left| \frac{(b-a)f_i}{af_i+g_i} \right| \right\} < 1,$$

and $F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[3,page 454] given by :

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1} \left(\begin{array}{c} (\alpha : h_1, \cdots, h_l, 1, \cdots, 1) : (\lambda_1 : 1), \cdots, (\lambda_l : 1); (-\sigma_1 : 1), \cdots, (-\sigma_k : 1) \\ \cdots \\ (\alpha + \beta : h_1, \cdots, h_l, 1, \cdots, 1) : -, \cdots, -; -, \cdots, - \end{array} \right.$$

$$\left. ; \tau_1(b-a)^{h_1}, \cdots, \tau_l(b-a)^{h_l}, -\frac{(b-a)f_1}{af_1+g_1}, \cdots, -\frac{(b-a)f_k}{af_k+g_k} \right) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}} \int_{L_1} \cdots \int_{L_{l+k}} \frac{\Gamma\left(\alpha + \sum_{j=1}^{l} h_j s_j + \sum_{j=1}^{k} s_{l+j}\right)}{\Gamma\left(\alpha + \beta + \sum_{j=1}^{l} h_j s_j + \sum_{j=1}^{k} s_{l+j}\right)} \prod_{j=1}^{l} \Gamma(\lambda_j + s_j) \prod_{j=1}^{k} \Gamma(-\sigma_j + s_{l+j})$$

$$\prod_{j=1}^{l+k} \Gamma(-s_j) z_1^{s_1} \cdots z_l^{s_l} z_{l+1}^{s_{l+1}} \cdots, z_{l+k}^{s_{l+k}} \, \mathrm{d}s_1 \cdots \mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j = L_{w\zeta_j\infty}(Re(\zeta_j) = v_j'')$ starting at the point $v_j'' - \omega\infty$ and terminating at the point $v_j'' + \omega\infty$ with $v_j'' \in \mathbb{R}(j = 1, \cdots, l)$ and each of the remaining contour $L_{l+1}, \cdots, L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l} \left[1 - \tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha} = \sum_{r=0}^{\infty} \frac{(\alpha)_r}{r!} z^r (|z| < 1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [2, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [7, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i = \prod_{j=1}^{l} \left[1 - \tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}}, \zeta_j^{(i)} > 0 (i = 1, \cdots, r) \; ; \theta_i' = \prod_{j=1}^{l} \left[1 - \tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}}, \zeta_j'^{(i)} > 0 (i = 1, \cdots, s)$$

$$\theta_i'' = \prod_{j=1}^{l} \left[1 - \tau_j(t-a)^{h_i} \right]^{-\zeta_j''^{(i)}}, \zeta_j''^{(i)} > 0 (i = 1, \cdots, u)$$

$$\theta_i''' = \prod_{j=1}^{l} \left[1 - \tau_j(t-a)^{h_i} \right]^{-\zeta_j'''^{(i)}}, \zeta_j'''^{(i)} > 0 (i = 1, \cdots, v) \tag{3.1}$$

$$X = m_1, n_1; \cdots; m_r, n_r; m_1', n_1'; \cdots; m_s', n_s'; 1, 0; \cdots; 1, 0; 1, 0; \cdots; 1, 0 \tag{3.2}$$

$$Y = p_1, q_1; \cdots; p_r, q_r; p_1', q_1'; \cdots; p_s', q_s'; 0, 1; \cdots; 0, 1; 0, 1; \cdots; 0, 1 \tag{3.3}$$

$$A = (a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}, 0, \cdots, 0, 0, \cdots, 0, 0, \cdots, 0; A_j)_{1,p} \tag{3.4}$$

$$B = (b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}, 0 \cdots, 0, 0 \cdots, 0, 0 \cdots, 0; B_j)_{1,q} \tag{3.5}$$

$$A' = (a_j'; 0, \cdots, 0, \alpha_j'^{(1)}, \cdots, \alpha_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0; A_j')_{1,p'} \tag{3.6}$$

$$B' = (b_j'; 0, \cdots, 0, \beta_j'^{(1)}, \cdots, \beta_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0; B_j')_{1,q'} \tag{3.7}$$

$$C = (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r}; (c_j'^{(1)}, \gamma_j'^{(1)}; C_j'^{(1)})_{1,p_1'}; \cdots; (c_j'^{(r)}, \gamma_j^{(s)}; C_j'^{(s)})_{1,p_s'}$$

$$(1, 0; 1); \cdots; (1, 0; 1); (1, 0; 1); \cdots; (1, 0; 1) \tag{3.8}$$

$$D = (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r}; (d_j'^{(1)}, \delta_j'^{(1)}; D_j'^{(1)})_{1,q_1'}; \cdots; (d_j'^{(s)}, \delta_j'^{(s)}; D_j'^{(s)})_{1,q_s'};$$

$$(0, 1; 1); \cdots; (0, 1; 1); (0, 1; 1); \cdots; (0, 1; 1) \tag{3.9}$$

$$K_1 = (1 - \alpha - \sum_{i=1}^{u} R_i a_i - \sum_{i=1}^{v} K_i a_i'; \mu_1, \cdots, \mu_r, \mu_1', \cdots, \mu_s', h_1, \cdots, h_l, 1, \cdots, 1) \tag{3.10}$$

$$K_2 = (1 - \beta - \sum_{i=1}^{u} R_i b_i - \sum_{i=1}^{v} K_i b_i'; \rho_1, \cdots, \rho_r, \rho_1', \cdots, \rho_s', 0, \cdots, 0, 0 \cdots, 0) \tag{3.11}$$

$$K_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{v} K_i \zeta_j'''^{(i)}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)},$$

$$0, \cdots, \underset{j}{1}, \cdots, 0, 0 \cdots, 0]_{1,l} \tag{3.12}$$

$$K_j' = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} K_i \lambda_j'''^{(i)}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)},$$

$$0, \cdots, 0, 0 \cdots, \underset{j}{1}, \cdots, 0]_{1,k} \tag{3.13}$$

$$L_1 = (1 - \alpha - \beta - \sum_{i=1}^{u} R_i(a_i + b_i) - \sum_{i=1}^{v}(a_i' + b_i')K_i; \mu_1 + \rho_1, \cdots, \mu_r + \rho_r, \mu_1' + \rho_1', \cdots, \mu_r' + \rho_r',$$

$$h_1, \cdots, h_l, 1, \cdots, 1) \tag{3.14}$$

$$L_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{v} \zeta_j'''^{(i)} K_i; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)}, 0, \cdots, 0, 0 \cdots, 0]_{1,l} \quad (3.15)$$

$$L_j' = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \lambda_j'''^{(i)} K_i; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0]_{1,k} \quad (3.16)$$

$$P_1 = (b-a)^{\alpha+\beta-1} \left\{ \prod_{j=1}^{h} (af_j + g_j)^{\sigma_j} \right\} \quad (3.17)$$

$$B_{u,v} = (b-a)^{\sum_{i=1}^{v} K_i(a_i' + b_i') + \sum_{i=1}^{u} (a_i + b_i) R_i} \left\{ \prod_{j=1}^{h} (af_j + g_j)^{- \sum_{i=1}^{v} K_i \lambda_i''' - \sum_{i=1}^{u} \lambda_i'' R_i} \right\} \quad (3.18)$$

$$\mathfrak{A} = A, A'; \mathfrak{B} = B, B' \quad (3.19)$$

We have the following integral : $\displaystyle\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^{l} \left[1 - \tau_j (t-a)^{h_i} \right]^{-\lambda_j} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$

$$S_L^{h_1, \cdots, h_u} \begin{pmatrix} z_1'' \theta_1'' (t-a)^{a_1} (b-t)^{b_1} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j''^{(1)}} \\ . \\ . \\ . \\ z_u'' \theta_u'' (t-a)^{a_u} (b-t)^{b_u} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j''^{(u)}} \end{pmatrix}$$

$$S_{N_1, \cdots, N_v}^{\mathfrak{M}_1, \cdots, \mathfrak{M}_v} \begin{pmatrix} z_1''' \theta_1''' (t-a)^{a_1'} (b-t)^{b_1'} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ z_v''' \theta_v''' (t-a)^{a_v'} (b-t)^{b_v'} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'''^{(v)}} \end{pmatrix}$$

$$I \begin{pmatrix} z_1 \theta_1 (t-a)^{\mu_1} (b-t)^{\rho_1} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ z_r \theta_r (t-a)^{\mu_r} (b-t)^{\rho_r} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(r)}} \end{pmatrix}$$

$$I \begin{pmatrix} z_1' \theta_1' (t-a)^{\mu_1'} (b-t)^{\rho_1'} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ z_s' \theta_s' (t-a)^{\mu_s'} (b-t)^{\rho_s'} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'^{(s)}} \end{pmatrix} \mathrm{d}t$$

$$= P_1 \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \sum_{R_1,\cdots,R_u=0}^{h_1 R_1+\cdots h_u R_u \leqslant L} a_v b_u B_{u,v} \prod_{k=1}^{u} z''^{R_k} \prod_{k=1}^{v} z'''^{K_k}$$

$$I_{p+p'+l+k+2,q+q'+l+k+1;Y}^{0,n+n'+l+k+2;X} \left(\begin{array}{c|c} \dfrac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \\ \cdots & \mathfrak{A}\ ,\ \mathrm{K}_1, \mathrm{K}_2, K_j, K_j' : C \\ \cdots & \cdot \\ \dfrac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \dfrac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \dfrac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \tau_l(b-a)^{h_l} & \cdot \\ \dfrac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \mathfrak{B}\ ,\ \mathrm{L}_1, L_j, L_j' : D \\ \cdots & \\ \dfrac{(b-a)f_k}{af_k+g_k} & \end{array} \right) \qquad (3.20)$$

where $\mathfrak{A}, \mathfrak{B}, C, D, X, K_1, K_2, K_j, K_j', L_1, L_j, L_j', \mathfrak{B}, \mathfrak{B}', P_1, P_u, B_u$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a, b \in \mathbb{R}(a < b); \mu_i, \mu_u', \rho_i, \rho_u', \lambda_j^{(i)}, \lambda_j'^{(u)}, h_v \in \mathbb{R}^+, f_i, g_j, \tau_v, \sigma_j, \lambda_v \in \mathbb{C}\ (i = 1, \cdots, r; j = 1, \cdots; k;$

$u = 1, \cdots, s; v = 1, \cdots, l), a_i, b_i, \lambda_j''^{(i)}, \zeta_j''^{(i)} \in \mathbb{R}^+, (i = 1, \cdots, u; j = 1, \cdots, k)$

$a_i', b_i', \lambda_j'''^{(i)}, \zeta_j'''^{(i)} \in \mathbb{R}^+, (i = 1, \cdots, v; j = 1, \cdots, k)$

(B) $m_j, n_j, p_j, q_j (j = 1, \cdots, r), n, p, q \in \mathbb{N}^*; \delta_j^{(i)} \in \mathbb{R}_+ (j = 1, \cdots, q_i; i = 1, \cdots, r)$

$\alpha_j^{(i)} \in \mathbb{R}_+ (j = 1, \cdots, p; i = 1, \cdots, r), \beta_j^{(i)} \in \mathbb{R}_+ (j = 1, \cdots, q; i = 1, \cdots, r), \gamma_j^{(i)} \in \mathbb{R}_+ (j = 1, \cdots, p_i; i = 1, \cdots, r)$

$a_j(j = 1, \cdots, p), b_j(j = 1, \cdots, q), c_j^{(i)}(j = 1, \cdots, p_i, i = 1, \cdots, r), d_j^{(i)}(j = 1, \cdots, q_i, i = 1, \cdots, r) \in \mathbb{C}$

The exposants $A_j(j = 1, \cdots, p), B_j(j = 1, \cdots, q), C_j^{(i)}(j = 1, \cdots, p_i; i = 1, \cdots, r), D_j^{(i)}(j = 1, \cdots, q_i; i = 1, \cdots, r)$

of various gamma function involved in (1.3) and (1.4) may take non integer values.

$m'_j, n'_j, p'_j, q'_j (j = 1, \cdots, s), n', p', q' \in \mathbb{N}^* ; \delta'^{(i)}_j \in \mathbb{R}_+ (j = 1, \cdots, q'_i; i = 1, \cdots, s)$

$\alpha'^{(i)}_j \in \mathbb{R}_+ (j = 1, \cdots, p'; i = 1, \cdots, s), \beta'^{(i)}_j \in \mathbb{R}_+ (j = 1, \cdots, q'; i = 1, \cdots, r), \gamma'^{(i)}_j \in \mathbb{R}_+ (j = 1, \cdots, p'_i; i = 1, \cdots, s)$
$a'_j (j = 1, \cdots, p'), b'_j (j = 1, \cdots, q'), c'^{(i)}_j (j = 1, \cdots, p'_i, i = 1, \cdots, s), d'^{(i)}_j (j = 1, \cdots, q'_i, i = 1, \cdots, s) \in \mathbb{C}$

The exposants

$A'_j (j = 1, \cdots, p'), B'_j (j = 1, \cdots, q'), C'^{(i)}_j (j = 1, \cdots, p'_i; i = 1, \cdots, s), D'^{(i)}_j (j = 1, \cdots, q'_i; i = 1, \cdots, s)$

of various gamma function involved in (1.9) and (1.10) may take non integer values.

(C) $\max_{1 \leqslant j \leqslant k} \left\{ \left| \dfrac{(b-a)f_i}{af_i + g_i} \right| \right\} < 1, \max_{1 \leqslant j \leqslant l} \left\{ \left| \tau_j (b-a)^{h_j} \right| \right\} < 1$

(D) $Re\left[\alpha + \displaystyle\sum_{j=1}^{r} \mu_j \min_{1 \leqslant k \leqslant m_i} \dfrac{d^{(j)}_k}{\delta^{(j)}_k} + \sum_{j=1}^{s} \mu'_i \min_{1 \leqslant k \leqslant m'_i} \dfrac{d'^{(j)}_k}{\delta'^{(j)}_k} \right] > 0$

$Re\left[\beta + \displaystyle\sum_{j=1}^{r} \rho_j \min_{1 \leqslant k \leqslant m_i} \dfrac{d^{(j)}_k}{\delta^{(j)}_k} + \sum_{j=1}^{s} \rho'_j \min_{1 \leqslant k \leqslant m'_i} \dfrac{d'^{(j)}_k}{\delta'^{(j)}_k} \right] > 0$

(E) $Re\left(\alpha + \displaystyle\sum_{i=1}^{v} K_i a'_i + \sum_{i=1}^{u} R_i a_i + \sum_{i=1}^{r} \mu_i s_i + \sum_{i=1}^{s} t_i \mu'_i \right) > 0$

$Re\left(\beta + \displaystyle\sum_{i=1}^{v} K_i b'_i + \sum_{i=1}^{u} R_i b_i + \sum_{i=1}^{r} v_i s_i + \sum_{i=1}^{s} t_i \rho'_i \right) > 0$

$Re\left(\lambda_j + \displaystyle\sum_{i=1}^{v} K_i \lambda'''^{(i)}_j + \sum_{i=1}^{u} R_i \lambda''^{(i)}_j + \sum_{i=1}^{r} s_i \zeta^{(i)}_j + \sum_{i=1}^{s} t_i \zeta'^{(i)}_j \right) > 0 (j = 1, \cdots, l);$

$Re\left(-\sigma_j + \displaystyle\sum_{i=1}^{v} K_i \lambda'''^{(i)} + \sum_{i=1}^{u} R_i \lambda''^{(i)}_j + \sum_{i=1}^{r} s_i \lambda^{(i)}_j + \sum_{i=1}^{s} t_i \lambda'^{(i)}_j \right) > 0 (j = 1, \cdots, k)$

(F) $U_i = \displaystyle\sum_{j=1}^{p} A_j \alpha^{(i)}_j - \sum_{j=1}^{q} B_j \beta^{(i)}_j + \sum_{j=1}^{p_i} C^{(i)}_j \gamma^{(i)}_j - \sum_{j=1}^{q_i} D^{(i)}_j \delta^{(i)}_j \leqslant 0, i = 1, \cdots, r$

$U'_i = \displaystyle\sum_{j=1}^{p'} A'_j \alpha'^{(i)}_j - \sum_{j=1}^{q'} B'_j \beta'^{(i)}_j + \sum_{j=1}^{p'_i} C'^{(i)}_j \gamma'^{(i)}_j - \sum_{j=1}^{q'_i} D'^{(i)}_j \delta'^{(i)}_j \leqslant 0, i = 1, \cdots, s$

(G) $\Delta_k = - \displaystyle\sum_{j=n+1}^{p} A_j \alpha^{(k)}_j - \sum_{j=1}^{q} B_j \beta^{(k)}_j + \sum_{j=1}^{m_k} D^{(k)}_j \delta^{(k)}_j - \sum_{j=m_k+1}^{q_k} D^{(k)}_j \delta^{(k)}_j + \sum_{j=1}^{n_k} C^{(k)}_j \gamma^{(k)}_j - \sum_{j=n_k+1}^{p_k} C^{(k)}_j \gamma^{(k)}_j$

$-\mu_i - \rho_i - \displaystyle\sum_{l=1}^{k} \lambda^{(i)}_j - \sum_{l=1}^{l} \zeta^{(i)}_j > 0 \quad (i = 1, \cdots, r)$

$\Delta'_k = - \displaystyle\sum_{j=n'+1}^{p'} A'_j \alpha'^{(k)}_j - \sum_{j=1}^{q'} B'_j \beta'^{(k)}_j + \sum_{j=1}^{m'_k} D'^{(k)}_j \delta'^{(k)}_j - \sum_{j=m'_k+1}^{q'_k} D'^{(k)}_j \delta'^{(k)}_j + \sum_{j=1}^{n'_k} C'^{(k)}_j \gamma'^{(k)}_j - \sum_{j=n'_k+1}^{p'_k} C'^{(k)}_j \gamma'^{(k)}_j$

$$-\mu_i' - \rho_i' - \sum_{l=1}^{k} \lambda_j'^{(i)} - \sum_{l=1}^{l} \zeta_j'^{(i)} > 0 \quad (i = 1, \cdots, s)$$

(H) $\left| arg \left(z_i \prod_{j=1}^{l} \left[1 - \tau_j(t-a)^{h_i} \right]^{-\zeta_j^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| < \frac{1}{2}\Delta_i \pi \ (a \leqslant t \leqslant b; i = 1, \cdots, r)$

$\left| arg \left(z_i' \prod_{j=1}^{l} \left[1 - \tau_j'(t-a)^{h_i'} \right]^{-\zeta_j'^{(i)}} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'^{(i)}} \right) \right| < \frac{1}{2}\Delta_i' \pi \ (a \leqslant t \leqslant b; i = 1, \cdots, s)$

Proof

To prove (3.20), first expressing a class of multivariable polynomials $S_{N_1, \cdots, N_v}^{\mathfrak{M}_1, \cdots, \mathfrak{M}_v}[.]$ defined by Srivastava [4] in serie with the help of (1.13), a class of multivariable polynomials $S_L^{h_1, \cdots, h_u}[.]$ defined by Srivastava et al [6] in serie with the help of (1.14) and we interchange the order of summations and t-integral (which is permissible under the conditions stated). Expressing the I-functions of r-variables and s-variables defined by Kumari et al [2] in terms of Mellin-Barnes type contour integral with the help of (1.2) and (1.4) respectively and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collect the power of $\left[1 - \tau_j(t-a)^{h_i} \right]$ with $(i = 1, \cdots, r; j = 1, \cdots, l)$ and collect the power of $(f_j t + g_j)$ with $j = 1, \cdots, k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r + s + k + l)$ dimensional Mellin-Barnes integral in multivariable I-function defined by Kumari et al [2], we obtain the equation (3.19).

Remarks

If a) $\rho_1 = \cdots, \rho_r = \rho_1' = \cdots, \rho_s' = 0$; b) $\mu_1 = \cdots, \mu_r = \mu_1' = \cdots, \mu_s' = 0$, we obtain the similar formulas that (3.20) with the corresponding simplifications.

4. Particular cases

a) If $A_j = B_j = C_j^{(i)} = D_j^{(i)} = A_j' = B_j' = C_j'^{(i)} = D_j'^{(i)} = 1$, the multivariable I-function defined by Nambisan et al [2] reduces to multivariable H-function defined by Srivastava et al [8].

We the following generalized Eulerian integral concerning the multivariable H-functions :

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^{l} \left[1 - \tau_j(t-a)^{h_i} \right]^{-\lambda_j} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$$

$$S_L^{h_1, \cdots, h_u} \begin{pmatrix} z_1'' \theta_1''(t-a)^{a_1}(b-t)^{b_1} \prod_{j=1}^{k}(f_j t + g_j)^{-\lambda_j''^{(1)}} \\ . \\ . \\ . \\ z_u'' \theta_u''(t-a)^{a_u}(b-t)^{b_u} \prod_{j=1}^{k}(f_j t + g_j)^{-\lambda_j''^{(u)}} \end{pmatrix}$$

$$S_{N_1, \cdots, N_v}^{\mathfrak{M}_1, \cdots, \mathfrak{M}_v} \begin{pmatrix} z_1''' \theta_1'''(t-a)^{a_1'}(b-t)^{b_1'} \prod_{j=1}^{k}(f_j t + g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ z_v''' \theta_v'''(t-a)^{a_v'}(b-t)^{b_v'} \prod_{j=1}^{k}(f_j t + g_j)^{-\lambda_j'''^{(v)}} \end{pmatrix}$$

$$H\begin{pmatrix} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(r)}} \end{pmatrix}$$

$$H\begin{pmatrix} z_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{pmatrix}\mathrm{d}t$$

$$= P_1\sum_{K_1=0}^{[N_1/\mathfrak{M}_1]}\cdots\sum_{K_v=0}^{[N_v/\mathfrak{M}_v]}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}a_vb_uB_{u,v}\prod_{k=1}^{u}z''^{R_k}\prod_{k=1}^{v}z'''^{K_k}$$

$$H_{p+p'+l+k+2,q+q'+l+k+1;Y}^{0,n+n'+l+k+2;X}\begin{pmatrix} \dfrac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} \\ \cdots \\ \cdots \\ \dfrac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} \\ \dfrac{z_1'(b-a)^{\mu_1'+\rho_1'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} \\ \cdots \\ \cdots \\ \dfrac{z_s'(b-a)^{\mu_s'+\rho_s'}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} \\ \tau_1(b-a)^{h_1} \\ \cdots \\ \cdots \\ \tau_l(b-a)^{h_l} \\ \dfrac{(b-a)f_1}{af_1+g_1} \\ \cdots \\ \cdots \\ \dfrac{(b-a)f_k}{af_k+g_k} \end{pmatrix}\begin{matrix} \mathfrak{A}\ ,K_1,K_2,K_j,K_j':C \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \mathfrak{B}\ ,L_1,L_j,L_j':D \end{matrix}\qquad (4.1)$$

under the same notations and conditions that (3.20) with $A_j = B_j = C_j^{(i)} = D_j^{(i)} = A_j' = B_j' = C_j'^{(i)} = D_j'^{(i)} = 1$

b) If $B(L; R_1, \cdots, R_u) = \dfrac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1 \theta_j' + \cdots + R_u \theta_j^{(u)}} \prod_{j=1}^{B'} (b_j')_{R_1 \phi_j'} \cdots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u \phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{R_1 \psi_j' + \cdots + R_u \psi_j^{(u)}} \prod_{j=1}^{D'} (d_j')_{R_1 \delta_j'} \cdots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u \delta_j^{(u)}}}$ (4.2)

then the general class of multivariable polynomial $S_L^{h_1, \cdots, h_u}[z_1, \cdots, z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [5]. We have

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^l \left[1 - \tau_j(t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^k (f_j t + g_j)^{\sigma_j}$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}} \left(\begin{array}{c} z_1'' \theta_1'' (t-a)^{a_1} (b-t)^{b_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j''^{(1)}} \\ . \\ . \\ . \\ z_u'' \theta_u'' (t-a)^{a_u} (b-t)^{b_u} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j''^{(u)}} \end{array} \right.$$

$$\left. \begin{array}{c} [(-L); R_1, \cdots, R_u][(a); \theta', \cdots, \theta^{(u)}] : [(b'); \phi']; \cdots ; [(b^{(u)}); \phi^{(u)}] \\ [(c); \psi', \cdots, \psi^{(u)}] : [(d'); \delta']; \cdots ; [(d^{(u)}); \delta^{(u)}] \end{array} \right)$$

$$S_{N_1, \cdots, N_v}^{\mathfrak{M}_1, \cdots, \mathfrak{M}_v} \left(\begin{array}{c} z_1''' \theta_1''' (t-a)^{a_1'} (b-t)^{b_1'} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'''^{(1)}} \\ . \\ . \\ . \\ z_v''' \theta_v''' (t-a)^{a_v'} (b-t)^{b_v'} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'''^{(v)}} \end{array} \right)$$

$$I \left(\begin{array}{c} z_1 \theta_1 (t-a)^{\mu_1} (b-t)^{\rho_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ . \\ . \\ . \\ z_r \theta_r (t-a)^{\mu_r} (b-t)^{\rho_r} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(r)}} \end{array} \right)$$

$$I \left(\begin{array}{c} z_1' \theta_1' (t-a)^{\mu_1'} (b-t)^{\rho_1'} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'^{(1)}} \\ . \\ . \\ . \\ z_s' \theta_s' (t-a)^{\mu_s'} (b-t)^{\rho_s'} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'^{(s)}} \end{array} \right) \mathrm{d}t$$

$$= P_1 \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \cdots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \sum_{R_1,\cdots,R_u=0}^{h_1 R_1 + \cdots h_u R_u \leqslant L} a_v b'_u B_{u,v} \prod_{k=1}^{u} z''^{R_k} \prod_{k=1}^{v} z'''^{K_k}$$

$$I_{p+p'+l+k+2,q+q'+l+k+1;Y}^{0,n+n'+l+k+2;X} \left(\begin{array}{c|c} \dfrac{z_1 (b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A} , K_1, K_2, K_j, K'_j : C \\ \cdots & \\ \cdots & \cdot \\ \dfrac{z_r (b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \dfrac{z'_1 (b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'_j{}^{(1)}}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \dfrac{z'_s (b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'_j{}^{(s)}}} & \cdot \\ \tau_1 (b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \tau_l (b-a)^{h_l} & \cdot \\ \dfrac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \\ \cdots & \mathfrak{B} , L_1, L_j, L'_j : D \\ \dfrac{(b-a)f_k}{af_k+g_k} & \end{array} \right) \qquad (4.3)$$

under the same conditions and notations that (3.20)

where $b'_u = \dfrac{(-L)_{h_1 R_1 + \cdots + h_u R_u} B(E; R_1, \cdots, R_u)}{R_1! \cdots R_u!}$, $B[E; R_1, \ldots, R_v]$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extented   the classes of multivariable polynomials defined by Srivastava et al [6] and Srivastava [4].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable I-functions defined by Kumari et al [2] and classes of multivariable polynomials defined by Srivastava [4] and Srivastava et al [6] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES

[1] B. L. J. Braaksma, "Asymptotic expansions and analytic continuations for a class of Barnes integrals,"Compositio Mathematical, vol. 15, pp. 239–341, 1964.

[2]   Prathima J. Nambisan V. and Kurumujji S.K. A Study of I-function of Several Complex Variables, International Journalof Engineering Mathematics Vol(2014) , 2014 page 1-12

[3] Saigo M. and Saxena R.K. Unified fractional integral formulas for the multivariable H-function I. J.Fractional Calculus 15 (1999), page 91-107.

[4] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191.

[5] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[6] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function. Rev. Roumaine Phys. 32(1987), page 685-692.

[7] Srivastava H.M. and Karlsson P.W. Multiple Gaussian Hypergeometric series. Ellis.Horwood. Limited. New-York, Chichester. Brisbane. Toronto , 1985.

[8] H.M. Srivastava and R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

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