Resolvability in generalized double Topological Space

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Abstract

In this paper the relationship between resolvability in generalized double topoligical spaces and pairwise resolvability in bigeneralized topological spaces has been studied.

1.Introduction:

In 1968 C.L. Chang [2] introduced the concept of fuzzy topological space. The concept of intuitionistic fuzzy set was introduced in 1986 by Atanassov [1] as a possible generalization of ordinary fuzzy sets. In 1997 coker [3] introduced the concept of intuitionistic fuzzy topological spaces as a generalization of fuzzy topological spaces. In 1989, Kandil introduced the concept of fuzzy bitopological space. The concepts of resolvability and irresolvability in a topological space were introduced and studied by E. Hewit in 1943.

2. Preliminaries

Now we introduce some basic definition. Throughout the paper by X we denote a non-empty set. In this section we shall present various fundamental definitions and proposition.

Definition 2.1 [4]

A double set A is an object having the form A = <x, A_1, A_2> where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A while A_2 is the set of non-members of A.

Throughout the remainder of this paper we use the simpler notation A = (A_1, A_2) for a double set.

Definition 2.2 [4]

Let the double sets A and B on X be of the form A = (A_1, A_2), B = (B_1, B_2) respectively. Furthermore, let $\{A_j : j \in J\}$ be an arbitrary family of double sets in X, where $A_j = (A_j^{(1)}, A_j^{(2)})$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$
(b) A = B if and only if $A \subseteq B$ and $B \subseteq A$
(c) $\bar{A} = (A_2, A_1)$ denotes the complement of A
(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$
(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$
(f) $\underline{\phi} = (\phi, X)$ and $\underline{X} = (X, \phi)$

Definition 2.3 [5]

A generalized double topology on a set X is a family τ of double sets in X satisfying the following axioms

$T_1 : \underline{\phi}, \underline{X} \in \tau$

$T_2 : \cup G_j \in \tau$ for any arbitrary family $\{G_j : j \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a generalized double topological space and double set in τ is known as a double open set. The complement $\bar{A}$ of an double open set A in generalized double topological space is called a double closed set in X.

Definition 2.4 [5]

Let (X, τ) be generalized double topological space and A = (A_1, A_2) be double set in X. Then the interior and closure of A are defined by

int (A) = ∪ { G: G is a double open sets in X and $G \subseteq A$}

cl (A) = ∩ { H: H is a double closed sets in X and $A \subseteq H$}

respectively.

3. Comparison of resolvability in generalized double topology with bigeneralized topology:

Definition: 3.1

A generalized topological space (X, T) is called resolvable if there exist a dense set A in (X, T) such that X – A is also a dense in (X, T) Otherwise (X, T) is called a irresolvable space.

Definition: 3.2

A bigeneralized topological space (X, T_1, T_2) is called a pair wise resolvable space if there exists a T_1 dense set A such that X – A is a T_2 dense set or a T_2 dense set B such that X – B is a T_1 dense set. Otherwise (X, T_1, T_2) is called a pair wise irresolvable space.

Definition: 3.3

A double set A in a generalized double topological space (X, T) is called dense if there exist no double closed set B such that $A \subset B \subset X$.

Definition: 3.4

A generalized double topological space (X, T) is said to be resolvable if there exists a dense double set A in (X, T) whose complement is also dense in (X, T).

Definition: 3.5

A generalized double topological space (X, T) is said to be irresolvable if it is not resolvable.

Theorem: 3.6

Let (X, T) be generalized double topological space. Let T = { (A_i, B_i)}. Let T_1 = {Sets formed by the first co-ordinates of elements of T }. ie, $T_1 = \{A_i\}$. Then T_1 is a generalized topological space.

Proof:

Since (ϕ, X) and $(X, \phi) \in T$, $\phi, X \in T_1$.

Also T is closed under arbitrary union. Hence for any collection of open sets { (A_i, B_i)} in T, $\cup (A_i, B_i) = (\cup A_i, \cap B_i) \in T$. Therefore, $\cup A_i \in T_1$. Hence for any collection of $\{A_i\}$ in T_1, $\cup A_i \in T_1$. ie, T_1 is closed under arbitrary union. Hence T_1 forms a generalized topological space.

Theorem: 3.7

Let (X, T) be generalized double topological space such that for any open set (A_i, B_i), $A_i \neq \phi$. Let T_1 be the generalized topological space formed by the first co-ordinates. If (X, T_1) is resolvable then (X, T) is resolvable.

Proof:

Let T = { (A_i, B_i)} be the generalized double topology. Let (X, T_1) be resolvable where T_1 is the first co-ordinate generalized topology. Then there exist A such that A and X – A are dense in (X, T_1).

Since A is dense in (X, T_1), $\forall i$ and $X - A_i \neq X$, $A \not\subset X - A_i$ ie, $\forall i$ and $A_i \neq \phi$, $A \supseteq X - A_i$. But $A_i \cap B_i = \phi$ $\forall i$, so $B_i \subseteq X - A_i$ $\forall i$. That implies $\forall i$ and $B_i \neq X$, $A \supseteq B_i$. Therefore, $(A, B) \not\subset (B_i, A_i)$ $\forall i$ and $B_i \neq X$ and for any B such that $A \cap B = \phi$.

Hence (A, B) is dense in (X, T).

Now, since X – A is dense in (X, T_1), $\forall i$ and $X - A_i \neq X$, $X - A \not\subset X - A_i$ .ie, $\forall i$ and $A_i \neq \phi$, $X - A \supseteq X - A_i$ ie, $A \subseteq A_i$. Hence $\forall i$ and $A_i \neq \phi$, $A \supseteq A_i$. Hence $(B, A) \not\subset (B_i, A_i)$ $\forall i$ and $A_i \neq \phi$ Hence (B, A) is dense in (X, T). Therefore, (X, T) is resolvable.

Result: 3.8

The converse of the above theorem is not true.

Let X = {a, b, c}. Let T = { $\underline{\phi}$, ({c}, {a}), ({a, b}, {c}), $\underline{X}$} Clearly T is generalized double topological space. Here (X, T) is resolvable space.

But (X, T_1) is not resolvable.

Result: 3.9

Theorem 3.7 is not true when there is an open set $(A_1, A_2) \neq (\phi, X)$ such that $A_1 = \phi$ consider the following example.

Let X = {a, b} and T = {$\underline{\phi}$, (ϕ, {a}), $\underline{X}$). Here T_1 = {ϕ , X}. Hence (X, T_1) is obviously resolvable. But (X, T) is a irresolvable space.

Theorem: 3.10

Let (X, T) be a generalized double topological space. Let T = {(A_i, B_i)}. Let T_2 = { sets formed by complement of second co-ordinate of elements of T) ie, $T_2 = \{X - B_i\}$. Then T_2 forms a generalized topology.

Proof:

Since (ϕ , X) and $(X, \phi) \in T$, $X, \phi \in T_2$. For any arbitrary collection of sets $\{X - B_i\}$ in T_2. $\cup \{X - B_i\} = X - \cap B_i$. Since T is closed under arbitrary union, $\cup (A_i, B_i) = (\cup A_i, \cap B_i) \in T$. Hence $X - \cap B_i \in T_2$. Therefore T_2 is closed under arbitrary union. Hence T_2 is a generalized topology.

Theorem: 3.11

Let (X, T) be generalized double topological space. Let T_2 be the generalized topology formed by the complement of second co-ordinates of T. If (X, T_2) is resolvable then (X, T) is resolvable.

Proof:

Let T = {(A_i, B_i)} be the generalized double topology. Let T_2 be the second co-ordinate generalized topology and (X, T_2) be resolvable. Then there exist a set A such that A and X – A are dense in (X, T_2).

Since A is dense in (X, T_2), $\forall i$ and $B_i \neq X$, $A \not\subset B_i$ Therefore $\forall i$ and $B_i \neq X$, $(A, X - A) \not\subset (B_i, A_i)$. Therefore (A, X – A) is dense in (X, T)

Also since X – A is dense in (X, T_2), $X - A \not\subset B_i$ $\forall i$ and $B_i \neq X$.

Therefore $(X - A, A) \not\subset (B_i, A_i)$.

Hence (X, T) is resolvable.

Result: 3.12

(X, T) is resolvable does not imply that (X, T_2) is resolvable, where T_2 is the generalized topology formed by the complement of second components of T. Consider the example.

Let X = {a, b, c}

Let T = {ϕ, X, ({c}, {b}), ({a, b}, {c}), ({c}, {a}), ({c}, {ϕ})}

Here (X, T) is resolvable. But (X, T_2) is irresolvable

Definition: 3.13

The Let (X, T) be generalized double topological space. Then (X, T_1, T_2) is called the induced bigeneralized topological space where T_1 and T_2 are generalized topologies formed by the first and the complement of second co-ordinates of T respectively.

Theorem: 3.14

Let (X, T) be an generalized double topological space where T = {(A_i, B_i)} such that for any non – empty open set (A_i, B_i), $A_i \neq \phi$. If the induced bigeneralized topological space (X, T_1, T_2) is pair wise resolvable, then (X, T) is resolvable.

Proof:

Let the induced bigeneralized topological space (X, T_1, T_2) be pair wise resolvable. Then there exist a dense set A in T_1 such that X – A is dense in T_2. ie, $\forall i$ and $X - A_i \neq X$, $A \supset X - A_i$ And $\forall i$ and $B_i \neq X$, $X - A \supset B_i$. Hence $\forall i$ and $A_i \neq \phi$ and $B_i \neq X$, $(X - A, A) \not\subset (B_i, A_i)$. Therefore, (X – A, A) is dense in (X, T)

Also $\forall i$ and $X - A_i \neq X$, $A \supset X - A_i$ Moreover $A_i \cap B_i = \phi$. Therefore $B_i \subseteq X - A_i$ Hence $\forall i$ and $X - A_i \neq X$, $A \supseteq B_i$. So $(A, X - A) \not\subset (B_i, A_i)$, $\forall i$ Hence (A, X – A) is dense in (X, T). Therefore, (X, T) is resolvable space.

Result: 3.15

converse of the above theorem is not true. For the example, Let X = {a, b, c}

Let T = {ϕ, X, ({c}, {b}), ({a, b}, {c}), ({c}, {a}), ({c}, ϕ}) . Here (X, T) is resolvable, but (X, T_1, T_2) is not pair wise resolvable.

Now to find the condition when the converse of the above theorem is true.

Theorem: 3.16

Let (X, T) be a resolvable space. Let (ρ, λ) be a double dense set whose complement is also dense. Let T = {(A_i, B_i)} be the collection of open sets. If $A_i \subset \rho \subset B_i\ \forall i$ or $A_i \subset \lambda \subset B_i\ \forall i$ then (X, T_1, T_2) is pair wise resolvable.

Proof:

Let $A_i \subset \rho \subset B_i\ \forall i$. Since (ρ, λ) dense in (X, T), $(\rho, \lambda) \not\subset (B_i, A_i)\ \forall i$ Therefore $\forall i$ $\rho \not\subset B_i$ or $\lambda \not\supseteq A_i$ ie, $\forall i$, $\rho \supset B_i$ or $\lambda \subset A_i$. But $\rho \subset B_i\ \forall i$. Hence $\lambda \subset A_i\ \forall i$. ie, $X - \lambda \supset X - A_i\ \forall i$. That implies $X - \lambda \not\subseteq X - A_i\ \forall i$. Hence $X - \lambda$ is dense in T_1.

Now since (λ, ρ) is dense in (X, T), $(\lambda,\rho) \not\subset (B_i, A_i)\ \forall i$. Therefore $\forall i$, $\lambda \not\subseteq B_i$ or $\rho \supseteq A_i$ ie, $\forall i$, $\lambda \supset B_i$ or $\rho \subset Ai$. But $A_i \subset \rho\ \forall i$ Hence $\lambda \supset B_i\ \forall i$. So λ is dense in T_2. Hence (X, T_1, T_2) is pair wise resolvable.

The proof is similar if $A_i \subset \lambda \subset B_i\ \forall i$

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