On Some Properties of β^*-Closed sets in topological spaces

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Abstract—*In this paper, the authors study the concept of properties a new class of closed sets called β^*-closed sets (briefly β^*-closed set). Also we investigate some of their properties*

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I. Introduction

Levine.N[6] introduced the concept of generalized closed (briey g-closed) sets in topological spaces. S.Arya and Nour[10], Bhattacharya.P and Lahiri.B.K[11],Levine.N[6], Maki et.al7] introduced and investigated generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α - open sets, semi pre-open sets which are some of the weak forms of open sets and the complements of these sets are called the same types of closed sets.

Extension research of generalized closedness was done in recent years as the notion of generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α - open sets, semi pre-open sets were investigated. The aim of this paper is to continue the study of generalized closed sets in general and in particular, the notion of generalized β^* closed sets and its various characterizations were studied.

II. Preliminaries

In this section we begin by recalling some definitions and properties Let (X, τ) be a topological spaces and A be a subset.The closure of A and interior of A are denoted by $cl(A)$ and $\text{int}(A)$ respectively. We recall some generalized open sets.

Definition 2.1: A subset A of a space X is g-closed if and only if cl(A) ⊂ G whenever A ⊂ G and G is open.

Definition 2.2: A map f : X → Y is called g-closed if each closed set F of X, f(F) is g-closed in Y .

Definition 2.3: A map f : X → Y is called semi-closed if each closed set F of X, f(F) is semiclosed in Y.

Definition 2.4 : A map f : X → Y is called α-open if each open set F of X, f(F) is α-set in Y.

Definition 2.5 : A map f : X → Y is called pre-closed if for each closed map F of X, f(F) is preclosed in Y.

Definition 2.6: A map f : X → Y is called regular-closed if for each set F of X, f(F) is regular closed in Y.

Definition 2.7: A map f : X → Y is said to be strongly continuous if $f^{-1}(V)$ is both open and closed in X for each subset V of Y .

Definition 2.8: A map f : X → Y is said to be generalized continuous if $f^{-1}(V)$ is g-open in X for each set V of Y

Definition 2.9 :A subset A of a topological space X is said to be $\beta*$ closed set in X if cl(int(A)) contained in U whenever U is G-open

III. Properties

In this section we study some of the properties of $\beta*$ closed set

Definition 3.1: A map f : X → Y is called $\beta*$ closed map if for each closed set F of X, f(F) is $\beta*$ closed set.

Theorem 3.2: If a map f : X → Y is closed and a map g : Y → Z is β^*-closed then f : X → Z is $\beta*$ -closed.

Proof : Let H be a closed set in X. Then f(H) is closed and (g ∘ F)(H) = g(f(H)) is $\beta*$ -closed as g is $\beta*$ -closed. Thus g ∘ f is $\beta*$ -closed.

Theorem 3.3: If f : X → Y is continuous and $\beta*$ -closed and A is a $\beta*$ -closed set of X then f_A : A → Y is continuous and $\beta*$ -closed.

Proof: If F is a closed set of A then F is a $\beta*$ closed set of X. From Theorem 3.4, It follows that $f_A(F) = f(F)$ is a $\beta*$ -closed set of Y. Hence f_A is $\beta*$ -closed. Also f_A is continuous.

Theorem 3.4: If f : X → Y is $\beta*$ -closed and A = $f^{-1}(B)$ for some closed set B of Y then f_A : A → Y.is $\beta*$ -closed .

Proof: Let F be a closed set in A. Then there is a closed set H in X such that F = A ∩ H. Then $f_A(F)$ = f(A ∩ H) = f(H) ∩ f(B). Since f is $\beta*$ -closed. f(H) is $\beta*$ -closed in Y. so f(H) ∩ B is $\beta*$ -closed in Y. Since the intersection of a $\beta*$ -closed and a closed set is a $\beta*$ -closed set. Hence f_A is $\beta*$ -closed.

Remark 3.5: If B is not closed in Y then the above theorem does not hold from the following example.

Example 3.6: Take B = {b,c}. Then A = $f^{-1}(B)$ = {b, c} and {c} is closed in A but f_A({b}) = {b} is not $\beta*$ -closed in Y .{a} is also not $\beta*$ -closed in B.

IV. $\beta*$ OPEN SETS AND $\beta*$ NEIGHBORHOODS

In this section we introduce $\beta*$ neighborhoods ($\beta*$ -nbhd) topological spaces by using the notion of $\beta*$ open sets and study some properties .

Definition 4.1: Let X be the point in topological space X , then the set of all $\beta*$ -neighborhood of a X is called $\beta*$ -nbhd system of X which is denoted by $\beta*$ -N(X)

Theorem 4.2 : Let X be the topological space and each $x \in X$ .Let $\beta^* - N(x,\tau)$ be the collection of all $\beta*$ -nbhd of X .then we have the following results

(i) $\forall x \in X, \beta^* - N(X) \neq \phi$

(ii) $N \in \beta^* - N(X) \Rightarrow x \in N$

(iii)

$$N \in \beta^* - N(X), M \supset N \Rightarrow M \in \beta^* - N(X)$$

(iv)

$N \in \beta^* - N(X) \Rightarrow \exists\ M \in \beta^* - N(X)$ such that M ⊂ N and $M \in \beta^* - N(Y), \forall Y \in M$

Proof : (i) Since X is β^* open set ,it is β^* -nbhd of every $\forall x \in X$, Hence there exists atleast one β^* -nbhd (namely X) for each $x \in X$ .Hence $\forall x \in X, \beta^* - N(X) \neq \phi$

(ii) if $N \in \beta^* - N(X)$,then N is a β^* -nbhd of x .then by definition β^* -nbhd(x)∈N

(iii) Let $N \in \beta^*$-nbhd and M⊃N, then there is a β^* -open set U such that $x \in U \subset N$ Since N⊂M , $x \in U \subset M$ and M is β^* -nbhd of X ,Hence $M \in \beta^* - N(X)$

(iv) If N $\in \beta^* - N(X)$,then there exists a β^* -open set such that $x \in M \subset N$,since M is a β^* -open set ,it is β^* -nbhd of each of its points. Therefore $M \in \beta^* - N(Y)$ for every $Y \in M$

Theorem 4.3: Let X be a nonempty set, for each x ∈ X, let β^* -N(x) be nonempty collection of subsets of X satisfying following conditions.

(i) N ∈ β^* -N(X, τ)) $\Rightarrow x \in N$.

(ii) Let τ consists of the empty set and all those non-empty subsets of U of X having the property that x ⊂ U implies that there exists an N ⊂ β^* -N(X) such that x ∈ N⊂ U, Then τ is a topology for X.

Proof : (i) φ ∈ τ by definition. We now show that x ∈ τ . Let x be any arbitrary element of X. Since β^* -N(x) is non empty, there is an N∈ β^* -N(X) and so x∈N.

Since N is a subset of X, we have x ∈ N ∈ X. Hence X ∈ τ .

(ii) Let U_λ ∈ τ for every λ ∈ Λ. If x ∈ U {U_λ : λ ∈ Λ}, then x ∈ $U_{\lambda x}$ for some λx∈Λ.

Since $U_{\lambda x}$ ∈ τ , there exists an N ∈ β^* -N(x) such that x ∈N∈ Uλx and consequently x ∈ N∈U {Uλ : λ ∈Λ}. Hence U { Uλ : λ ∈ Λ} ∈ τ .It follows that τ is topology for X.

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