

Interference Problem of Machines Embedded with Fuzziness

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Abstract

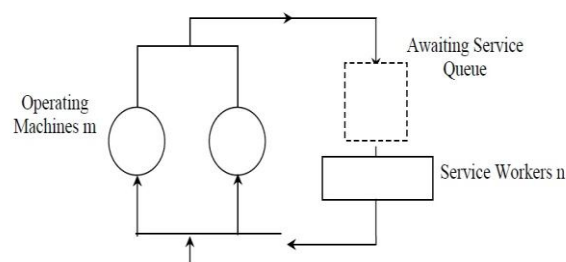
Machine interference is a significant problem in many manufacturing system and client server computing. Machine interference problem involve many parameters like break down rate, service rate, machine production rate, etc. Due to uncontrollable factors parameters in the machine interference problem may be fuzzy. This paper, proposes a methodology for constructing system performance measures, where breakdown rate and service rate are trapezoidal fuzzy numbers.

Function principle is used as arithmetic operations of fuzzy trapezoidal numbers. Numerical example is solved successfully to illustrate the validity of the proposed approach. Since the system characteristics being expressed as a fuzzy trapezoidal numbers more information is provided for used by Management. By extending the fuzzy environment, the fuzzy queues can be represented more accurately by using the proposed approach, and the analysis of results for such queuing model will be useful and significant for system designers and practitioners.

Keywords: Machine Interference problem, Function principle, Graded Mean Integration Representation

1. INTRODUCTION

Consider a system consisting of n technicians who support m machines subject to stochastic failure where $n \leq m$. Whenever a machine breaks down, it is repaired by a technician, each repair keeps a technician busy for a period of time during which they cannot service other broken machines. The problem of operating such a system efficiently is commonly referred to as the Machine Repair Problem (MRP) or machine interference problem. This type of problem can arise not only in maintenance operations but also in manufacturing applications and in client server computing.



A MRP System of m machines and n workers

Machine interference is sometimes used to describe situations where machines may physically get in each other's way during operation rather than while awaiting service. This includes for example the mechanical motion of robotic arms that are closely spaced on an assembly line. We do not consider these subjects or those that deals with "machine repair" in more general settings.

Analysis of MRP model typically begins by deriving the steady-state probability distribution $P_i \in \{0, 1, 2, \dots, n\}$, that describes the long run probability of i machines being in the failed (or) down state at any given point in time. For example, the machine operating lifetimes are exponentially distributed with mean time of $1/\lambda$ and that the service durations are likewise exponentially distributed with mean time of $1/\mu$. Further assume that the service facility has ample buffer space for machines to queue up while awaiting service machines are served in First come, First served order, and that a machine returns to operation "as good as new" after being served. Under the assumption the system is easily modelled as a finite population M/M/m queue with n sources. The steady state distribution can be used in turn to derive a variety of performance measures for the system such as average number of machines waiting for service, average number of machines down, average down time duration of a machine and average duration of waiting time for repair etc. This descriptive analysis takes the model parameter (m ,

n, λ, μ) as given and then describes the system performance according to certain metrics. Efficient methods have been developed for analyzing machine interference problem with its parameters like breakdown rate and service rate are known exactly. One commonly used type of solution methods is the queuing theory approach in that the machine interference problem is modeled as a finite calling, population queuing system. The machines breakdowns are treated as customers and the repair persons are servers in the system. We can derive system performance measures of the machine interference problem and its variants when their parameters are known exactly. However, there are cases that these parameters may not be presented precisely due to uncontrollable factors.

Specifically, in many practical applications, the statistical data may be obtained subjectively. The breakdown pattern and repair pattern are more suitably described by linguistic terms such as fast, moderate or slow rather than by probability distributions based on statistical theory. To deal with imprecise information in making decision, Zadeh introduced the concept of fuzziness. Today, fuzzy set theory is well known for modelling imprecise data and the interest of many researchers is the discussion of fuzzy queues. Thus, fuzzy queues are potentially much more useful and realistic than the commonly used crisp queues. Li and Lee investigated the analytical results for two typical fuzzy queues $M/F/1/\infty$ and $FM/FM/1/\infty$ where F represents fuzzy time and FM represents fuzzified exponential distributions using a general approach based on Zadeh's extension principle. Negi and Lee proposed a procedure using α -cuts and two variable simulation using α -cuts and two variable simulations to analyse fuzzy queues. Using parametric programming Kao constructed the membership functions of the system characteristic for fuzzy queues and successfully applied them to four simple fuzzy queue models $M/F/1/\infty$, $F/F/1/\infty$ and $FM/FM/1/\infty$. Recently Chen developed $FM/FM/1/K$ and $FM/FM^{(k)}/1/\infty$ fuzzy systems using the same approach.

Clearly when the machine breakdown or service rate are fuzzy, the system performance measures of the machine interference problem will be fuzzy as well. To conserve the fuzziness of input information completely, the performance measure should be fuzzy. In this paper, we introduce fuzzy machine interference problem in which the breakdown rate and service rate are all trapezoidal fuzzy numbers.

In order to simplify the calculation of trapezoidal fuzzy numbers, Chen's Function Principle is introduced to calculate the fuzzy system performance measures of our proposed model.

Function principle is proposed as the fuzzy arithmetic operations of fuzzy numbers in 1985. Also the principle is proven that it does not change the type of membership function under fuzzy arithmetic operations of fuzzy numbers. In the fuzzy sense, it is reasonable to discuss the grade of each point of support set of fuzzy numbers for representing fuzzy numbers. Therefore Chen and Hsieh's Graded Mean Integration Representation Method adopted grade as the important degree of each point of support set of generalized fuzzy number. We use it to defuzzify the trapezoidal fuzzy system performance measure. First we shall see some of the basic aspects of Fuzzy sets and other related concepts.

2. FUZZY SET

In a universe of discourse X , a fuzzy subset \tilde{A} on X is defined by the membership function $\mu_{\tilde{A}}(x)$ which maps each element x into X to a real number in the interval $[0, 1]$. $\mu_{\tilde{A}}(x)$ denotes the grade or degree of membership and it is usually denoted as $\mu_{\tilde{A}} : X \rightarrow [0, 1]$.

2.1 Fuzzy Number

The fuzzy number \tilde{A} is said to be a trapezoidal fuzzy number if it is fully determined by (a_1, a_2, a_3, a_4) of crisp numbers such that $a_1 < a_2 < a_3 < a_4$ with membership function, representing a trapezoid of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{x - a_4}{a_3 - a_4}, & a_3 \leq x \leq a_4 \end{cases}$$

where a_1, a_2, a_3 and a_4 are the lower limit, lower mode, upper mode and upper limit respectively of the fuzzy number. When $a_2 = a_3$, the trapezoidal fuzzy number becomes a triangular fuzzy number.

3. MACHINE INTERFERENCE FUZZY MODEL

Consider a conventional machine interference model that consists of m machines and n repairmen. At any instant of time, a particular machine is in either good or bad condition. When a machine

breaks down it must be repaired by anyone of the available n repairmen. Normally a repair man is in charge of more than one machine at a time. When a machine breaks at the time when all repair men are busy it has to wait and is interfered by the machine being repaired. Suppose the mean time to repair a machine is $1/\mu$ and the mean time between failures for a single machine is $1/\lambda$. The machine interference problem is a queueing model with finite calling population in which machines are customers. Suppose the breakdown rate λ and service rate μ are represented as fuzzy sets $\tilde{\lambda}, \tilde{\mu}$ respectively. Let $\eta_{\tilde{\lambda}}$ and $\eta_{\tilde{\mu}}$ denote their membership function, we then have

$$\tilde{\lambda} = \{(\lambda, \eta_{\tilde{\lambda}}(\lambda) / \lambda \in X)\}$$

$$\tilde{\mu} = \{(\mu, \eta_{\tilde{\mu}}(\mu) / \mu \in Y)\}$$

where X and Y are the crisp universal sets of the breakdown rate and the service rate respectively. The performance measures of the system obtained by using Function Principle arithmetic operation of $\tilde{\lambda}, \tilde{\mu}$.

4. MEASURES RELATED TO PERFORMANCE

In this study, we consider the following fuzzy performance measures that are commonly used in traditional queueing theory.

- (i) Operator utilization
- (ii) Machine availability
- (iii) Average number of machines waiting for service
- (iv) Average number of machines down
- (v) Average downtime duration of a machine
- (vi) Average duration of waiting time for repair.

Once a machine is repaired it returns to good condition and is again susceptible to breakdown. The length of time that a machine remains in good condition follows an exponential distribution with breakdown rate $\tilde{\lambda}$ and repair rate $\tilde{\mu}$. Both $\tilde{\lambda}$ and $\tilde{\mu}$ are trapezoidal fuzzy numbers. Each machine has gross production rate \tilde{G} that would be achieved if each machine were always available.

Operator fuzzy utilization $\tilde{\rho} = 1 - \tilde{P}_0$.

Machine availability is given by $\tilde{\eta} = \tilde{\rho} \otimes \tilde{\mu} \otimes m \otimes \tilde{\lambda}$, where \otimes denote coordinate wise multiplication. In particular, if $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ then $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

Production rate of each finished items is given by $\tilde{H} = \tilde{\rho} \otimes \tilde{\mu} \otimes h \otimes \tilde{\lambda}$

Fuzzy Average number of machines waiting for service is given by $\tilde{L} = m \otimes (1! \tilde{\eta})! \tilde{\rho}$

Fuzzy Average number of machines down is given by $\tilde{N} = m \otimes (1! \tilde{\eta})$

Fuzzy Average downtime duration of a machine is given by $\tilde{T} = 1! \tilde{\eta} \otimes \tilde{\eta} \otimes \tilde{\lambda}$

Fuzzy Average duration of waiting time for repair is given by $\tilde{W} = (1! \tilde{\eta}) \otimes \tilde{\eta} \otimes \tilde{\lambda} \otimes 1! \tilde{\mu}$

Fuzzy Average number of failures per unit is given by $\tilde{\lambda} \otimes \tilde{\eta} \otimes m$

Since all the system performance measures are described by trapezoidal fuzzy numbers the value conserves completely all of the fuzziness of the breakdown rate, service rate. However manager or practitioners would prefer one crisp value rather than fuzzy number. In order to overcome this problem we defuzzify the fuzzy performance measures using Graded Mean Integration Representation Method based on the integral value of Graded Mean h-level of generated fuzzy number.

Let \tilde{B} be a trapezoidal fuzzy number and be denoted by $\tilde{B} = (b_1, b_2, b_3, b_4)$ then we get the Graded Mean Integration Representation of \tilde{B} as

$$P(\tilde{B}) = \frac{\int_0^1 h \left(\frac{b_1 + b_4 + (b_2 - b_1 - b_4 + b_3)h}{2} \right) dh}{\int_0^1 h dh} = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

5. NUMERICAL EXAMPLE

To demonstrate the validity of the proposed approach, a numerical example inspired by Gross and Harris is solved. The W.E. Finish Machine Shop Company have five turret lathes. These machines are broken down periodically and the company has one repairman to service the lathes when they breakdown. When the lathe is fixed, the time until the next breakdown is exponentially distributed with a fuzzy rate that can be represented by a trapezoidal fuzzy number $\tilde{\lambda} = (3, 4, 5, 6)$. The repair time for each repairman is exponentially distributed with a fuzzy rate that can be represented a trapezoidal fuzzy number $\tilde{\mu} = (7, 8, 9, 10)$. The shop manager wishes to know the average number of lathes operational at any given time, the expected

“down time” of a lathe that requires repair and expected idle time of each repairman is given by

$\tilde{\mu} = (0.3, 0.44, 0.625, 0.857)$, where $*$ represent co-ordinate wise division. In particular, if

$\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ then

$\tilde{A} * \tilde{B} = (a_1 / b_1, a_2 / b_2, a_3 / b_3, a_4 / b_4)$.

Fuzzy time of operator utilization $\tilde{\rho} = (0.8608, 0.9464, 0.982, 0.994)$

Fuzzy Machine availability $\tilde{\eta} = (0.201, 0.3028, 0.4419, 0.6629)$

Fuzzy number of machines waiting for service $\tilde{L}_q = (0.6911, 1.8085, 2.5396, 3.1342)$

Fuzzy average number of machines down $\tilde{N} = (1.6855, 2.7905, 3.486, 3.995)$

Using Graded mean Integration Method we find that, $\rho = 0.951, \eta = 0.392, L_q = 2.0869,$

$N = 3.0389$

6. CONCLUSION

When the breakdown rate and service rate are fuzzy numbers, the performance measures of the machine interference system is also fuzzy numbers. By using Function Principle as a fuzzy arithmetical operator of fuzzy trapezoidal numbers, the system performance measures can be derived.

Clearly fuzzy average number of machines down is [1.6855, 3.995] indicating that the expected waiting time of lathes for repair will never below 1.6855 or exceed 3.995 approximately. Consider the fuzzy average number of lathes waiting for repair \tilde{L}_q is [0.6911, 3.1342]. Similarly, fuzzy machine availability is [0.201, 0.6629]. The above information obtained from the proposed approach completely maintain the fuzziness of input data, thus they can describe the machine interference problem more accurately. It will be useful in designing machine interference system. In this paper thus, all fuzzy performance measures are expressed by a fuzzy number that completely conserves the fuzziness of input information when some parameter in the machine interference model are fuzzy.

REFERENCES

1. B. Atkinson, Some related paradoxes of queuing theory: new cases and unifying explanation, Journal of the Operational Research Society 51(2000) 921–935.
2. C. Kao, C.C. Li, S.P. Chen, Parametric programming to the analysis of fuzzy queues, Fuzzy Sets and Systems 107 (1999) 93–100.
3. D. Gross, C.M. Harris, Fundamentals of Queueing Theory, third ed., John Wiley, New York, 1998.

4. D. Gross, J.F. Ince, The machine repair problem with heterogeneous populations, Operations Research 29 (1981) 532–549.
5. D.I. Cho, M. Parlar, A survey of maintenance models for multi-unit systems, European Journal of Operational Research 51 (1991) 1–23.
6. D.S. Negi, E.S. Lee, Analysis and simulation of fuzzy queues, Fuzzy Sets and Systems 46 (1992) 321–330.
7. E.A. Elsayed, An optimum repair policy for the machine interference problem, Journal of the Operational Research Society 32 (1981) 793–801.
8. H.A. Taha, Operations Research: An Introduction, seventh ed., Prentice-Hall, Englewood Cliffs, NJ, 2003.
9. H.J. Zimmermann, Fuzzy Set Theory and its Applications, fourth ed., Kluwer–Nijhoff, Boston, 2001.
10. J.B. Jo, Y. Tsujimura, M. Gen, G. Yamazaki, Performance evaluation of network models based on fuzzy queuing system, Japanese Journal of Fuzzy Theory and Systems 8 (1996) 393–408.
11. L.A. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems 1 (1978) 3–28.