A Procedure for Division by Some Small Primes

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This note gives a procedure for division of a number by some small primes up to 151.

1. Introduction.

While dealing with division in early school days, we are told that a positive integer N is divisible by 3 (respectively 9) if and only if the sum of the digits forming the number is divisible by 3 (respectively 9). We are not normally told why this happens although there is a very valid reason behind it. In the same way we also learn that N is divisible by 11 if and only if the sum and difference of the digits forming N taken alternatively in order is divisible by 11.Divisibility of N by 2 or 5 is simple. For 2 the digit in thr units place of N is divisible by 2 while for 5 this digit has to be either 0 or 5. In [2] there is given a procedure for the division of N by 19. Proceeding on the lines of the procedure given in [2], we give procedures for division of N by several small primes up to 151.

2. The Procedure

Let N be a positive integer, x the number of tens (and mot the digit in the tens place) in it and y be the digit in the units place in N. Then $N = 10x + y$. Let $p > 1$ and k be positive integers such that p divides $10k + 1$.

Theorem 1. Let $N_1 = x - ky$. Then p divides N if and only if p divides N_1.

Proof. Observe that $N - 10N_1 = 10x + y - 10(x - ky) = (10k + 1)y$. Therefore p divides $N - 10N_1$. Since $\gcd(10k + 1, 10) = 1$ and p divides $10k + 1$, $\gcd(10, p) = 1$. If p divides N, then p divides $N - (N - 10N_1) = 10N_1$ and, therefore , p divides N_1. On the other hand, if p divides N_1, then p divides $N - 10N_1 + 10N_1 = N$.

Corollary 2.

(a) 31 divides N if and only if 31 divides $x - 3y$;

(b) 41 divides N if and only if 41 divides $x - 4y$;

(c) 17 divides N if and only if 17 divides $x - 5y$;

(d) 61 divides N if and only if 61 divides $x - 6y$;

(e) 71 divides N if and only if 71 divides $x - 7y$;

(f) 101 divides N if and only if 101 divides $x - 10y$;

(g) 37 divides N if and only if 37 divides $x - 11y$;

(h) 131 divides N if and only if 131 divides $x - 13y$;

(i) 47 divides N if and only if 47 divides $x - 14y$;

(j) 151 divides N if and only if 151 divides $x - 15y$;

Now suppose that $p > 1, k$ are positive integers such that p divides $10k - 1$. With N, x, y as already stated, let $N_2 = x + ky$. Proceeding precisely on the lines of proof of Theorem 1, we can also prove the following:

Theorem 3. The number p divides N if and only if p divides N_2.

Corollary 4.

(a) 29 divides N if and only if 29 divides $x + 3y$;

(b) 13 divides N if and only if 13 divides $x + 4y$;

(c) 7 divides N if and only if 7 divides $x + 5y$;

(d) 59 divides N if and only if 59 divides $x + 6y$;

(e) 79 divides N if and only if 79 divides $x + 8y$;

(f) 89 divides N if and only if 89 divides $x + 9y$;

(g) 109 divides N if and only if 109 divides $x + 11y$;

(h) 17 divides N if and only if 17 divides $x + 12y$

(i) 43 divides N if and only if 43 divides $x + 13y$;

(j) 139 divides N if and only if 139 divides $x + 14y$;

(k) 149 divides N if and only if 1493 divides $x + 15y$.

3. Some Examples

As illustrations on the use of Theorems 1 and 3 or their corollaries and using recursive argument we decide the divisibility of some numbers by $17, 37, 29$ and 47.

Example Decide if the number 8179341256 is divisible by (a) 17, (b) 29, (c) 37, (d) 47.

Solution. (a) Adding $12 \times (units\ digit)$ recursively, we get

$$N_2 = 817934125 + 6 \times 12 = 817934197 \to 81793419 + 7 \times 12 = 81793503 \to 8179350 + 3 \times 12 = 8179386 \to 817938 + 6 \times 12 = 818010 \to 81801 \to 8180 + 1 \times 12 = 8192 \to 819 + 2 \times 12 = 843 \to 84 + 3 \times 12 = 120 \to 12$$

Since the last value of N_2 is 12 which is not divisible by 17, the given number is not divisible by 17.

(b) Adding $3 \times (units\ digit)$ recursively, we get

$N_2 = 817934125 + 6 \times 3 = 817934143 \to 81793414 + 3 \times 3 = 81793423 \to 8179342 + 3 \times 3 = 8179351 \to 817935 + 1 \times 3 = 817938 \to 81793 + 8 \times 3 = 81817 \to 8181 + 7 \times 3 = 8202 \to 820 + 2 \times 3 = 826 \to 82 + 6 \times 3 = 100 \to 10.$

Since the last value of N_2 is10 which is not divisible by 29, the given number is not divisible by 29.

(c) Subtracting $11 \times (units\ digit)$ recursively, we get

$$N_1 = 817934125 - 6 \times 11 = 817934059 \to 81793405 - 9 \times 11 = 81793406 \to 8179340 - 6 \times 11 = 8179274 \to 817927 - 4 \times 11 = 817883 \to 81788 - 3 \times 11 = 81755 \to 8175 - 5 \times 11 = 8120 \to 812 \to 81 - 2 \times 11 = 59.$$

The last value of N_1 is 59 which is not divisible by 37. Therefore the given number is not divisible by 37.

(d) Subtracting $14 \times (units\ digit)$ recursively, we get

$$N_1 = 817934125 - 6 \times 14 = 817934041 \to 8179340 - 1 \times 14 = 8179326 \to 817932 - 6 \times 14 = 817848 \to 81784 - 8 \times 14 = 81672 \to 8167 - 2 \times 14 = 8139 \to 813 - 9 \times 14 = 687 \to 68 - 7 \times 14 = -30.$$

The last value of N_1 is -30 which is not divisible by 47. Therefore the given number is not divisible by 47.

The procedure of division as discussed above involves multiplication of a single digit number by a number of $1\ or\ 2$ digits and then addition of the result in a number. The processes of addition and (such a simple) multiplication are much simpler than the process of usual division. As such the procedures for checking divisibility given here are preferable to the usual process of division.

References

1. I. Niven, H. S, Zuckerman and H. L. Montgomery, An Introduction to The Theory of Numbers, Wiley- India, 2010.

2. Ya. I. Perelman, Algebra Can be Fun, Mir Publishers, Moscow, 1979.