# Soft W-Hausdorff Spaces

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***Abstract:*** ***In*** *this paper the concept of W-Hausdorffness in soft topological spaces is introduced by referring the definition of Fuzzy Hausdorffness introduced by Warren, R.H. [4].*

***Keywords:*** *Soft set, Soft topological space, Soft W-Hausdorff space, Fuzzy set, Fuzzy topological space .*

## I. Introduction

Most of the real life problems have various uncertainties. A number of theories have been proposed for dealing with uncertainties in an efficient way. In 1999, Molodstov[6] initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainty. In 2011, Shabir and Naz[7] defined soft topological spaces and studied separation axioms. In section II of this paper, preliminary definitions regarding soft sets and soft topological spaces are given. In section III of this paper, the concept of W-Hausdorffness in soft topological spaces is introduced by referring the definition of Fuzzy Hausdorffness introduced by Warren, R.H. [4].
Throughout this paper, X denotes initial universe and E denotes the set of parameters for the universe X.

## II. Preliminary Definitions

***Definition: 2.1 [6]***
Let X be an initial universe and E be a set of parameters. Let P(X) denotes the power set of X and A be a nonempty subset of E. A pair (F, A) denoted by $F_A$ is called a soft set over X, where F is a mapping given by F: A → P(X).In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular e ∈ A, F(e) may be considered the set of e-approximate elements of the soft set (F, A) and if e ∉ A, then F(e) = ϕ
i.e. $F_A$={F(e): e ∈ A ⊆ E; F: A → P(X)}.
The family of all these soft sets over X with respect to the parameter set E is denoted by $SS(X)_E$ .

***Definition 2.2 [5]***
Let $F_A, G_B \in SS(X)_E$. Then $F_A$ is soft subset of $G_B$ , denoted by $F_A \tilde{\subseteq} G_B$, if
(1) A ⊆ B, and
(2) F(e) ⊆ G(e), ∀e ∈ A.
In this case, $F_A$ is said to be a soft subset of $G_B$ and $G_B$ is said to be a soft superset of $F_A$, $G_B \tilde{\supseteq} F_A$

***Definition 2.3 [5]***
Two soft subsets $F_A$ and $G_B$ over a common universe X are said to be soft equal if $F_A$ is a soft subset of $G_B$ and $G_B$ is a soft subset of $F_A$.

***Definition 2.4 [1]***
The complement of a soft set (F, A) denoted by (F, A)′ is defined by (F, A)′ = (F′, A) ,F′ : A ⟶ P(X) is a mapping given by F′(e) =X – F(e); ∀ e ∈ A and F′ is called the soft complement function of F.Clearly (F′)′ is the same as F and ((F, A)′)′= (F, A).

***Definition 2.5 [5]***
A soft set (F, A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or $\phi_A$ if for all e ∈ A, F(e) = ϕ (null set).

***Definition 2.6 [5]***
A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or $X_A$ if for all e ∈ A, F(e) = X. Clearly we have $X'_A = \phi_A$ and $\phi'_A = X_A$.

***Definition 2.7 [5]***
The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where C = A ∪ B and for all e ∈ C,

$$H(e) = \begin{cases} F(e), e \in A - B, \\ G(e), e \in B - A, \\ F(e) \cup G(e), e \in A \cap B \end{cases}$$

***Definition 2.8 [5]***
The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where C = A ∩ B and for all e ∈ C, H(e) = F(e) ∩ G(e).

***Definition 2.9 [7]***

Let $(X, \tau, E)$ be a soft topological space, $(F, E) \in SS(X)_E$ and Y be a non null subset of X. Then the soft subset of (F, E) over Y denoted by $(F_Y, E)$ is defined as follows:

$$F_Y(e) = Y \cap F(e), \forall e \in E$$

In other words, $(F_Y, E) = Y_E \cap (F, E)$.

***Definition 2.10 [7]***

Let $(X, \tau, E)$ be a soft topological space and Y be a non null subset of X. Then $\tau_Y = \{(F_Y, E): (F, E) \in \tau\}$ is said to be the relative soft topology on Y and $(Y, \tau_Y, E)$ is called a soft subspace of $(X, \tau, E)$

***Definition 2.11 [2]***

Let $F_A \in SS(X)_E$ and $G_B \in SS(Y)_K$. The cartesian product $F_A \otimes G_B$ is defined by $(F_A \otimes G_B)(e, k) = F_A(e) \times G_B(k), \forall (e,k) \in A \times B$. According to this definition $F_A \otimes G_B$ is a soft set over X×Y and its parameter set is $E \times K$.

***Definition 2.12 [2]***

Let $(X, \tau_X, E)$ and $(Y, \tau_Y, K)$ be two soft topological spaces. The soft product topology $\tau_X \otimes \tau_Y$ over X×Y with respect to E×K is the soft topology having the collection $\{F_E \otimes G_K / F_E \in \tau_X, G_K \in \tau_Y\}$ as the basis.

***Definition 2.13 [3]***

A fuzzy set in X is a map f: X →[0, 1] = I. The family of fuzzy sets in X is denoted by $I^X$. Following are some basic operations on fuzzy sets. For the fuzzy sets f and g in X,

(1) $f = g \Leftrightarrow f(x) = g(x)$ for all $x \in X$.
(2) $f \leq g \Leftrightarrow f(x) \leq g(x)$ for all $x \in X$.
(3) $(f \vee g)(x) = \max\{f(x), g(x)\}$ for all $x \in X$.
(4) $(f \wedge g)(x) = \min\{f(x), g(x)\}$ for all $x \in X$.
(5) $f^C(x) = 1 - f(x)$ for all $x \in X$. Here $f^C$ denotes the complement of f.
(6) For a family $\{f_\lambda / \lambda \in \Lambda\}$ of fuzzy sets defined on a set X
(i) $(\vee_{\lambda \in \Lambda} f_\lambda)(x) = \vee_{\lambda \in \Lambda} f_\lambda(x)$
(ii) $(\wedge_{\lambda \in \Lambda} f_\lambda)(x) = \wedge_{\lambda \in \Lambda} f_\lambda(x)$
(7) For any $\alpha \in I$, the constant fuzzy set αin X isa fuzzy set in X defined by $\alpha(x) = \alpha$ for all$x \in X$ and is denoted by $\alpha_x$. $0_x$ denotes null fuzzy set in X and $1_x$ denotes universal fuzzyset in X.

***Definition 2.14 [3]***

A fuzzy topological space is a pair $(X, \tau)$ where Xis a non empty set and τ is a family of fuzzy sets on X satisfying the following properties:

(1) the constant functions $0_x$ and $1_x$ belongs to τ.
(2) $f, g \in \tau$ implies $f \wedge g \in \tau$
(3) $f_\lambda \in \tau$ for each $\lambda \in \Lambda$ implies $\vee_{\lambda \in \Lambda} f_\lambda \in \tau$.

Then τ is called a fuzzy topology on X. Every member of τ is called fuzzy open and g is called fuzzy closed in $(X, \tau)$ if $g^C \in \tau$.

***Definition 2.15 [4]***

A fuzzy topological space $(X, \tau)$ is said to be fuzzy Hausdorff if $\forall x, y \in X$, $x \neq y$, there exist $f, g \in \tau$ such that $f(x) = 1$, $g(y) = 1$ and $f \wedge g = 0_X$.

## III. SOFT W-HAUSDORFF SPACES

***Definition 3.1***

A soft topological space $(X, \tau, E)$ is said to be soft W-Hausdorff space of type 1 denoted by $(SW - H)_1$ if for every $e_1, e_2 \in E, e_1 \neq e_2$ there exist$F_A, G_B \in \tau$, such that $F_A(e_1) = X, G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$

***Theorem 3.2***

Soft subspace of a $(SW - H)_1$space is $(SW - H)_1$.

***Proof***

Let $(X, \tau, E)$ be a $(SW - H)_1$ space. Let Y be a non null subset of X. Let $(Y, \tau_Y, E)$ be a soft subspace of $(X, \tau, E)$ where $\tau_Y = \{(F_Y, E): (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist$F_A, G_B \in \tau$,such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Therefore $((F_A)_Y, E), ((G_B)_Y, E) \in \tau_Y$

Also
$$(F_A)_Y(e_1) = Y \cap F_A(e_1) = Y \cap X = Y$$
$$(G_B)_Y(e_2) = Y \cap G_B(e_2) = Y \cap X = Y$$
$$((F_A)_Y \cap (G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e) = Y \cap (F_A \cap G_B)(e) = Y \cap \tilde{\phi}(e) = Y \cap \phi = \phi$$
$$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$$

Hence $(Y, \tau_Y, E)$ is $(SW - H)_1$

***Definition 3.3***

Let $(X, \tau, E)$ be a soft topological space and $H \subseteq E$. Then $(X, \tau_H, H)$ is called soft p-subspace of $(X, \tau, E)$ relative to the parameter set H where $\tau_H = \{(F_A)/H: H \subseteq A \subseteq E, F_A \in \tau\}$ and $(F_A)/H$ is the restriction map on H.

***Theorem 3.4***

Soft p-subspace of a $(SW - H)_1$ space is $(SW - H)_1$.

***Proof***

Let $(X, \tau, E)$ be a $(SW - H)_1$ space. Let $H \subseteq E$. Let $(X, \tau_H, H)$ be a soft p-subspace of $(X, \tau, E)$ relative to the parameter set H where

$\tau_H = \{(F_A)/H: H \subseteq A \subseteq E, F_A \in \tau\}$. Consider $h_1, h_2 \in H$, $h_1 \neq h_2$.Then $h_1, h_2 \in E$. Therefore, there exist $F_A, G_B \in \tau$ such that $F_A(h_1) = X$, $G_B(h_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Therefore $(F_A)/H, (G_B)/H \in \tau_H$.

Also $((F_A)/H)(h_1) = F_A(h_1) = X$

$((G_B)/H)(h_2) = G_B(h_2) = X$ and

$((F_A)/H) \cap ((G_B)/H) = (F_A \cap G_B)/H$

$= \tilde{\phi}/H$

$= \tilde{\phi}$

Hence $(X, \tau_H, H)$ is $(SW-H)_1$.

***Theorem 3.5***

Product of two $(SW-H)_1$spaces is$(SW-H)_1$.

***Proof***

Let $(X,\tau_X,E)$ and $(Y,\tau_Y,K)$ be two $(SW-H)_1$ spaces. Consider two distinct points $(e_1,k_1)$, $(e_2,k_2) \in E \times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since $(X,\tau_X,E)$ is $(SW-H)_1$, there exist $F_A, G_B \in \tau_X$such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Therefore $F_A \otimes Y_K$, $G_B \otimes Y_K \in \tau_X \otimes \tau_Y$

$(F_A \otimes Y_K)(e_1,k_1) = F_A(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_B \otimes Y_K)(e_2,k_2) = G_B(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e,k) \in (E \times K)$, $(F_A \otimes Y_K)(e,k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$ (since$F_A \cap G_B = \tilde{\phi} \Rightarrow$

$F_A(e) \cap G_B(e) = \phi$ )

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e,k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$

Assume$k_1 \neq k_2$. Since $(Y,\tau_Y,K)$ is $(SW-H)_1$, there exist $F_A, G_B \in \tau_Y$, such that $F_A(k_1) = Y, G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$.

Therefore $X_E \otimes F_A$, $X_E \otimes G_B \in \tau_X \otimes \tau_Y$

$(X_E \otimes F_A)(e_1,k_1) = X_E(e_1) \times F_A(k_1)$

$= X \times Y$

$(X_E \otimes G_B)(e_2,k_2) = X_E(e_2) \times G_B(k_2)$

$= X \times Y$

If for any $(e,k) \in E \times K$, $(X_E \otimes F_A)(e,k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$ (Since$F_A \cap G_B = \tilde{\phi} \Rightarrow$

$F_A(k) \cap G_B(k) = \phi$ )

$\Rightarrow X_E(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e,k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_X \otimes \tau_Y, E \times K)$ is $(SW-H)_1$.

***Definition 3.6***

A soft topological space $(X, \tau, E)$ is said to be soft W-Hausdorff space of type 2 denoted by $(SW-H)_2$ if for every $e_1, e_2 \in E, e_1 \neq e_2$ there exists $F_E, G_E \in \tau$, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$

***Theorem 3.7***

Soft subspace of a $(SW-H)_2$space is $(SW-H)_2$.

***Proof***

Let $(X, \tau, E)$ be a $(SW-H)_2$ space. Let Y be a non-null subset of X. Let $(Y, \tau_Y, E)$ be a soft subspace of $(X, \tau, E)$ where $\tau_Y = \{(F_Y, E): (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist$F_E, G_E \in \tau$,such that$F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Therefore $((F_E)_Y, E), ((G_E)_Y, E) \in \tau_Y$

Also $(F_E)_Y(e_1) = Y \cap F_E(e_1)$

$= Y \cap X$

$= Y$

$(G_E)_Y(e_2) = Y \cap G_E(e_2)$

$= Y \cap X$

$= Y$

$((F_E)_Y \cap (G_E)_Y)(e) = ((F_E \cap G_E)_Y)(e)$

$= Y \cap (F_E \cap G_E)(e)$

$= Y \cap \tilde{\phi}(e)$

$= Y \cap \phi$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence $(Y,\tau_Y,E)$ is $(SW-H)_2$.

***Theorem 3.8***

Soft p-subspace of a $(SW-H)_2$ space is $(SW-H)_2$.

***Proof***

Let $(X, \tau, E)$ be a $(SW-H)_2$ space. Let $H \subseteq E$. Let $(X,\tau_H,H)$ be a soft p-subspace of $(X, \tau, E)$ relative to the parameter set H where $\tau_H = \{(F_A)/H: H \subseteq A \subseteq E, F_A \in \tau\}$. Consider $h_1, h_2 \in H$, $h_1 \neq h_2$ then $h_1, h_2 \in E$ there exist $F_E, G_E \in \tau$ such that $F_E(h_1) = X$, $G_E(h_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Therefore $(F_E)/H, (G_E)/H \in \tau_H$.

Also $((F_E)/H)(h_1) = F_E(h_1) = X$

$((G_E)/H)(h_2) = G_E(h_2) = X$ and

$((F_E)/H) \cap ((G_E)/H) = (F_E \cap G_E)/H$

$= \tilde{\phi}/H$

$= \tilde{\phi}$

Hence $(X,\tau_H,H)$ is $(SW-H)_2$.

***Theorem3.9***

Product of two $(SW-H)_2$ spaces is$(SW-H)_2$..

***Proof***

Let (X,$\tau_X$,E) and (Y,$\tau_Y$,K) be two $(SW-H)_2$ spaces. Consider two distinct points $(e_1, k_1)$, $(e_2, k_2) \in E \times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X,$\tau_X$,E) is $(SW-H)_2$, there exist $F_E$ ,$G_E \in \tau_X$, such that $F_E(e_1) = X, G_E(e_2) = X$ and $F_E \cap G_E = \widetilde{\phi}$.

Therefore $F_E \otimes Y_K$, $G_E \otimes Y_K \in \tau_X \otimes \tau_Y$

$(F_E \otimes Y_K)\ (e_1, k_1) = F_E(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_E \otimes Y_K)\ (e_2, k_2) = G_E(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any (e,k) $\in (E \times K)$, $(F_E \otimes Y_K)$ (e,k) $\neq \phi$

$\Rightarrow F_E(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_E(e) \times Y \neq \phi$

$\Rightarrow F_E(e) \neq \phi$

$\Rightarrow G_E(e) = \phi(\text{Since}F_E \cap G_E = \widetilde{\phi} \Rightarrow F_A(e) \cap G_E(e) = \phi\ )$

$\Rightarrow G_E(e) \times Y_K(k) = \phi$

$\Rightarrow (G_E \otimes Y_K)(e, k)\ = \phi$

$\Rightarrow (F_E \otimes Y_K) \cap (G_E \otimes Y_K) = \widetilde{\phi}$

Similarly, one can prove the case when $k_1 \neq k_2$.

Hence $(X \times Y,\ \tau_X \otimes \tau_Y, E \times K)$ is $(SW-H)_2$.

## IV. CONCLUSION

In this paper the concept of Soft W-Hausdorff spaces is introduced and some basic properties regarding this concept are proved.

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