Fourier Motzkin Elimination Algorithm of Hexagonal Fuzzy Number for Solving Fuzzy Linear Programming Problem

K.Arumugam[#], R.Abdul Saleem[#1], K.Sugantha[#]

[#]*Department of Mathematics, A.V.C.College(Autonomous)*
Mannampandal, Mayiladuthurai, Tamilnadu, India.

Abstract—*In this paper, we introduce a hexagonal fuzzy number solving a Fuzzy LPP using Fourier Motzkin elimination method. The main objective of this paper is first we change the fuzzy linear programming problem into fuzzy linear system of equations. Then a Fourier Motzkin elimination algorithm used to discussed to solve the above converted fuzzy linear system of equations.*

Keywords—*Hexagonal fuzzy number, Fourier Motzkin elimination method, fuzzy linear programming problem.*

I. INTRODUCTION

A method discovered by Fourier[4] in 1826 for manipulating linear inequalities can be adapted to solve Linear programming models. The theoretical insight given by this method is demonstrated as well as its clear geometrical interpretation. It has been rediscovered a number of times by different authors: Motzkin [7](the name Fourier Motzkin algorithm is often used for this method) Dantzig and Cottle [5]andKhun[6]

.

The Fourier' method is used for solving a system of linear constraints of the form $a_1x_1 + a_2x_2 + \cdots + a_nx_n \succ b$ on the set of real numbers (or more generally on an ordered field) where $\succ$ is $>, \geq$ or $=$ and $a_1, a_2, a_3, \ldots a_{n,} b$ are real numbers. The Fourier Motzkin elimination consists in successive elimination of the unknowns. Each step transforms the constraints system S_n with the unknowns $x_1, x_2, \ldots x_n$ to a new system S_{n-1} in which one of the unknowns say $x_n$does not occur anymore suchthatx_n has been eliminated.

The concept of fuzzy numbers and arithmetic operations with these numbers was first introduced and investigated by Zadeh[9]. One of the major applications of fuzzy number arithmetic is treating linear systems and their parameters that are all partially respected by fuzzy number. Friedmantal [8] introduced a general modal for solving a fuzzy $n \times n$ linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy $n \times n$ linear system by a crisp $2n \times 2n$ linear system.

In this paper, Applying Fourier Motzkin elimination algorithm in fuzzy linear programming problem and obtaining the values of the fuzzy variables. Here all the variables are considered as fuzzy variables.

This paper is outlined as follows: section 2 discusses the basic definitions. In section 3 , we presented a concept hexagonal fuzzy number. In section 4, Fourier Motzkin elimination method in fuzzy linear systems is described. and also Fourier Motzkin elimination algorithm for solving fuzzy linear system is given. In section 5, we illustrate the effectiveness of the proposed method in numerical example. Finally in section 6 ,our conclusion is included.

II. PRELIMINARIES

2.1Fuzzy set[10]

If X is a collection of object denoted generically by x, then fuzzy set $\widetilde{A}$ in$X$defined to be a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set. The membership function maps each element of X value between [0,1].

2.2 Fuzzy number[1]

A fuzzy number is generalized regular real number .which refers to a connected set of possible values of weights between 0 and 1.This weights is called membership function.

A fuzzy number $\tilde{A}$is a convex normalized fuzzy set on the real line R such that:

- There exist at least one $x \in R$with$\mu_{\tilde{A}}(x) = 1$.
- $\mu_{\tilde{A}}(x)$is piecewise continuous.

2.3 Fuzzy linear system of equations[7]

Consider the $m \times n$ fuzzy linear system of equations

$$a_{11} \times \tilde{x}_{11} + a_{12} \times \tilde{x}_{12} + \cdots + a_{1n} \times \tilde{x}_{1n} = \tilde{b}_1$$
$$a_{21} \times \tilde{x}_{21} + a_{22} \times \tilde{x}_{22} + \cdots + a_{2n} \times \tilde{x}_{2n} = \tilde{b}_2$$
$$\vdots$$
$$a_{m1} \times \tilde{x}_{m1} + a_{m2} \times \tilde{x}_{m2} + \cdots + a_{mn} \times \tilde{x}_{mn} = \tilde{b}_m$$

The matrix form of the above equation is $A \times \tilde{x} = \tilde{b}$, where the coefficient matrix A is (a_{ij}), where $i = 1\ to\ m\ j = 1\ to\ n$, $\tilde{x}$ is a fuzzy variable and $\tilde{b}$ is also fuzzy variable.

Raking function:[12]

The ranking function is approach of ordering fuzzy number which is an efficient. The ranking function is denoted by $F(\mathfrak{R}) \longrightarrow \mathfrak{R}$, where $\mathfrak{R}: F(\mathfrak{R}) \longrightarrow \mathfrak{R}$, and $F(\mathfrak{R})$ is the set of fuzzy numbers defined on a real line, where a natural order exist. suppose that $\tilde{A}_H$ and $\tilde{B}_H$ be two hexagonal fuzzy number. We have the following comparison

1.$\tilde{A}_H < \tilde{B}_H$ iff $\mathfrak{R}(\tilde{A}_H) < R(\tilde{B}_H)$

2. $\tilde{A}_H > \tilde{B}_H$ iff $\mathfrak{R}(\tilde{A}_H) > R(\tilde{B}_H)$

3.$\tilde{A}_H = \tilde{B}_H$ iff $\mathfrak{R}(\tilde{A}_H) = \mathfrak{R}(\tilde{B}_H)$

III. HEXAGONAL FUZZY NUMBER

3.1. Definition[3]

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by

$\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$.where

$a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membershipfunction is given be below

$$\mu_{\tilde{A}_H(x)} = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & for\, a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_{3-}a_2}\right) & for\, a_2 \le x \le a_3 \\ 1 & for\, a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & for\, a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & for\, a_5 \le x \le a_6 \\ 0 & otherwise \end{cases}$$

3.2 Definition[3]

An hexagonal fuzzy number denoted by $\tilde{A}_H$ is defined as

$\tilde{A}_w = (P_1(u), Q_1(v), Q_2(v), P_2(u))$ for $u \epsilon [0,0.5]$ and $v \in [0.5, w]$.

where

- $P_1(u)$ is bounded left continuous non decreasing function over $[0,0.5]$.
- $Q_1(v)$ is bounded left continuous non decreasing function over $[0.5, w]$.
- $Q_2(v)$ is bounded leftcontinuous non increasing function over $[w, 0.5]$.
- $P_2(u)$ is bounded left continuous non increasing function over $[0.5,0]$.

Note

.If w=1,then the hexagonal fuzzy number is called normal hexagonal fuzzy number. Here $\tilde{A}_w$ representing a fuzzy number in which“w” is a maximum membership value

3.3.Arithmetic operations on hexagonal fuzzy number[1]:

Following are the three operations that can be performed on the hexagonal fuzzy number

Suppose that $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$.and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$.are th two hexagonal fuzzy number

Then

- **Addition:**

$$\tilde{A}_H + \tilde{B}_H = = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$$

- **Subtraction:**

$$\tilde{A}_H - \tilde{B}_H = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$$

- **Multiplication:**

$\tilde{A}_H * \tilde{B}_H = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$.

3.4. Positive hexagonal fuzzy number[2]

A positive hexagonal fuzzy number $\tilde{A}_H$ is denoted by $a_i's > 0$.for all $i = 1,2,3,4,5,6$.

Example: $\tilde{A}_H = (11,12,13,15,16,17)$.

3.5. Negative hexagonal fuzzy number[2]

A negative hexagonal fuzzy number $\tilde{A}_H$ is denoted by $a_i's < 0$.for all $i = 1,2,3,4,5,6$.

Example: $\tilde{A}_H = (-7, -6, -5, -3, -2, -1)$.

3.5.1.Remark:

A negative of hexagonal fuzzy number can be written as the negative multiplication of positive hexagonal fuzzy number.

Example: $\tilde{A}_H = (-7, -6, -5, -3, -2, -1)$

$= -(7,6,5,3,2,1)$

3.6.Magnitude of a hexagonal fuzzy number[11]

The magnitude of a hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is defined as

Mag $(\tilde{A}_H) = \frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}$

IV. FOURIER MOTZKIN ELIMINATION METHOD IN FUZZY LINEAR SYSTEMS

Consider a fuzzy linear system $A\tilde{x} \leq \tilde{b}$,$A \in R^{m,n}$,$\tilde{x}, \tilde{b}$ are fuzzy variables and let $I := \{1,2,3 \cdots, m\}$. The fuzzy linear system in the following form:

$$a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 + \cdots a_{1n}\tilde{x}_n \leq \tilde{b}_1$$
$$a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 + \cdots a_{2n}\tilde{x}_n \leq \tilde{b}_2$$
$$\vdots$$
$$a_{m1}\tilde{x}_1 + a_{m2}\tilde{x}_2 + \cdots a_{mn}\tilde{x}_n \leq \tilde{b}_m \quad (4.1)$$

Eliminate $\tilde{x}_1$ from the system (1) for each $I$where $a_{i1} \neq 0$, we multiply the ith inequality by$1/|a_{i1}|$ .This gives an equilant system:

$$\tilde{x}_1 + a_{i2}^{'}\tilde{x}_2 + \cdots a_{in}^{'}\tilde{x}_n \leq b_i^{'} \quad (i \in I^+)$$
$$a_{i2}\tilde{x}_2 + \cdots + a_{in}\tilde{x}_n \leq \tilde{b}_i (i \in I^0)$$
$$-\tilde{x}_1 + a_{i2}^{'}\tilde{x}_2 + \cdots + a_{in}^{'}\tilde{x}_n \leq \tilde{b}_i^{'} (i \in I^-) \quad (4.2)$$

Where $I^+ = \{i: a_{i1} > 0\}$, $I^- = \{ i: a_{i1} < 0 \}$,, $I^0 = \{ i: a_{i1} < 0 \}$
$a_{ij}^{'} = a_{ij}/|a_{i1}|and\tilde{b}_i^{'} = \tilde{b}_i/|a_{i1}|$.

Thus, the row index set $I = \{1,2,3,\cdots,m\}$ is partitioned in to subsets $I^+, I^0$and I^-, some of which may be empty. It follows that $\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n$ is a solution of the original system (4.1) if and only if $\tilde{x}_2, \tilde{x}_3, \cdots, \tilde{x}_n$ satisfy

$$\sum_{j=2}^{n} a_{kj}^{'}\tilde{x}_j - \tilde{b}_k^{'} \leq \tilde{b}_i^{'} - \sum_{j=2}^{n} a_{ij}^{'}\tilde{x}_j \ (i \in I^+, k \in I^-),$$
$$\sum_{j=2}^{n} a_{ij}\tilde{x}_j \leq \tilde{b}_i (i \in I^0) \quad (4.3)$$

and$\tilde{x}_1$satisfies

$$\max_{k \in I^-}\left(\sum_{j=2}^{n} a_{kj}^{'}\tilde{x}_j - \tilde{b}_k^{'}\right) \leq \tilde{x}_1 \leq \min_{i \in I^+}\left(\tilde{b}_i^{'} - \sum_{j=2}^{n} a_{ij}^{'}\tilde{x}_j\right) \quad (4.4)$$

If I^- $(resp. I^+)$ is empty, then the first set of constraints in (4.3) vanishes and the maximum (resp.minimum) in (4.4) is interpreted as ∞ (resp.$-\infty$). If I^0 is empty and either I^-or $I^+$is empty too, then we terminate :the general solution of $A\tilde{x} \leq \tilde{b}$ is obtained by choosing $\tilde{x}_2, \tilde{x}_3, \cdots, \tilde{x}_n$arbitrarily and choosing $\tilde{x}_1$ according to (4.4). The constraint (4.4) says that $\tilde{x}_1$ lies in a certain interval which is determined by$\tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \cdots, \tilde{x}_n$. The polyhedron defind by (4.3) is the projection of p along the $\tilde{x}_1$ _axis, i.e, into the space of the variables $\tilde{x}_2, \tilde{x}_3, \cdots, \tilde{x}_n$. One may then proceed similarly and eliminate $\tilde{x}_2, \tilde{x}_3$, etc,.eventually one obtains a systems $l \leq \tilde{x}_n \leq u$. If$l < u$, then one concludes that $A\tilde{x} \leq \tilde{b}$ has no solution , otherwise one may choose $\tilde{x}_n \in [l, u]$, and then choose $\tilde{x}_{n-1}$ in an interval which depends on $\tilde{x}_n$, this back substitution procedure produces a solution $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)$ to $A\tilde{x} \leq \tilde{b}$ is, Moreover ,every solution of $A\tilde{x} \leq \tilde{b}$ may be produced in this way(if the system is inconsistent, then ths might possibly be discovered at an early stage and one terminates). The number of constraints may grow exponentially fast as fuzzy variables are eliminated using Fourier Motzkin elimination. Actually, a main problem in practice is that the number of inequalities becomes "*too large*" during the elimination process, even when redundant inequalities are removed. It is therefore of interest to known situations where the projected linear systems are not very large or, at least, have some interesting structure.

4.1. Fourier Motzkin elimination algorithm:

Step 1. Formulate the fuzzy linear programming problem from the given problem.
Step 2. Then change the objective function as inequality and join it with the constraints. Now we get the fuzzy linear system of equation.(for max problem, use'$\leq$' and min problem'$\geq$'inequality).
Step 3. Now change all the inequalities of the fuzzy linear system into '$\geq$'for minimization problem'$\leq$' for maximization problem.
Step 4. Now we are going to eliminate one by one in the $(\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n.)$

- Divide each equation by its modulus value of $\tilde{x}_1$ coefficient or all the equations.
- Now we have three classes of $\tilde{x}_1$coefficient,i.e ,'-1' or '$+1$' or '0' in the fuzzy linear equations.
- Adding or subtracting any two classes of equations to eliminate $\tilde{x}_1$.

Step 5.Repeat step 4 untilall the 'n'fuzzy variable are eliminated.
Step 6 .After eliminating all the 'n'fuzzy variables ,we get the $\tilde{Z}$ values and substitute the $\tilde{Z}$ in above ,we get the values of fuzzy variables in back to back substitution.

V. NUMERICAL EXAMPLE

Problem:

Consider the fuzzy linear programming problem

Max $\tilde{z} = 5\tilde{x}_1 + 3\tilde{x}_2$

Subject to the constraints

$3\tilde{x}_1 + 5\tilde{x}_2 \leq (13,14,15,15,16,17)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (8,9,10,10,11,12)$

and the non-negative restrictions

$\tilde{x}_1 \geq (0,0,0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0,0,0)$.

Solution

Formulation of the problem

Max $\tilde{z} = 5\tilde{x}_1 + 3\tilde{x}_2$

Subject to the constraints

$3\tilde{x}_1 + 5\tilde{x}_2 \leq (13,14,15,15,16,17)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (8,9,10,10,11,12)$

and the non-negative constraints

$\tilde{x}_1 \geq (0,0,0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0,0,0)$.

First we have to include the objective function in the constraints to form a fuzzy linear system of equations. For maximization problem, change the equal '$=$'in the objective as'$\leq$'and join with it all constraints

$\tilde{z} \leq 5\tilde{x}_1 + 3\tilde{x}_2$
$3\tilde{x}_1 + 5\tilde{x}_2 \leq (13,14,15,15,16,17)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (8,9,10,10,11,12)$
$\tilde{x}_1 \geq (0,0,0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0,0,0)$. (5.1)

Equation (5.1) is a fuzzy linear system of equation now change all the inequalities in the systems as '$\leq$' for maximization.

$-5\tilde{x}_1 - 3\tilde{x}_2 + \tilde{z} \leq 0$
$3\tilde{x}_1 + 5\tilde{x}_2 \leq (13,14,15,15,16,17)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (8,9,10,10,11,12)$
$-\tilde{x}_1 \leq (0,0,0,0,0,0)$
$-\tilde{x}_2 \leq (0,0,0,0,0,0)$. (5.2)

Now we are going to eliminate $\tilde{x}_1$ and dividing each co efficient of the system (5.2) by its co efficient of $\tilde{x}_1$,
we have

$-\tilde{x}_1 - 0.6\tilde{x}_2 + 0.2\tilde{z} \leq 0$
$\tilde{x}_1 + 1.66\tilde{x}_2 \leq (4.33,4.66,5,5,5.33,5.66)$
$\tilde{x}_1 + 0.4\tilde{x}_2 \leq (1.6,1.8,2,2,2.2,2.4)$
$-\tilde{x}_1 \leq (0,0,0,0,0,0)$
$-\tilde{x}_2 \leq (0,0,0,0,0,0)$. (5.3)

Now we have in three classes of equations in fuzzy linear system (5.3) we get the co efficient of $\tilde{x}_1$, in the first class of equations is '-1',in the second class equations '$+1$' and in the third class of equations is'0'. Now adding first class of equations with the second class of equations to eliminate $\tilde{x}_1$,

$1.06\tilde{x}_2 + 0.2\tilde{z} \leq (4.33,4.66,5,5,5.33,5.66)$
$-0.2\tilde{x}_2 + 0.2\tilde{z} \leq (1.6,1.8,2,2,2.2,2.4)$
$1.66\tilde{x}_2 \leq (4.33,4.66,5,5,5.33,5.66)$
$0.4\tilde{x}_2 \leq (1.6,1.8,2,2,2.2,2.4)$
$-\tilde{x}_2 \leq (0,0,0,0,0,0)$ (5.4)

Now eliminate $\tilde{x}_2$using the same procedure

$\tilde{x}_2 + 0.189\tilde{z} \leq (4.08,4.39,4.72,4.72,5.03,5.34)$
$-\tilde{x}_2 + \tilde{z} \leq (8,9,10,10,11,12)$
$\tilde{x}_2 \leq (2.61,2.81,3.01,3.01,3.21,3.41)$
$\tilde{x}_2 \leq (4,4.5,5,5,5.5,6)$
$-\tilde{x}_2 \leq (0,0,0,0,0,0)$ (5.5)

Now adding first class of equation with second class of equation

$1.189\tilde{z} \leq (12.08,13.39,14.72,14.72,16.03,17.34)$
$\tilde{z} \leq (10.61,11.81,13.01,13.01,14.21,15.41)$
$\tilde{z} \leq (12,13.5,15,15,16.5,18)$
$0.189\tilde{z} \leq (4.08,4.39,4.72,4.72,5.03,5.34)$
$0 \leq (2.61,2.81,3.01,3.01,3.21,3.41)$
$0 \leq (4,4.5,5,5,5.5,6)$ (5.6)

There is no possibility to eliminate $\tilde{z}$in(5.6) so stop the process.
We have from (5.6)

$\tilde{z} \leq (10.16,11.26,12.38,12.38,13.48,14.58)$
$\tilde{z} \leq (10.61,11.81,13.01,13.01,14.21,15.41)$
$\tilde{z} \leq (12,13.5,15,15,16.5,18)$
$\tilde{z} \leq (21.59,23.23,24.97,26.61,28.25)$
Now choosing minimum value for $\tilde{z}$to satisfy all the above conditions.

$\therefore \tilde{z} = (10.16,11.26,12.38,12.38,13.48,14.58)$

Substitute $\tilde{z}$ in (5.5)

$\tilde{x}_2 + 0.189(10.16,11.26,12.38,12.38,13.48,14.58) \leq (4.08,4.40,4.72,4.72,5.03,5.34)$
$-\tilde{x}_2 + (10.16,11.26,12.38,12.38,13.48,14.58) \leq (8,9,10,10,11,12)$
$\tilde{x}_2 \leq (2.61,2.81,3.01,3.01,3.21,3.41)$
$\tilde{x}_2 \leq (4,4.5,5,5,5.5,6) - \tilde{x}_2 \leq (0,0,0,0,0,0)$

We get
$\tilde{x}_2 \leq (4.08,4.39,4.72,4.72,5.03,5.34) - (1.92,2.13,2.34,2.55,2.75)$
$-\tilde{x}_2 \leq (8,9,10,10,11,12) - (10.16,11.26,12.38,12.38,13.48,14.58)$
$\tilde{x}_2 \leq (2.61,2.81,3.01,3.01,3.21,3.41)$
$\tilde{x}_2 \leq (4,4.5,5,5,5.5,6)$
$-\tilde{x}_2 \leq (0,0,0,0,0,0)$

$\tilde{x}_2 \leq (1.33,1.84,2.38,2.38,2.9,3.42)$
$\tilde{x}_2 \geq (6.58,4.48.2.38,2.38,0.26,-1.84)$
$\tilde{x}_2 \leq (2.61,2.81,3.01,3.01,3.21,3.41)$
$\tilde{x}_2 \leq (4,4.5,5,5,5.5,6)$
$\tilde{x}_2 \geq (0,0,0,0,0,0)$.

From the above equations
$(6.58,4.48.2.38,2.38,0.26,-1.84) \leq \tilde{x}_2 \leq (1.33,1.84,2.38,2.38,2.9,3.42)$

The magnitude of $\tilde{x}_2$ on both nearly 2.37.

$\therefore\ \tilde{x}_2 = (1.33,1.84,2.38,2.38,2.9,3.42)$

Substituting $\tilde{x}_2$ and $\tilde{z}$ in equation (5.3)

$-\tilde{x}_1 - 0.6(1.33,1.84,2.38,2.38,2.9,3.42) + 0.2(10.16,11.26,12.38,12.38,13.48,14.58) \leq 0$
$\tilde{x}_1 + 1.66(1.33,1.84,2.38,2.38,2.9,3.42) \leq (4.33,4.66,5,5,5.33,5.66)$
$\tilde{x}_1 + 0.4(1.33,1.84,2.38,2.38,2.9,3.42) \leq (1.6,1.8,2,2,2.2,2.4)$
$-\tilde{x}_1 \leq (0,0,0,0,0,0)$
$-(1.33,1.84,2.38,2.38,2.9,3.42) \leq (0,0,0,0)$

We get
$-\tilde{x}_1 - (0.798,1.104,1.428,1.428,1.74,2.052) + (2.032,2.252,2.476,2.476,2.696,2.916) \leq 0$
$\tilde{x}_1 + (2.21,3.05,3.95,4.81,5.68) \leq (4.33,4.66,5,5,5.33,5.66)$
$\tilde{x}_1 + (.532,.736,.952,.952,1.16,1.368) \leq (1.6,1.8,2,2,2.2,2.4)$
$-\tilde{x}_1 \leq (0,0,0,0,0,0)$
$-(1.33,1.84,2.38,2.38,2.9,3.42) \leq (0,0,0,0,0,0)$.

Then
$\tilde{x}_1 \geq (2.118,1.592,1.048,1.048,0.512,-0.02)$
$\tilde{x}_1 \leq (-1.35,-0.15,1.05,1.05,2.28,3.45)$
$\tilde{x}_1 \leq (0.232,0.64,1.048,1.048,1.464)$
$\tilde{x}_1 \geq (0,0,0,0)$
From the above equation we get
$(2.118,1.592,1.048,1.048,0.512,-0.02) \leq \tilde{x}_1 \leq (-1.35,-0.15,1.05,1.05,2.28,3.45)$
Then magnitude of $\tilde{x}_1$ on both side nearly 1.05
$\therefore \tilde{x}_1 = (-1.35,-0.15,1.05,1.05,2.28,3.45)$.
We take

$\boldsymbol{\tilde{x}_1 = (-1.35,-0.15,1.05,1.05,2.28,3.45) = 1.05.}$

$\boldsymbol{\tilde{x}_2 = (1.33,1.84,2.38,2.38,2.9,3.42) = 2.37}$

$\boldsymbol{\tilde{z} = (10.16,11.26,12.38,12.38,13.48,14.58)}$
=12.37

Conclusion

In this paper we concentrate with Fourier Motzkin elimination method for solving fuzzy linear programming problem .In our approach a hexagonal fuzzy number is applied for fuzzy number in right hand side of the constriants and also addition and subtraction operations on hexagonal fuzzy number used for this method . More ever this paper useful for future works on fuzzy linear programming problem.

References

[1] Dr. Mrs. A . Sahayasudha and Mrs . M.Ravathy (2016) “A new Ranking on hexagonal fuzzy number. International journal of fuzzy logic systems,vol.6,No.4,pp1-8.

[2] Nasseri.H (2008), “Fuzzy Numbers :positiveandnegative”,International Mathematical Fourn,vol 3,No.36,pp1777- 1780.

[3] P. Rajarajeswari, A. sahayasudha and R.karthika. “A new operation on Hexagonalfuzzy number”.International Journal of fuzzy logic system,vol.3,No.3,2013.pp 15-26.

[4] J.B.J. Fourier, Soiution d’ une question particuliere.ducalculdesnegalities,1826,andextractsfrom ,”Histoiredel’,Acadamic’’,1823,1824,Oeuvres,pp;317-328.

[5]. George Bernard Dantzig and RichardCottle, The Basic George B.Dantzigstanforduniversity press, California, 2008.

[6] H.W. Kuhn, 1956. Solvability and consistency for linear equations and Inequalities, Amer.math.Monthly, 43, 217-232.

[7] T.S. Motzkin,.Beitragezurtheorie der linear en ungleichugen, Doctoral thesis.univercity of Bessel.(1934).

[8] M.Friedman, M. Ming and A. Kandal,. Fuzzy linear systems, Fuzzy sets and systems, 961998), 201-209.

[9] L.A.Zadeh,.. The concept of a linguistic variable and its applications to approxi;matereasonings- Parts I,II,III, Inform.sci., 8(1975), 199-249;8(1975), 301-357;9(1976),43-80.

[10].Bellman, R.E. and Zadeh, L.A..Decision making in a fuzzy environment. Management sciences, 17(1970), pp:141-164.

[11] A. Thamaraiselvi and R,santhi, “optimal soltion of fuzzy transportation problem using hexagonal fuzzy numbers” International jornlofscientificand engineering Reasearchvol.6(2015), pp(40-45).

[12] A,sahayasudha and K.R vijalashmi, “A modern approach in Ranking of symmetric hexagonal fuzzy number”,Arayabhatta Journal of Mathematics and Informatics Vol ,2016 pp:42-49.