# Fixed Point Theorems for Self-Mappings in a Menger Space using Contractive Control Function under CLR / JCLR-Property

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***Abstract***. *In this paper, using contractive control function and CLR / JCLR property two fixed point theorems for self-mappings in a Menger space are mainly proved and also considered a variant of those theorems. Examples are provided in support of the theorems.*

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**Key words and phrases**: *Menger space, triangular norm, weakly compatible mappings, common fixed point.*

## 1. INTRODUCTION

K. P. R. Sastry et. al[2] proved a fixed point theorem for four self-mappings on a complete Menger space using compatible, weakly compatible and continuity concepts. Now, this result is extended to six self-mappings and proved under weaker conditions using CLR / JCLR property.

Throughout this paper $\mathbb{R}$ and $\mathbb{R}^+$ stand for the set of all reals and the set of all non-negative reals respectively.

## 2. PRELIMINARIES AND BASIC RESULTS

**Definition 2.1**. ([3]) A function $F:\mathbb{R}\to\mathbb{R}^+$ is said to be a distribution function if and only if

(i) F is non-decreasing (i.e. monotonic increasing),
(ii) F is left continuous and
(iii) inf{F(u) : $u\in\mathbb{R}$ } = 0 and sup{F(u) : $u\in\mathbb{R}$ } = 1.

D denotes the family of all distribution functions on $\mathbb{R}$ . H is a special element of D defined by

$$H(u)=\begin{cases}0 & if\ \ u\le 0\\ 1 & if\ \ u>0.\end{cases}$$

( H is called the Heaviside function.)

**Definition 2.2.** ([3]) Let X be a non-empty set and D denotes the set of all distribution functions. The ordered pair (X, F ) is called a probabilistic metric space if and only if F is a mapping from X × X into D satisfying the following conditions:

(i) $F_{x,y}$ (u) = H(u) if and only if y = x;
(ii) $F_{x,y}$ (u) = $F_{y,x}$ (u);
(iii) $F_{x,y}$ (0) = 0, and
(iv) if $F_{x,z}$ (u) = 1, $F_{z,y}$ (v) = 1 then $F_{x,y}$ (u + v ) = 1
for all x, y, z in X and u, v>0.
($F_{x,y}$ is the distribution function F (x, y) associated with (x, y )).

Every metric space (X, d) can be viewed as a probabilistic metric space by taking $F_{x,y}$ (u) = H(u - d(x, y)) for all x, y in X.

**Definition 2.3.** ([3]) A mapping * : [0, 1] × [0, 1] - [0, 1] is said to be a triangular norm (known as t-norm) if and only if * satisfies the following conditions. For all a,b,c in [0,1],

(i) *(a, 1) = a and *(0, 0) = 0;
(ii) *(a, b) = *(b, a);
(iii) *(a, b) $\leq$ *(c, d) whenever one of a, b is $\leq$c and the other is $\leq$ d, and
(iv) *(*(a, b), c) = *(a, *(b, c)).
(*(a, b) is denoted by a * b)

If further * is continuous on [0, 1] × [0, 1] (under the usual metric) then it is called a continuous triangular norm.

**Definition 2.4**. ([3]) A Menger space is an ordered triad (X, F, *) where * is a triangular norm and (X, F ) is a probabilistic metric space satisfying the following condition :

$$F_{x,z}(u+v) \geq *(F_{x,y}(u), F_{y,z}(v)) = F_{x,y}(u) * F_{y,z}(v)$$

for all x, y, z in X and u, v >0.

**Definition 2.5.** ([4]) Self maps A and B of a Merger space (X, F, *) are said to be weakly compatible if and only if they commute at their coincidence points; i.e., if Ax = Bx for some $x \in X$ then ABx = BAx.

**Definition 2.6.** ([2]) A mapping $\zeta : \mathbb{R}^+ \to \mathbb{R}^+$ is such that $\zeta$ is strictly increasing and for some $\alpha \in (1,2)$, $(\alpha-1)\zeta(t) > t$ for all t > 0, then $\zeta$ is called a contractive control function.

**Notation**: Z stands for the class of all contractive control functions.

**Definition 2.7.** ([1]) Let (X, F, *) be a Menger space and A, S be self mappings on X. The pairs {A, S} and {B, T} share common property (E.A) if and only If there exist sequences $\{x_n\}$ and $\{y_n\}$ and a point $z \in X$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z.$$

**Definition 2.8.** ([5]) Let (X,F,*) be a Menger space, where * denotes a continuous t-norm and f, g, h, k be self mappings on X. The pairs {f, g} and {h, k} are said to satisfy the "common limit in the range of g" ($CLR_g$)- property if and only if there exist sequences $\{x_n\}$ and $\{y_n\}$ and a point $u \in X$ such that

$$\lim_{n\to\infty} F_{fx_n,gu}(t) = \lim_{n\to\infty} F_{gx_n,gu}(t) = \lim_{n\to\infty} F_{hy_n,gu}(t) = \lim_{n\to\infty} F_{ky_n,gu}(t) = 1, \text{ for all } t > 0.$$

Similar is the case with $CLR_f$, $CLR_h$, $CLR_k$-properties where gu is replaced by fu, hu, ku in the above equality quantities.

**Definition 2.9.** ([6]) Let (X,F,*) be a Menger space, where * denotes a continuous t-norm and f, g, h, k be self mappings on X. The pairs {f, g} and {h, k} are said to satisfy the "joint common limit in the range of g" ($JCLR_{gk}$)- property if and only if there exist sequences $\{x_n\}$ and $\{y_n\}$ and a point $u \in X$ such that and $ku = gu$ and

$$\lim_{n\to\infty} F_{fx_n,gu}(t) = \lim_{n\to\infty} F_{gx_n,gu}(t) = \lim_{n\to\infty} F_{hy_n,gu}(t) = \lim_{n\to\infty} F_{ky_n,gu}(t) = 1, \text{ for all } t > 0.$$

Similar is the case with respect to other relevant combinations.

## 3. Main Results

K. P. R. Sastry et. al[2] proved the following:

**Result 3.1**. Let A, B, S and T be self mappings on a complete Menger space (X,F, *) where * is the min t-norm and satisfying:

(i) A(X)$\subseteq$ T(X) and B(X)$\subseteq$ S(X);

(ii) $F_{Ax,By}(t) \geq F_{Sx,Ty}(\zeta(t)) * F_{Ax,Sx}(\zeta(t)) * F_{By,Ty}(\zeta(t)) * F_{Ax,Ty}(\alpha\zeta(t)) * F_{By,Sx}(\alpha\zeta(t))$ fo r all $x, y \in X$, $t > 0$, $\zeta \in Z$ and for some $\alpha \in (1,2)$;

(iii) the ordered pair (A;S) is compatible and fB;Tg is weakly compatible or vice-versa;

(iv) one mapping of the compatible pair is continuous.

Then A, B, S and T have a unique common fixed point in X.

Now we extend the above one to six self mappings and prove under weaker conditions.

**Theorem 3.2**: Let A, B, S, T, L and M be self mappings on a Menger space (X,F, *), where * is the min t-norm and satisfying:

(i) AL(X)$\subseteq$ T(X) and BM(X)$\subseteq$ S(X);

(ii) $F_{ALx,BMy}(t) \geq F_{Sx,Ty}(\zeta(t)) * F_{ALx,Sx}(\zeta(t)) * F_{BMy,Ty}(\zeta(t)) * F_{ALx,Ty}(\alpha\zeta(t)) * F_{BMy,Sx}(\alpha\zeta(t))$

for all $x, y \in X$, $t > 0$, $\zeta \in Z$ and for some $\alpha \in (1,2)$;

(iii) the pair {AL, S} and {BM, T} are weakly compatible;

(iv) AL=LA and either SA=AS or SL=LS;

(v) BM=MB and either TB=BT or TM=MT;

(vi) the pair {AL, S} and {BM, T} share one of the $CLR_{AL}$, $CLR_{BM}$, $CLR_S$, $CLR_T$-property.

Then A, B, S, T, L and M have a unique common fixed point in X.

**Proof**:

**Case I**: Suppose {AL, S} and {BM, T} share $CLR_{AL}$ or $CLR_S$-property. So, by the definition, there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} ALx_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} BMy_n = \lim_{n\to\infty} Ty_n = Su, \text{ for some } u \in X.$$

Taking $x = u$ and $y = y_n$ in (ii), we get that

$$F_{ALu,BMy_n}(t) \geq F_{Su,Ty_n}(\zeta(t)) * F_{ALu,Su}(\zeta(t)) * F_{BMy_n,Ty_n}(\zeta(t)) * F_{ALu,Ty_n}(\alpha\zeta(t)) * F_{BMy_n,Su}(\alpha\zeta(t))$$

As $n \to \infty$, we get that

$$\begin{aligned} F_{ALu,Su}(t) &\geq F_{Su,Su}(\zeta(t)) * F_{ALu,Su}(\zeta(t)) * F_{Su,Su}(\zeta(t)) * F_{ALu,Su}(\alpha\zeta(t)) * F_{Su,Su}(\alpha\zeta(t)) \\ &= F_{ALu,Su}(\zeta(t)) * F_{ALu,Su}(\alpha\zeta(t)) \\ &\geq F_{ALu,Su}(\zeta(t)) \\ &\geq F_{ALu,Su}(\zeta^2(t)) \geq \text{L} \geq F_{ALu,Su}(\zeta^n(t)) \to 1 \text{ as } n \to \infty (\text{since } 0<(\alpha-1)<1). \end{aligned}$$

Hence $ALu=Su$.

Since AL(X)$\subseteq$ T(X), there is a $v \in X$ such that $ALu = Tv$.

Taking $x = x_n$ and $y = v$ in (ii), we get that

$$F_{ALx_n,BMv}(t) \geq F_{Sx_n,Tv}(\zeta(t)) * F_{ALx_n,Sx_n}(\zeta(t)) * F_{BMv,Tv}(\zeta(t)) * F_{ALx_n,Tv}(\alpha\zeta(t)) * F_{BMv,Sx_n}(\alpha\zeta(t))$$

As $n \to \infty$, we get that

$$\begin{aligned} F_{Tv,BMv}(t) &\geq F_{Tv,Tv}(\zeta(t)) * F_{Tv,Tv}(\zeta(t)) * F_{BMv,Tv}(\zeta(t)) * F_{Tv,Tv}(\alpha\zeta(t)) * F_{BMv,Tv}(\alpha\zeta(t)) \\ &= F_{BMv,Tv}(\zeta(t)) * F_{BMv,Tv}(\alpha\zeta(t)) \\ &\geq F_{BMv,Tv}(\zeta(t)) \\ &\geq F_{BMv,Tv}(\zeta^2(t)) \geq \text{L} \geq F_{BMv,Tv}(\zeta^n(t)) \to 1 \text{ as } n \to \infty. \end{aligned}$$

Then $BMv=Tv$. Thus ALu=Su=BMv=Tv=z (say).

Since {AL, S} and {BM, T} are weakly compatible, ALSu = SALu and BMTv= TBMv. i.e, ALz = Sz and BMz = Tz.

Taking $x = z$ and $y = v$ in (ii), we get that

$$F_{ALz,BMv}(t) \geq F_{Sz,Tv}(\zeta(t)) * F_{ALz,Sz}(\zeta(t)) * F_{BMv,Tv}(\zeta(t)) * F_{ALz,Tv}(\alpha\zeta(t)) * F_{BMv,Sz}(\alpha\zeta(t)).$$

i.e, $F_{ALz,z}(t) \geq F_{ALz,z}(\zeta(t)) * F_{ALz,z}(\alpha\zeta(t)) * F_{z,ALz}(\alpha\zeta(t)) = F_{ALz,z}(\zeta(t))$

So, $ALz = Sz (\Rightarrow ALz = Sz = z)$.

Similarly, by taking $x = u$ and $y = z$ in (ii), we get that $BMz = z (\Rightarrow BMz = Tz = z)$.

Thus $ALz = BMz = Sz = Tz = z$.

Suppose SA=AS.
Since AL=LA, we have ALAz = AALz = Az and SAz = ASz = Az.
Taking $x = Az$ and $y = v$ in (ii), we get that $Az = z$.
Since $ALz = z$, follow that $Lz = z(\Rightarrow Az = Lz = Sz = z)$.

Suppose SL=LS.
Since AL=LA, we have ALLz =LALz = Lz and SLz = LSz = Lz.
Taking $x = Lz$ and $y = v$ in (ii), we get that $Az = z$.
Since $ALz = z$, follow that $Az = z(\Rightarrow Az = Lz = Sz = z)$.

Suppose TB=BT.
Since BM=MB, we have BMBz = BBMz = Bz and TBz = BTz = Bz.
Taking $x = u$ and $y = Bz$ in (ii), we get that $Bz = z$.
Since $BMz = z$, follow that $Mz = z \Rightarrow Bz = Mz = Tz = z$.

Suppose TM=MT.
Since BM=MB, we have BMMz = MBMz = Mz and TMz = MTz = Mz.
Taking $x = u$ and $y = Mz$ in (ii), we get that $Mz = z$.
Since $BMz = z$, follow that $Bz = z \Rightarrow Bz = Mz = Tz = z$.
Thus $Az = Bz = Lz = Mz = Sz = Tz = z$.

**Case II**: Suppose {AL, S} and {BM, T} share $CLR_{BM}$ or $CLR_T$-property. So, by the definition, there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} ALx_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} BMy_n = \lim_{n\to\infty} Ty_n = Tv, \text{ for some } v \in X.$$

Proceeding on similar lines of the previous case, we first get BMv=Tv. Since BM(X)$\subseteq$ S(X), there is a $u \in X$ such that $BMv = Su$. Using (ii) we get that $ALu = Su$. Thus $ALu = Su = BMv = Tv = z(say)$. Now, using (iii), we get that $ALz = Sz$ and $BMz = Tz$. Using (ii), we get that $ALz = BMz = Sz = Tz = z$.

From this stage, the proof is same given above. Hence, we get that z is a common fixed point of A, B, S, T, L and M in X.

Uniqueness follows trivially.

Hence, z is the unique common fixed point of A, B, S, T, L and M in X.

Now, by taking L=M=I(the identity mapping on X), we get the following:

**Corollary 3.3**: Let A, B, S and T be self-mappings on a Menger space (X,F, *), where * is the min t-norm and satisfying:

(i) A(X)$\subseteq$ T(X) and B(X)$\subseteq$ S(X);

(ii) $F_{Ax,By}(t) \geq F_{Sx,Ty}(\zeta(t)) * F_{Ax,Sx}(\zeta(t)) * F_{By,Ty}(\zeta(t)) * F_{Ax,Ty}(\alpha\zeta(t)) * F_{By,Sx}(\alpha\zeta(t))$ for all $x, y \in X$, $t > 0$, $\zeta \in Z$ and for some $\alpha \in (1,2)$;

(iii) the pair {A, S} and {B, T} are weakly compatible;

(iv) the pair {A, S} and {B, T} share one of the $CLR_A$, $CLR_B$, $CLR_S$, $CLR_T$-property.

Then A, B, S and T have a unique common fixed point in X.

We give an example in support of our Theorem(3.2).

**Example 3.4**: (X, F, *) is a Menger space, where X=[0, $\infty$) with the usual metric and $F:\mathbb{R}\to[0,1]$ is defined by $F_{x,y}(t)=\dfrac{t}{t+|x-y|}$ for all $x,y\in\mathbb{R}$, $t>0$ and * is the min t-norm, i.e, $a*b=\min\{a,b\}$ for all $a,b\in[0,1]$.

Define $\zeta:\mathbb{R}^+\to\mathbb{R}^+$ by $\zeta(t)=2t$.

Let A, B, S, T, L and M be the self-mappings on X, defined by

$$A(x)=\begin{cases}0 & if\ x\le 16,\\ 1 & if\ x>16,\end{cases}\qquad S(x)=\begin{pmatrix}0 & if\ x\le 16\\ x^{\frac{1}{2}} & if\ x>16,\end{pmatrix}$$

$Bx=0, Mx=\dfrac{x}{3}, Lx=x$ and $Tx=x^2$, for all $x\in X$.

Take $x_n=y_n=\dfrac{1}{n}$.

Then $\lim_{n\to\infty}ALx_n=\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}BMy_n=\lim_{n\to\infty}Ty_n=0=S(0)$.

So {AL, S} and {BM, T} shares $CLR_S$-property.

**Case 1**: When $x\le 16$ and $y\in X$.

L.H.S= $F_{ALx,BMy}(t)=F_{0,0}(t)=1$.

Clearly, L.H.S $\ge$ R.H.S.

**Case 2**: When $x>16$ and $y\in X$.

L.H.S = $F_{ALx,BMy}(t)=F_{1,0}(t)=\dfrac{t}{t+1}$.

$$\begin{aligned}
\text{R.H.S} &= F_{Sx,Ty}(\zeta(t))*F_{ALx,Sx}(\zeta(t))*F_{BMy,Ty}(\zeta(t))*F_{ALx,Ty}(\alpha\zeta(t))*F_{BMy,Sx}(\alpha\zeta(t))\\
&=F_{x^{1/2},y^2}(2t)*F_{1,x^{1/2}}(2t)*F_{0,y^2}(2t)*F_{1,y^2}(\alpha 2t)*F_{0,x^{1/2}}(\alpha 2t)\\
&=\min\left\{\frac{2t}{2t+\left|x^{1/2}-y^2\right|},\frac{2t}{2t+(x^{1/2}-1)},\frac{2t}{2t+y^2},\frac{2\alpha t}{2\alpha t+\left|1-y^2\right|},\frac{2\alpha t}{2\alpha t+x^{1/2}}\right\}\\
&\le \frac{2t}{2t+(x^{1/2}-1)}\\
&\le \frac{t}{t+\dfrac{(x^{1/2}-1)}{2}}\\
&\le \frac{t}{t+1}=\text{L.H.S.}
\end{aligned}$$

The other conditions of the theorem are trivially satisfied. Clearly, ‘0’ is the unique common fixed point of A, B, S, T, L and M in X.

**Remark 3.5**: If we assume JCLR-property instead of CLR we can omit condition (i) in Theorem(3.2).

We now prove the following:

**Theorem 3.6:** Let A, B, S, T, L and M be self-mappings on a Menger space (X,F, *), where * is the min t-norm and satisfying:

(i) the pair {AL, S} and {BM, T} shares $JCLR_{ST}$-property;

(ii) $F_{ALx,BMy}(t) \geq F_{Sx,Ty}(\zeta(t)) * F_{ALx,Sx}(\zeta(t)) * F_{BMy,Ty}(\zeta(t)) * F_{ALx,Ty}(\alpha\zeta(t)) * F_{BMy,Sx}(\alpha\zeta(t))$
for all $x, y \in X$, $t > 0$, $\zeta \in Z$ and for some $\alpha \in (1,2)$;

(iii) the pair {AL, S} and {BM, T} are weakly compatible;

(iv) AL=LA and either SA=AS or SL=LS;

(v) BM=MB and either TB=BT or TM=MT.

Then A, B, S, T, L and M have a unique common fixed point in X.

**Proof:**

Suppose {AL, S} and {BM, T} shares JCLR$_{ST}$-property. Then there exist sequences {$x_n$} and {$y_n$} in X and $u \in X$ such that $Su = Tu$ and

$$\lim_{n\to\infty} ALx_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} BMy_n = \lim_{n\to\infty} Ty_n = Su(=Tu).$$

Taking $x = u$ and $y = y_n$ in (ii), we get that ALu=Su (ALu = Su =Tu).

Taking $x = x_n$ and $y = u$ in (ii), we get that Tu=BMu.

Thus ALu = Su = BMv = Tv = z(say).

Since {AL, S} and {BM, T} are weakly compatible, ALSu = SALu and BMTu = TBMu. i.e, ALz = Sz and BMz = Tz.

Taking $x = z$ and $y = u$ in (ii), we get that ALz=Sz=z.

Taking $x = u$ and $y = z$ in (ii), we get that BMz = Tz =z.

Hence, ALz=Sz=BMz=Tz=z.

From this stage, the proof runs as in Theorem (3.2).

We now give an example in support of our Theorem(3.6).

**Example 3.7**: (X, F, *) is a Menger space, where X=[0, $\infty$) with the usual metric and $F : ¡ \to [0,1]$ is defined by $F_{x,y}(t) = \dfrac{t}{t + |x - y|}$ for all $x, y \in ¡$, $t > 0$ and * is the min t-norm, i.e, $a * b = \min\{a,b\}$ for all $a, b \in [0,1]$.

Define $\zeta : ¡^{+} \to ¡^{+}$ by $\zeta(t) = 2t$.

Let A, B, S, T, L and M be the self-mappings on X, defined by

$$A(x) = \begin{cases} 0 & if \ x \leq 25, \\ 1 & if \ x > 25, \end{cases} \qquad S(x) = \begin{pmatrix} 0 & if \ x \leq 25 \\ x^{\frac{1}{2}} & if \ x > 25, \end{pmatrix}$$

$Bx = 0, Mx = \dfrac{x}{7}, Lx = x$ and $Tx = x^2$, for all $x \in X$.

Take $x_n = y_n = \dfrac{1}{n}$. Then $\lim_{n\to\infty} ALx_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} BMy_n = \lim_{n\to\infty} Ty_n = 0 = S(0) = T(0).$

So {AL, S} and {BM, T} shares JCLR$_{ST}$-property.

**Case 1**: When $x \leq 25$ and $y \in X$.

L.H.S= $F_{ALx,BMy}(t) = F_{0,0}(t) = 1$.

Clearly, L.H.S $\geq$ R.H.S.

**Case 2**: When $x > 25$ and $y \in X$ .

L.H.S = $F_{ALx,BMy}(t) = F_{1,0}(t) = \dfrac{t}{t+1}$.

$$\text{R.H.S} = F_{Sx,Ty}(\zeta(t)) * F_{ALx,Sx}(\zeta(t)) * F_{BMy,Ty}(\zeta(t)) * F_{ALx,Ty}(\alpha\zeta(t)) * F_{BMy,Sx}(\alpha\zeta(t))$$

$$= F_{x^{1/2},y^2}(2t) * F_{1,x^{1/2}}(2t) * F_{0,y^2}(2t) * F_{1,y^2}(\alpha 2t) * F_{0,x^{1/2}}(\alpha 2t)$$

$$= \min\left\{ \frac{2t}{2t+\left|x^{1/2}-y^2\right|}, \frac{2t}{2t+(x^{1/2}-1)}, \frac{2t}{2t+y^2}, \frac{2\alpha t}{2\alpha t+\left|1-y^2\right|}, \frac{2\alpha t}{2\alpha t+x^{1/2}} \right\}$$

$$\leq \frac{2t}{2t+(x^{1/2}-1)}$$

$$\leq \frac{t}{t+\dfrac{(x^{1/2}-1)}{2}}$$

$$\leq \frac{t}{t+1} = \text{L.H.S.}$$

The other conditions of the theorem are trivially satisfied. Clearly, ‘0’ is the unique common fixed point of A, B, S, T, L and M in X.

We have the following theorem where the pairs share common property (E.A). The proof runs on similar lines of the Theorem (3.2).

**Theorem 3.8**: Let A, B, S, T, L and M be self-mappings on a Menger space (X,F, *), where * is the min t-norm and satisfying:

(i) A(X) $\subseteq$ T(X) and B(X) $\subseteq$ S(X);

(ii) $F_{ALx,BMy}(t) \geq F_{Sx,Ty}(\zeta(t)) * F_{ALx,Sx}(\zeta(t)) * F_{BMy,Ty}(\zeta(t)) * F_{ALx,Ty}(\alpha\zeta(t)) * F_{BMy,Sx}(\alpha\zeta(t))$
for all $x, y \in X$, $t > 0$, $\zeta \in Z$ and for some $\alpha \in (1,2)$;

(iii) the pair {AL, S} and {BM, T} are weakly compatible;

(iv) AL=LA and either SA=AS or SL=LS;

(v) BM=MB and either TB=BT or TM=MT;

(vi) the pair {AL, S} and {BM, T} shares common property (E.A);

(vii) one of AL(X), BM(X), S(X), T(X) is a complete subspace of X.

Then A, B, S, T, L and M have a unique common fixed point in X.

Example 3.9: (X, F, *) is a Menger space, where X=[0, $\infty$) with the usual metric and $F : ¡ \rightarrow [0,1]$ is defined by $F_{x,y}(t) = \dfrac{t}{t+|x-y|}$ for all $x, y \in ¡$, $t > 0$ and * is the min t-norm, i.e, $a * b = \min\{a,b\}$ for all $a, b \in [0,1]$.

Define $\zeta : ¡^{+} \rightarrow ¡^{+}$ by $\zeta(t) = 2t$.

Let A, B, S, T, L and M be the self-mappings on X, defined by

$$A(x) = \begin{cases} 0 & if\ x \leq 9, \\ 1 & if\ x > 9, \end{cases} \qquad S(x) = \begin{cases} 0 & if\ x \leq 9, \\ x^{\frac{1}{2}} & if\ x > 9, \end{cases}$$

$Bx = 0, Mx = \dfrac{x}{5}, Lx = x$ and $Tx = x^2$, for all $x \in X$.

Take $x_n = y_n = \dfrac{1}{n}$.

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