On characterization of fuzzy η-continuous functions where η∈ {α, p, q, semi, pre, β, strongly pre}

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Abstract
In this paper we characterize the C-fuzzy α-continuous function, C-fuzzy p-continuous function, C-fuzzy q-continuous function, C-fuzzy semi-continuous function, C-fuzzy pre-continuous function, C-fuzzy β-continuous function and C-fuzzy strongly pre-continuous function.

Mathematical Subject Classification 2010: 54A40, 3E72.

Key Words: *Fuzzy topology, C-fuzzy α-continuous, C-fuzzy p-continuous, C-fuzzy, q-continuous, C-fuzzy semi-continuous, C-fuzzy pre-continuous, C-fuzzy β-continuous and C-fuzzy strongly pre-continuous.*

1. Introduction

Sutha.et. al. introduced and studied the concepts of C-fuzzy α-open sets,C-fuzzy semi-open sets, C-fuzzy pre-open sets, C-fuzzy β-open sets, C-fuzzy strongly pre-opensets, C-fuzzy p-sets, C-fuzzy q-sets and C-fuzzy α-closed sets, C-fuzzy semi-closed sets, C-fuzzy pre-closed sets, C-fuzzy β-closed sets and C-fuzzy strongly pre-closedsets.In this paper we characterize theC-fuzzy α-continuous function, C-fuzzy p-continuous function, C-fuzzy q-continuous function, C-fuzzy semi-continuous function, C-fuzzy pre-continuous function, C-fuzzy β-continuous function andC-fuzzy strongly pre-continuous function.

The concepts that are needed in this paper are discussed in the second section. The third section is devoted to applications of C-fuzzy α-open, C-fuzzy semi-open, C-fuzzy pre-open, C-fuzzy β-open, C-fuzzy strongly pre-open,C-fuzzy p-sets,C-fuzzy q-sets and C-fuzzy α-closed, C-fuzzy semi-closed, C-fuzzy pre-closed, C-fuzzy β-closed, C-fuzzy strongly pre-closedsets to fuzzy continuous functions.

Throughout this paper (X, τ) is a fuzzy topological space in the sense of Chang[4].

2. Preliminaries

Definition 2.1

Let (X,τ) be a fuzzy topological space. Then a fuzzy subset λ of X is called(i) fuzzy regular open in (X, τ) if *Int Cl* λ = λ **[1]**

(ii) fuzzyα-open in (X, τ) if λ≤*Int Cl Int*λ**[2]**

(iii) fuzzy semi-open in (X, τ) if λ≤*Cl Int*λ**[1]**

(iv) fuzzy pre-open in (X, τ) if λ≤*Int Cl* λ**[2]**

A fuzzy subset λ of a fuzzy topological space (X, τ) is said to befuzzy α-closed(resp. fuzzy semi closed, fuzzy pre closed) if 1−λ isfuzzy α-open(resp.fuzzy semi-open, fuzzy pre-open).

Definition 2.2

Let λ be a fuzzy subset of a fuzzy topological space (X, τ). Let C: [0, 1] →[0, 1] be a complement function. Then λ is

(i) C-fuzzy α-open if C(λ) is fuzzy α-closed,

(ii) C-fuzzy semi-open if C(λ) is fuzzy semi-closed,

(iii) C-fuzzy pre-open if C(λ) is fuzzy pre-closed,

(iv) C-fuzzy β-open if C(λ) is fuzzy β-closed,

(v) C-fuzzy strongly pre-open if C(λ) is fuzzy strongly pre-closed.**[9]**

Definition 2.3

Let λ be a fuzzy subset of a fuzzy topological space (X, τ).

Let C: [0, 1] →[0, 1] be a complement function. Then λ is

(i) C-fuzzy α-closed if Cλis fuzzy α-open,

(ii) C-fuzzy semi-closed if Cλ is fuzzy semi-open,

(iii) C-fuzzy pre-closed if Cλ is fuzzy pre-open,

(iv) C-fuzzy β-closed if Cλ is fuzzy β-open,

(v) C-fuzzy strongly pre-closed set if Cλis fuzzy strongly pre-open set.**[10]**

Definition 2.4

Let λ be a fuzzy subset of a fuzzy topological space (X, τ). LetC: [0, 1] →[0, 1] be a complement function. Then λ is a

(i) C-fuzzy p-set if C(λ) is a fuzzy p-set,

(ii) C-fuzzy q-set if C(λ) is a fuzzy q-set.**[11]**

Definition 2.5

f : (X, τ) → (Y, σ) is called a fuzzy semi-continuous function if $f^{-1}(\mu)$ is a fuzzy semi-open subset of X for each fuzzy open subset μ of Y.**[1]**

Definition 2.6

f : (X, τ) → (Y, σ)is fuzzy α-continuous if the inverse image of every fuzzy open set in (Y, σ) is fuzzy α-open in (X, τ). **[2]**

Definition 2.7

f : (X, τ) → (Y, σ) is called a fuzzy pre-continuous function if $f^{-1}(\mu)$ is a fuzzy pre-open set in (X, τ) for each fuzzy open set μ in (Y, σ).**[2]**

Definition 2.8

f : (X, τ) → (Y, σ)is fuzzy β-continuous if the inverse image of every fuzzy open set in (Y, σ) is fuzzy β-open in (X, τ). **[5]**

Definition 2.9

f : (X, τ) → (Y, σ)is fuzzy p-continuous if $f^{-1}(\mu)$ is a fuzzy p-set in (X, τ) for each fuzzy open set μ in (Y, σ). **[8]**

Definition 2.10

f : (X, τ) → (Y, σ)is fuzzy q-continuous if $f^{-1}(\mu)$ is a fuzzy q-set in (X, τ) for each fuzzy open set μ in (Y, σ). **[8]**

Definition 2.11

f : (X, τ) → (Y, σ) is called a fuzzy strongly pre-continuous function if $f^{-1}(\mu)$ is a fuzzy strongly pre-open set in (X, τ) for each fuzzy open set μ in (Y, σ).**[7]**

3. Onfuzzy η-continuous functions

Theorem 3.1

Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutive condition. Then f is fuzzy semi-continuous if and only if for every fuzzy open set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy semi-closed in X.

Proof

Suppose f is fuzzy semi-continuous. Then by using Definition 2.5, $f^{-1}(\lambda)$ is fuzzy semi-open in X for every fuzzy open set λ in Y. Since C satisfies the involutive condition, $C(C(f^{-1}(\lambda)))$ is fuzzy semi-open in X. Then byDefinition 2.3(ii), $C(f^{-1}(\lambda))$ is C-fuzzy semi-closed. Conversly, assume that, $C(f^{-1}(\lambda))$ is C-fuzzy semi-closed in X for every fuzzy open set λ in Y. Again by Definition 2.3(ii), $C(C(f^{-1}(\lambda)))$ is fuzzy semi-open in X. Since C satisfies the involutivecondition, $f^{-1}(\lambda)$ fuzzy semi-open in X. Again by using Definition 2.5, f is fuzzy semi-continuous.

Theorem 3.2

Suppose f :X→Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutivecondition. Then f is fuzzy α-continuous if and only if for every fuzzy open set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy α-closed in X.

Proof

Suppose f is fuzzy α-continuous. Then by using Definition 2.6, $f^{-1}(\lambda)$ is fuzzy α-open in X for every fuzzy open set λ in Y. Since C satisfies the involutivecondition, $C(C(f^{-1}(\lambda)))$ is fuzzy α-open in X. Then by Definition 2.3(i), $C(f^{-1}(\lambda))$ is C-fuzzy α-closed in X. Conversly, assume that, $C(f^{-1}(\lambda))$ is C-fuzzy α-closed in X for every fuzzy open set λ in Y. Again by Definition 2.3(i), $C(C(f^{-1}(\lambda)))$ is fuzzy α-open in X. Since C satisfies the involutive condition, $f^{-1}(\lambda)$ fuzzy α-open in X. Again by using Definition 2.6, f is fuzzy α-continuous.

Theorem 3.3

Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutivecondition. Then f is fuzzy pre-continuous if and only if for every fuzzy open set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy pre-closed in X.

Proof

Suppose f is fuzzy pre-continuous. Then by using Definition 2.7, $f^{-1}(\lambda)$ is fuzzy pre-open in X for every fuzzy open set λ in Y. Since C satisfies the involutivecondition, $C(C(f^{-1}(\lambda)))$ is fuzzy pre-open in X. Then by Definition 2.3(iii), $C(f^{-1}(\lambda))$ is C-fuzzy pre-closed in X. Conversly, assume that, $C(f^{-1}(\lambda))$ isC-fuzzy pre-closed in X for every fuzzy open set λ in Y. Again by Definition 2.3(iii), $C(C(f^{-1}(\lambda)))$ is fuzzy pre-open in X. Since C satisfies the involutivecondition, $f^{-1}(\lambda)$ fuzzy pre-open in X. Again by using Definition 2.7, f isfuzzy pre-continuous.

Theorem 3.4

Suppose $f : X \rightarrow Y$ be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] $\rightarrow$ [0, 1] satisfies the involutive condition. Then f is fuzzy β-continuous if and only if for every fuzzy open set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy β-closed in X.

Proof

Suppose f is fuzzy β-continuous. Then by using Definition 2.8, $f^{-1}(\lambda)$ is fuzzy β-open in X for every fuzzy open set λ in Y. Since C satisfies the involutivecondition, $C(C(f^{-1}(\lambda)))$ is fuzzy β-open in X. Then by Definition 2.3(iv),$C(f^{-1}(\lambda))$ is C-fuzzy β-closed. Conversly, assume that, $C(f^{-1}(\lambda))$ is C-fuzzy β-closed in X for every fuzzy open set λ in Y. Again by Definition 2.3(iv), $C(C(f^{-1}(\lambda)))$ is fuzzy β-open in X. Since C satisfies the involutivecondition, $f^{-1}(\lambda)$ fuzzy β-open in X. Again by using Definition 2.8, f is fuzzy β-continuous.

Theorem 3.5

Suppose $f : X \rightarrow Y$ be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] $\rightarrow$ [0, 1] satisfies the involutivecondition. Then f is fuzzy p-continuous if and only if for every fuzzy open set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy p-set in X.

Proof

Suppose f is fuzzy p-continuous. Then by using Definition 2.9, $f^{-1}(\lambda)$ is fuzzy p-set in X for every fuzzy open set λ in Y. Since C satisfies the involutivecondition,$C(C(f^{-1}(\lambda)))$ is fuzzy p-set. Then by Definition 2.4(i), $C(f^{-1}(\lambda))$ is C-fuzzy p-set. Conversly, assume that, $C(f^{-1}(\lambda))$ is C-fuzzy p-set for every fuzzy open set λ in Y. Againby using Definition 2.4(i), $C(C(f^{-1}(\lambda)))$ is fuzzy p-set. Since C satisfies the involutivecondition, $f^{-1}(\lambda)$ fuzzy p-set in X. Again by using Definition 2.9, f is fuzzy p-continuous.

Theorem 3.6

Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutivecondition. Then f is fuzzy q-continuous if and only if for every fuzzy open set λ in Y, C(f^{-1}(λ)) is C-fuzzy q-set in X.

Proof

Suppose f is fuzzy q-continuous. Then by using Definition 2.10, f^{-1}(λ) is fuzzy q-set in X for every fuzzy open set λ in Y. Since C satisfies the involutivecondition, C(C(f^{-1}(λ))) is fuzzy q-set. Then by Definition 2.4(ii), C(f^{-1}(λ)) is C-fuzzy q-set. Conversly, assume that, C(f^{-1}(λ)) is C-fuzzy q-closed for every fuzzy open set λ in Y. Again by using Definition 2.4(ii), C(C(f^{-1}(λ))) is fuzzy q-set. Since C satisfies the involutivecondition, f^{-1}(λ) fuzzy q-set in X. Again by using Definition 2.10, f is fuzzy q-continuous.

Theorem 3.7

Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutivecondition. Then f is fuzzy strongly pre-continuous if and only if for every fuzzy open set λ in Y, C(f^{-1}(λ)) is C-fuzzy strongly pre-closed in X.

Proof

Suppose f is fuzzy strongly pre-continuous. Then by using Definition 2.11,f^{-1}(λ) is fuzzy strongly pre-open in X for every fuzzy open set λ in Y. SinceC satisfies the involutivecondition, C(C(f^{-1}(λ))) is fuzzy strongly pre-open in X. Then by Definition 2.3(v), C(f^{-1}(λ)) is C-fuzzy strongly pre-closed in X.Conversly,assume that, C(f^{-1}(λ)) is C-fuzzy strongly pre-closed in X for every fuzzy open set λ in Y. Again by Definition 2.3(v), C(C(f^{-1}(λ))) is fuzzy strongly pre-open in X. Since C satisfies the involutivecondition, f^{-1}(λ) fuzzy strongly pre-open in X. Again by using Definition 2.11, f is fuzzy strongly pre-continuous.

Theorem 3.8

Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutivecondition. Then f is fuzzy semi-continuous if and only if for every fuzzy closed set λ in Y, C(f^{-1}(λ)) is C-fuzzy semi-closed in X.

Proof

Suppose f is fuzzy semi-continuous. Then by Definition 2.5, $f^{-1}(\lambda)$ is fuzzy semi-closed in X for every fuzzy closed set λ in Y. Since C satisfies the involutivecondition, $C(C(f^{-1}(\lambda)))$ is fuzzy semi-closed in X. Then byDefinition 2.2(ii), $C(f^{-1}(\lambda))$ is C-fuzzy semi-open. Conversly, assume that, $C(f^{-1}(\lambda))$ is C-fuzzy semi-open in X for every fuzzy open set λ in Y. Again by Definition 2.2(ii), $C(C(f^{-1}(\lambda)))$ is fuzzy semi-closed. Since C satisfies the involutive condition, $f^{-1}(\lambda)$ fuzzy semi-closed in X. Again by Definition 2.5, f is fuzzy semi-continuous.

Theorem 3.9

Suppose $f : X \to Y$ be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: $[0, 1] \to [0, 1]$ satisfies the involutivecondition. Then f is fuzzy α-continuous if and only if for every fuzzy closed set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy α-open in X.

Proof

Suppose f is fuzzy α-continuous. Then by Definition 2.6, $f^{-1}(\lambda)$ is fuzzy α-closed in X for every fuzzy closed set λ in Y. Since C satisfies the involutivecondition, $C(C(f^{-1}(\lambda)))$ is fuzzy α-closed in X. Then by Definition 2.2(i),$C(f^{-1}(\lambda))$ is C-fuzzy α-open. Conversly, assume that, $C(f^{-1}(\lambda))$ is C-fuzzyα-open in X for every fuzzy open set λ in Y. Again by Definition 2.2(i), $C(C(f^{-1}(\lambda)))$ is fuzzy α-closed. Since C satisfies the involutivecondition, $f^{-1}(\lambda)$ fuzzy α-closed in X. Again by Definition 2.6, f is fuzzy α-continuous.

Theorem 3.10

Suppose $f : X \to Y$ be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: $[0, 1] \to [0, 1]$ satisfies the involutivecondition. Then f is fuzzy pre-continuous if and only if for every fuzzy closed set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy pre-open in X.

Proof

Suppose f is fuzzy pre-continuous. Then by Definition 2.7, $f^{-1}(\lambda)$ is fuzzy pre-closed in X for every fuzzy closed set λ in Y. Since C satisfies the involutivecondition, $C(C(f^{-1}(\lambda)))$ is fuzzy pre-closed in X. Then by Definition 2.2(iii), $Cf^{-1}(\lambda)$ is C-fuzzy pre-open. Conversly, assume that, $C(f^{-1}(\lambda))$ isC-fuzzy pre-open in X for every fuzzy open set λ in Y. Again by Definition 2.2(iii), $C(C(f^{-1}(\lambda)))$ is fuzzy pre-closed. Since C satisfies the involutivecondition, $f^{-1}(\lambda)$ fuzzy pre-closed in X. Again by Definition 2.7, f is fuzzy pre-continuous.

Theorem 3.11

Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutivecondition. Then f is fuzzy β-continuous if and only if for every fuzzy closed set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy β-open in X.

Proof

Suppose f is fuzzy β-continuous. Then by Definition 2.8, $f^{-1}(\lambda)$ is fuzzy β-closed in X for every fuzzy closed set λ in Y. Since C satisfies the involutivecondition, $C(C(f^{-1}(\lambda)))$ is fuzzy β-closed in X. Then by Definition 2.2(iv), $C(f^{-1}(\lambda))$ is C-fuzzy β-open. Conversly, assume that, $C(f^{-1}(\lambda))$ is C-fuzzyβ-open in X for every fuzzy open set λ in Y. Again by Definition 2.2(iv),$C(C(f^{-1}(\lambda)))$ is fuzzy β -closed. Since C satisfies the involutivecondition, $f^{-1}(\lambda)$ fuzzy β-closed in X. Again by Definition 2.8, f is fuzzy β-continuous.

Theorem 12.12

Suppose f : X →Y be a mapping from a fuzzy topological space (X, τ) to a fuzzy topological space (Y, σ) and C: [0, 1] → [0, 1] satisfies the involutivecondition. Then f is fuzzy strongly pre-continuous if and only if for every fuzzy closed set λ in Y, $C(f^{-1}(\lambda))$ is C-fuzzy strongly pre-open in X.

Proof

Suppose f is fuzzy strongly pre-continuous. Then by Definition 2.11, $f^{-1}(\lambda)$ is fuzzy strongly pre-closed in X for every fuzzy closed set λ in Y. Since C satisfies the involutivecondition, $C(C(f^{-1}(\lambda)))$ is fuzzy strongly pre-closed in X. Then by Definition 2.2(v), $C(f^{-1}(\lambda))$ is C-fuzzy strongly pre-open. Conversly, assume that, $C(f^{-1}(\lambda))$ is C-fuzzy strongly pre-open in X for every fuzzy open set λ in Y. Again by Definition 2.2(v), $C(C(f^{-1}(\lambda)))$ is fuzzy strongly pre-closed. Since C satisfies the involutivecondition, $f^{-1}(\lambda)$ fuzzy strongly pre-closed in X. Again by Definition 2.11, f is fuzzy strongly pre-continuous.

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