

# On general Eulerian integral of certain product of Kumari's multivariable I-function the classes of polynomials and generalized hypergeometric function

F.Y. AYANT<sup>1</sup>

<sup>1</sup> Teacher in High School , France

**ABSTRACT**

The object of this paper is to establish an general Eulerian integral involving the product of the multivariable I-function defined by Kumari et al [2], the general classes of multivariable polynomials and a generalized hypergeometric function which provide unification and extension of numerous results. We will study the particular case concerning the multivariable H-function defined by Srivastava et al [9] and the Srivastava-Daoust polynomial [6].

Keywords: Eulerian integral, multivariable I-function, Lauricella function of several variables, multivariable H-function, generalized hypergeometric function.

**2010 Mathematics Subject Classification.** 33C45, 33C60, 26D20

## 1. Introduction

In this paper, we evaluate a general Eulerian integral concerning the product of the multivariable I-function defined by Kumari et al [2], a generalized hypergeometric function and the classes of multivariable polynomials.

The multivariable I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, \dots, z_r) = I_{p,q;p_1,q_1;\dots;p_r,q_r}^{0,n;m_1,n_1;\dots;m_r,n_r} \left( \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix} \middle| \begin{matrix} (a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)}; A_j)_{1,p} : \\ \\ \\ \\ (b_j; \beta_j^{(1)}, \dots, \beta_j^{(r)}; B_j)_{1,q} : \end{matrix} \right. \tag{1.1}$$

$$\left. \begin{matrix} (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \dots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r} \\ \\ (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \dots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r} \end{matrix} \right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \phi(s_1, \dots, s_r) \prod_{i=1}^r \theta_i(s_i) z_i^{s_i} ds_1 \dots ds_r \tag{1.2}$$

where  $\phi(s_1, \dots, s_r), \theta_i(s_i), i = 1, \dots, r$  are given by :

$$\phi(s_1, \dots, s_r) = \frac{\prod_{j=1}^n \Gamma^{A_j} (1 - a_j + \sum_{i=1}^r \alpha_j^{(i)} s_j)}{\prod_{j=n+1}^p \Gamma^{A_j} (a_j - \sum_{i=1}^r \alpha_j^{(i)} s_j) \prod_{j=1}^q \Gamma^{B_j} (1 - b_j + \sum_{i=1}^r \beta_j^{(i)} s_j)} \tag{1.3}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma^{C_j^{(i)}} (1 - c_j^{(i)} + \gamma_j^{(i)} s_i) \prod_{j=1}^{m_i} \Gamma^{D_j^{(i)}} (d_j^{(i)} - \delta_j^{(i)} s_i)}{\prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}} (c_j^{(i)} - \gamma_j^{(i)} s_i) \prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}} (1 - d_j^{(i)} + \delta_j^{(i)} s_i)} \tag{1.4}$$

For more details, see Nambisan et al [2].

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i = \sum_{j=1}^p A_j \alpha_j^{(i)} - \sum_{j=1}^q B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)} \delta_j^{(i)} \leq 0, i = 1, \dots, r \tag{1.5}$$

The integral (2.1) converges absolutely if

$$|arg(z_k)| < \frac{1}{2} \Delta_k \pi, k = 1, \dots, r \text{ where}$$

$$\Delta_k = - \sum_{j=n+1}^p A_j \alpha_j^{(k)} - \sum_{j=1}^q B_j \beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)} > 0 \tag{1.6}$$

The generalized polynomials of multivariables defined by Srivastava [5], is given in the following manner :

$$S_{N_1, \dots, N_v}^{\mathfrak{M}_1, \dots, \mathfrak{M}_v} [y_1, \dots, y_v] = \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \dots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \dots \frac{(-N_v)_{\mathfrak{M}_v K_v}}{K_v!} A[N_1, K_1; \dots; N_v, K_v] y_1^{K_1} \dots y_v^{K_v} \tag{1.7}$$

where  $\mathfrak{M}_1, \dots, \mathfrak{M}_v$  are arbitrary positive integers and the coefficients  $A[N_1, K_1; \dots; N_v, K_v]$  are arbitrary constants, real or complex.

Srivastava and Garg [7] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1, \dots, h_u} [z_1, \dots, z_u] = \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} (-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{z_1^{R_1} \dots z_u^{R_u}}{R_1! \dots R_u!} \tag{1.8}$$

The coefficients are  $B[E; R_1, \dots, R_u]$  arbitrary constants, real or complex.

We will note  $a_v = \frac{(-N_1)_{\mathfrak{M}_1 K_1}}{K_1!} \dots \frac{(-N_v)_{\mathfrak{M}_v K_v}}{K_v!} A[N_1, K_1; \dots; N_v, K_v]$  and

$$b_u = \frac{(-E)_{F_1 L_1 + \dots + F_u L_u} B(E; L_1, \dots, L_u)}{L_1! \dots L_u!} \tag{1.9}$$

## 2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [8 ,page 39 eq.30]

$$\frac{\prod_{j=1}^P \Gamma(A_j)}{\prod_{j=1}^Q \Gamma(B_j)} {}_P F_Q [(A_P); (B_Q); -(x_1 + \dots + x_r)] = \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \frac{\prod_{j=1}^P \Gamma(A_j + s_1 + \dots + s_r)}{\prod_{j=1}^Q \Gamma(B_j + s_1 + \dots + s_r)} \Gamma(-s_1) \dots \Gamma(-s_r) x_1^{s_1} \dots x_r^{s_r} ds_1 \dots ds_r \tag{2.1}$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of  $\Gamma(A_j + s_1 + \dots + s_r)$  are separated from those of  $\Gamma(-s_j), j = 1, \dots, r$ . The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of  $\Gamma(-s_j), j = 1, \dots, r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^l [1 - \tau_j (t-a)^{h_j}]^{-\lambda_j} \prod_{j=1}^k (f_j t + g_j)^{\sigma_j} dt = (b-a)^{\alpha+\beta-1} B(\alpha, \beta) \prod_{j=1}^k (af_j + g_j)^{\sigma_j}$$

$$F_{1:0, \dots, 0; 0, \dots, 0}^{1:1, \dots, 1; 1, \dots, 1} \left( \begin{matrix} (\alpha : h_1, \dots, h_l, 1, \dots, 1) : (\lambda_1 : 1), \dots, (\lambda_l : 1); (-\sigma_1 : 1), \dots, (-\sigma_k : 1) \\ \dots \\ (\alpha + \beta : h_1, \dots, h_l, 1, \dots, 1) : -, \dots, -; -, \dots, - \end{matrix} \right)$$

$$; \tau_1 (b-a)^{h_1}, \dots, \tau_l (b-a)^{h_l}, -\frac{(b-a)f_1}{af_1 + g_1}, \dots, -\frac{(b-a)f_k}{af_k + g_k} \quad (2.2)$$

where  $a, b \in \mathbb{R} (a < b), \alpha, \beta, f_i, g_i, \sigma_i, \tau_j, h_j \in \mathbb{C}, \lambda_j \in \mathbb{R}^+ (i = 1, \dots, k; j = 1, \dots, l)$

$$\min(Re(\alpha), Re(\beta)) > 0, \max_{1 \leq j \leq l} \{ |\tau_j (b-a)^{h_j}| \} < 1, \max_{1 \leq j \leq k} \left\{ \left| \frac{(b-a)f_j}{af_j + g_j} \right| \right\} < 1,$$

and  $F_{1:0, \dots, 0; 0, \dots, 0}^{1:1, \dots, 1; 1, \dots, 1}$  is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust [6, page 454] given by :

$$F_{1:0, \dots, 0; 0, \dots, 0}^{1:1, \dots, 1; 1, \dots, 1} \left( \begin{matrix} (\alpha : h_1, \dots, h_l, 1, \dots, 1) : (\lambda_1 : 1), \dots, (\lambda_l : 1); (-\sigma_1 : 1), \dots, (-\sigma_k : 1) \\ \dots \\ (\alpha + \beta : h_1, \dots, h_l, 1, \dots, 1) : -, \dots, -; -, \dots, - \end{matrix} \right)$$

$$; \tau_1 (b-a)^{h_1}, \dots, \tau_l (b-a)^{h_l}, -\frac{(b-a)f_1}{af_1 + g_1}, \dots, -\frac{(b-a)f_k}{af_k + g_k} \Bigg) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \prod_{j=1}^l \Gamma(\lambda_j) \prod_{j=1}^k \Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}} \int_{L_1} \dots \int_{L_{l+k}} \frac{\Gamma(\alpha + \sum_{j=1}^l h_j w_j + \sum_{j=1}^k w_{l+j})}{\Gamma(\alpha + \beta + \sum_{j=1}^l h_j w_j + \sum_{j=1}^k w_{l+j})} \prod_{j=1}^l \Gamma(\lambda_j + w_j) \prod_{j=1}^k \Gamma(-\sigma_j + w_{l+j})$$

$$\prod_{j=1}^{l+k} \Gamma(-w_j) z_1^{w_1} \dots z_l^{w_l} z_{l+1}^{w_{l+1}} \dots, w_{l+k}^{w_{l+k}} dw_1 \dots dw_{l+k} \quad (2.3)$$

Here the contour  $L'_j s$  are defined by  $L_j = L_{w\zeta_j \infty} (Re(\zeta_j) = v''_j)$  starting at the point  $v''_j - \omega\infty$  and terminating at the point  $v''_j + \omega\infty$  with  $v''_j \in \mathbb{R} (j = 1, \dots, l)$  and each of the remaining contour  $L_{l+1}, \dots, L_{l+k}$  run from  $-\omega\infty$  to  $\omega\infty$

(2.2) can be easily established by expanding  $\prod_{j=1}^l [1 - \tau_j (t-a)^{h_j}]^{-\lambda_j}$  by means of the formula :

$$(1-z)^{-\alpha} = \sum_{r=0}^{\infty} \frac{(\alpha)_r}{r!} z^r (|z| < 1) \quad (2.4)$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [6, page 454].

### 3. Eulerian integral

In this section , we evaluate a general Eulerian integral with the product of two multivariable Aleph-functions, class of multivariable polynomials and generalized hypergeometric function. We note

$$\theta_i = \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j^{(i)}}, \zeta_j^{(i)} > 0 (i = 1, \dots, r); \theta'_i = \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j'^{(i)}}, \zeta_j'^{(i)} > 0 (i = 1, \dots, s)$$

$$\theta''_i = \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j''^{(i)}}, \zeta_j''^{(i)} > 0 (i = 1, \dots, u)$$

$$\theta'''_i = \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j'''^{(i)}}, \zeta_j'''^{(i)} > 0 (i = 1, \dots, v) \tag{3.1}$$

$$X = m_1, n_1; \dots; m_r, n_r; 1, 0; \dots; 1, 0; 1, 0; \dots; 1, 0; 1, 0; \dots; 1, 0 \tag{3.2}$$

$$Y = p_1, q_1; \dots; p_r, q_r; 0, 1; \dots; 0, 1; 0, 1; \dots; 0, 1; 0, 1; \dots; 0, 1 \tag{3.3}$$

$$A = (a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)}, 0, \dots, 0, 0, \dots, 0, 0, \dots, 0; A_j)_{1,p} \tag{3.4}$$

$$B = (b_j; \beta_j^{(1)}, \dots, \beta_j^{(r)}, 0, \dots, 0, 0, \dots, 0, 0, \dots, 0; B_j)_{1,q} \tag{3.5}$$

$$C = (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \dots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r}; (1, 0; 1) \dots; (1, 0; 1)$$

$$(1, 0; 1); \dots; (1, 0; 1); (1, 0; 1); \dots; (1, 0; 1) \tag{3.6}$$

$$D = (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \dots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r}; (0, 1; 1); \dots; (0, 1; 1);$$

$$(0, 1; 1); \dots; (0, 1; 1); (0, 1; 1); \dots; (0, 1; 1) \tag{3.7}$$

$$K_1 = (1 - \alpha - \sum_{i=1}^u R_i a_i - \sum_{i=1}^v K_i a'_i; \mu_1, \dots, \mu_r, \mu'_1, \dots, \mu'_s, h_1, \dots, h_l, 1, \dots, 1) \tag{3.8}$$

$$K_2 = (1 - \beta - \sum_{i=1}^u R_i b_i - \sum_{i=1}^v K_i b'_i; \rho_1, \dots, \rho_r, \rho'_1, \dots, \rho'_s, 0, \dots, 0, 0, \dots, 0) \tag{3.9}$$

$$K_P = [1 - A_j; 0, \dots, 0, 1, \dots, 1, 0, \dots, 0, 0, \dots, 0]_{1,P} \tag{3.10}$$

$$K_j = [1 - \lambda_j - \sum_{i=1}^u R_i \zeta_j''^{(i)} - \sum_{i=1}^v K_i \zeta_j'''^{(i)}; \zeta_j^{(1)}, \dots, \zeta_j^{(r)}, \zeta_j'^{(1)}, \dots, \zeta_j'^{(s)},$$

$$0, \dots, 1, \dots, 0, 0, \dots, 0]_{1,l} \tag{3.11}$$

$$K'_j = [1 + \sigma_j - \sum_{i=1}^u R_i \lambda_j^{''(i)} - \sum_{i=1}^v K_i \lambda_j^{'''(i)}; \lambda_j^{(1)}, \dots, \lambda_j^{(r)}, \lambda_j^{'(1)} \dots, \lambda_j^{'(s)}, 0, \dots, 0, 0 \dots, 1, \dots, 0]_{1,k} \tag{3.12}$$

$$L_1 = (1 - \alpha - \beta - \sum_{i=1}^u R_i (a_i + b_i) - \sum_{i=1}^v K_i (a'_i + b'_i); \mu_1 + \rho_1, \dots, \mu_r + \rho_r, \mu'_1 + \rho'_1, \dots, \mu'_r + \rho'_r, h_1, \dots, h_l, 1, \dots, 1) \tag{3.13}$$

$$L_Q = [1 - B_j; 0, \dots, 0, 1, \dots, 1, 0, \dots, 0, 0 \dots, 0]_{1,Q} \tag{3.14}$$

$$L_j = [1 - \lambda_j - \sum_{i=1}^u R_i \zeta_j^{''(i)} - \sum_{i=1}^v K_i \zeta_j^{'''(i)}; \zeta_j^{(1)}, \dots, \zeta_j^{(r)}, \zeta_j^{'(1)} \dots, \zeta_j^{'(s)}, 0, \dots, 0, 0 \dots, 0]_{1,l} \tag{3.15}$$

$$L'_j = [1 + \sigma_j - \sum_{i=1}^u R_i \lambda_j^{''(i)} - \sum_{i=1}^v K_i \lambda_j^{'''(i)}; \lambda_j^{(1)}, \dots, \lambda_j^{(r)}, \lambda_j^{'(1)} \dots, \lambda_j^{'(s)}, 0, \dots, 0, 0, \dots, 0]_{1,k} \tag{3.16}$$

$$B_{u,v} = (b - a)^{\sum_{i=1}^v K_i (a'_i + b'_i) + \sum_{i=1}^u (a_i + b_i) R_i} \left\{ \prod_{j=1}^h (a f_j + g_j)^{-\sum_{i=1}^v K_i \lambda_i^{'''} - \sum_{i=1}^u \lambda_i^{''} R_i} \right\} \tag{3.17}$$

We have the general Eulerian integral

$$\int_a^b (t - a)^{\alpha - 1} (b - t)^{\beta - 1} \prod_{j=1}^l [1 - \tau_j (t - a)^{h_j}]^{-\lambda_j} \prod_{j=1}^k (f_j t + g_j)^{\sigma_j} S_L^{h_1, \dots, h_u} \left( \begin{matrix} z_1'' \theta_1'' (t - a)^{a_1} (b - t)^{b_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{''(1)}} \\ \vdots \\ z_u'' \theta_u'' (t - a)^{a_u} (b - t)^{b_u} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{''(u)}} \end{matrix} \right) S_{N_1, \dots, N_v}^{\mathfrak{M}_1, \dots, \mathfrak{M}_v} \left( \begin{matrix} z_1''' \theta_1''' (t - a)^{a'_1} (b - t)^{b'_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{'''(1)}} \\ \vdots \\ z_v''' \theta_v''' (t - a)^{a'_v} (b - t)^{b'_v} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{'''(v)}} \end{matrix} \right) I \left( \begin{matrix} z_1 \theta_1 (t - a)^{\mu_1} (b - t)^{\rho_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \vdots \\ z_r \theta_r (t - a)^{\mu_r} (b - t)^{\rho_r} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(r)}} \end{matrix} \right)$$



$a_j(j = 1, \dots, p), b_j(j = 1, \dots, q), c_j^{(i)}(j = 1, \dots, p_i, i = 1, \dots, r), d_j^{(i)}(j = 1, \dots, q_i, i = 1, \dots, r) \in \mathbb{C}$

The exponents  $A_j(j = 1, \dots, p), B_j(j = 1, \dots, q), C_j^{(i)}(j = 1, \dots, p_i; i = 1, \dots, r), D_j^{(i)}(j = 1, \dots, q_i; i = 1, \dots, r)$  of various gamma function involved in (1.3) and (1.4) may take non integer values.

$$(C) \max_{1 \leq j \leq k} \left\{ \left| \frac{(b-a)f_i}{af_i + g_i} \right| \right\} < 1$$

$$(D) \operatorname{Re} \left[ \alpha + \sum_{i=1}^r \mu_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > 0 \text{ and } \operatorname{Re} \left[ \beta + \sum_{i=1}^r \rho_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right] > 0$$

$$(E) \operatorname{Re} \left( \alpha + \sum_{i=1}^v K_i a'_i + \sum_{i=1}^u R_i a_i + \sum_{i=1}^r \mu_i s_i + \sum_{i=1}^l h_i w_i \right) > 0; \operatorname{Re} \left( \beta + \sum_{i=1}^v K_i b'_i + \sum_{i=1}^u R_i b_i + \sum_{i=1}^r \rho_i s_i \right) > 0$$

$$\operatorname{Re} \left( \lambda_j + \sum_{i=1}^v K_i \lambda_j'''^{(i)} + \sum_{i=1}^u R_i \lambda_j''^{(i)} + \sum_{i=1}^r s_i \zeta_j^{(i)} \right) > 0 (j = 1, \dots, l)$$

$$\operatorname{Re} \left( -\sigma_j + \sum_{i=1}^v K_i \lambda_j'''^{(i)} + \sum_{i=1}^u R_i \lambda_j''^{(i)} + \sum_{i=1}^r s_i \lambda_j^{(i)} \right) > 0 (j = 1, \dots, k)$$

$$(F) U_i = \sum_{j=1}^p A_j \alpha_j^{(i)} - \sum_{j=1}^q B_j \beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)} \gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)} \delta_j^{(i)} \leq 0, i = 1, \dots, r$$

$$(G) \Delta_k = - \sum_{j=n+1}^p A_j \alpha_j^{(k)} - \sum_{j=1}^q B_j \beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)} \delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)} \gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)} \gamma_j^{(k)}$$

$$-\mu_i - \rho_i - \sum_{l=1}^k \lambda_l^{(i)} > 0 \quad (i = 1, \dots, r)$$

$$(H) \left| \arg \left( z_i \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j^{(i)}} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| < \frac{1}{2} \Delta_i \pi \quad (a \leq t \leq b; i = 1, \dots, r)$$

(I)  $P \leq Q + 1$ . The equality holds, when , in addition,

$$\text{either } P > Q \text{ and } \left| \left( z'_i \sum_{i=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j^{(i)}} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right|^{\frac{1}{Q-P}} < 1 \quad (a \leq t \leq b)$$

$$\text{or } P \leq Q \text{ and } \max_{1 \leq i \leq k} \left[ \left| \left( z'_i \sum_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j^{(i)}} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| \right] < 1 \quad (a \leq t \leq b)$$

**Proof**

To prove (3.18), first expressing a class of multivariable polynomials  $S_{N_1, \dots, N_v}^{\mathfrak{M}_1, \dots, \mathfrak{M}_v} [.]$  defined by Srivastava [5] in serie with the help of (1.7), a class of multivariable polynomials  $S_L^{h_1, \dots, h_u} [.]$  defined by Srivastava et al [7] in serie with the help of (1.8) and we interchange the order of summations and t-integral (which is permissible under the

conditions stated). Expressing the I-functions of r-variables defined by Prasad [1] in terms of Mellin-Barnes type contour integral with the help of (1.2) and the generalized hypergeometric function  ${}_pF_Q(\cdot)$  in Mellin-Barnes contour integral with the help of (2.1) and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collect the power of  $[1 - \tau_j(t - a)^{h_i}]$  with  $(i = 1, \dots, r; j = 1, \dots, l)$  and collect the power of  $(f_j t + g_j)$  with  $j = 1, \dots, k$ . Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the  $(r + s + k + l)$  dimensional Mellin-Barnes integral to multivariable Aleph-function, we obtain the equation (3.18).

#### 4. Particular cases

a) If  $A_j = B_j = C_j^{(i)} = D_j^{(i)} = 1$ , the multivariable I-function defined by Nambisan et al [2] reduces to multivariable H-function defined by Srivastava et al [9].

The following generalized Eulerian integral concerning the multivariable H-function under the same notations and conditions that (3.18) with  $A_j = B_j = C_j^{(i)} = D_j^{(i)} = 1$  :

$$\int_a^b (t - a)^{\alpha - 1} (b - t)^{\beta - 1} \prod_{j=1}^l [1 - \tau_j(t - a)^{h_i}]^{-\lambda_j} \prod_{j=1}^k (f_j t + g_j)^{\sigma_j}$$

$$S_L^{h_1, \dots, h_u} \left( \begin{matrix} z_1'' \theta_1'' (t - a)^{a_1} (b - t)^{b_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \vdots \\ z_u'' \theta_u'' (t - a)^{a_u} (b - t)^{b_u} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(u)}} \end{matrix} \right)$$

$$S_{N_1, \dots, N_v}^{\mathfrak{M}_1, \dots, \mathfrak{M}_v} \left( \begin{matrix} z_1''' \theta_1''' (t - a)^{a'_1} (b - t)^{b'_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)'}} \\ \vdots \\ z_v''' \theta_v''' (t - a)^{a'_v} (b - t)^{b'_v} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(v)'}} \end{matrix} \right)$$

$$H \left( \begin{matrix} z_1 \theta_1 (t - a)^{\mu_1} (b - t)^{\rho_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \vdots \\ z_r \theta_r (t - a)^{\mu_r} (b - t)^{\rho_r} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(r)}} \end{matrix} \right)$$

$${}_pF_Q \left[ (A_P); (B_Q); - \sum_{i=1}^s z'_i \theta'_i (t - a)^{\mu'_i} (b - t)^{\rho'_i} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(i)'}} \right] dt =$$

$$(b - a)^{\alpha + \beta - 1} \frac{\prod_{j=1}^Q \Gamma(B_j)}{\prod_{j=1}^P \Gamma(A_j)} \prod_{j=1}^s (a f_j + g_j)^{\sigma_j} \sum_{K_1=0}^{[N_1/\mathfrak{M}_1]} \dots \sum_{K_v=0}^{[N_v/\mathfrak{M}_v]} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} \prod_{i=1}^v z_i^{\mathfrak{M}_i K_i} \prod_{k=1}^u z_i^{\mathfrak{M}_i R_k} a_v b_u B_{u,v}$$







under the same notations and conditions that (3.22)

$$\text{where } b'_u = \frac{(-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u)}{R_1! \dots R_u!}, B[E; R_1, \dots, R_v] \text{ is defined by (4.2)}$$

**Remark:**

By the following similar procedure, the results of this document can be extended a class of multivariable polynomials defined by Srivastava et al [7] and Srivastava [5]. The formula (3.18) is an extension of result concerning the multivariable H-function defined by Srivastava et al [9]. For more details, see Saigo et al [4].

## 5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of the multivariable I-function defined by Kumari et al [2], the classes of multivariable polynomials and generalized hypergeometric function with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

## REFERENCES

- [1] B. L. J. Braaksma, "Asymptotic expansions and analytic continuations for a class of Barnes integrals," *Compositio Mathematica*, vol. 15, pp. 239–341, 1964.
- [2] Prathima J. Nambisan V. and Kurumujji S.K. A Study of I-function of Several Complex Variables, *International Journal of Engineering Mathematics* Vol(2014) , 2014 page 1-12
- [3] Saigo M. and Saxena R.K. Unified fractional integral formulas for the multivariable H-function I. *J.Fractional Calculus* 15 (1999), page 91-107.
- [4] Saigo M. and Saxena R.K. Unified fractional integral formulas for the multivariable H-function III. *J.Fractional Calculus* 20 (2001), page 45-68.
- [5] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, *Pacific. J. Math.* 177(1985), page183-191.
- [6] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet function. *Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math* 31(1969) page 449-457.
- [7] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function. *Rev. Roumaine Phys.* 32(1987), page 685-692.
- [8] Srivastava H.M. and Karlsson P.W. *Multiple Gaussian Hypergeometric series*. Ellis.Horwood. Limited. New-York, Chichester. Brisbane. Toronto , 1985.
- [9] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. *Comment. Math. Univ. St. Paul.* 24(1975), p.119-137.

Personal adress : 411 Avenue Joseph Raynaud  
Le parc Fleuri , Bat B  
83140 , Six-Fours les plages  
Tel : 06-83-12-49-68  
Department : VAR  
Country : FRANCE