W-Hausdorffness in Soft Bitopological Spaces

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Abstract: *In this paper the concept of W-Hausdorffness in soft bitopological spaces is introduced in three different ways by referring the definition of soft W- Hausdorffness introduced by Sruthi, Vijayalakshmi and Kalaichelvi [9].*

Keywords: *Soft set, Soft topological space, Soft W-Hausdorff space, Soft Bitopological space.*

I. INTRODUCTION

Soft set theory is one of the recent topics gaining significance in finding rational and logical solutions to various real life problems which involve uncertainty, impreciseness and vagueness. In 1999, Molodstov[6] initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainty. In 2011, Shabir and Naz[8] defined soft topological spaces and studied separation axioms. In 1963, Kelly[4], first initiated the concept of bi topological space. He defined a bitopological space (X, τ_1, τ_2) to be a set X equipped with two topologies τ_1 and τ_2 on X and initiated the systematic study of bitopological space. Also he studied separation properties of bitopological space.

In section II of this paper, preliminary definitions regarding soft sets, soft topological spaces and soft bitopological spaces are given. In section III of this paper, the concept of W-Hausdorffness in soft bitopological spaces is introduced in three different ways by referring the definition of soft W-Hausdorffness introduced by Sruthi, Vijayalakshmi, Kalaichelvi[9].

II. PRELIMINARY DEFINITIONS

Throughout this paper, X denotes initial universe and E denotes the set of parameters for the universe X.

Definition: 2.1 [6]

Let X be an initial universe and E be a set of parameters. Let P(X) denotes the power set of X and A be a nonempty subset of E. A pair (F, A) denoted by F_A is called a soft set over X, where F is a mapping given by F: A → P(X).In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, F(e) may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \phi$

i.e. $F_A=\{F(e): e \in A \subseteq E;\ F: A \to P(X)\}$.

The family of all these soft sets over X with respect to the parameter set E is denoted by $SS(X)_E$.

Definition 2.2[8]

Let $F_A, G_B \in SS(X)_E$. Then F_A is soft subset of G_B , denoted by $F_A \tilde{\subseteq} G_B$, if

(1) $A \subseteq B$, and

(2) $F(e) \subseteq G(e), \forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$

Definition 2.3 [8]

Two soft subsets F_A and G_B over a common universe X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4 [2]

The complement of a soft set (F, A) denoted by (F, A)′ is defined by (F, A)′ = (F′, A) ,F′ : A ⟶ P(X) is a mapping given by F′(e) =X – F(e); ∀ e ∈ A and F′ is called the soft complement function of F.Clearly (F′)′ is the same as F and ((F, A)′)′= (F, A).

Definition 2.5 [8]

A soft set (F, A) over X is said to be a NULL softset denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ (null set).

Definition 2.6 [8]

A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A$, F(e) = X. Clearly we have $X_A' = \phi_A$ and $\phi_A' = X_A$.

Definition 2.7 [8]

The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), e \in A - B, \\ G(e), e \in B - A, \\ F(e) \cup G(e), e \in A \cap B \end{cases}$$

Definition 2.8 [8]
The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 2.9 [8]
Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X, if
(1) ϕ, X belong to τ
(2) the union of any number of soft sets in τ belongs to τ
(3) the intersection of any two soft sets in τ belongs to τ

Definition 2.10 [1]
Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and Y be a non null subset of X. Then the soft subset of (F, E) over Y denoted by (F_Y, E) is defined as follows:
$F_Y(e) = Y \cap F(e), \forall e \in E$
In other words, $(F_Y, E) = Y_E \cap (F, E)$.

Definition 2.11 [1]
Let (X, τ, E)be a soft topological space and Y be a non null subset of X. Then $\tau_Y = \{(F_Y, E): (F, E) \in \tau\}$ is said to be the relative soft topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E)

Definition 2.12 [6]
Let $F_A \in SS(X)_E$ and $G_B \in SS(Y)_K$. The cartesian product $F_A \otimes G_B$is defined by $(F_A \otimes G_B)(e, k) = F_A(e) \times G_B(k), \forall (e,k) \in A \times B$.According to this definition $F_A \otimes G_B$is a soft set over X×Y and its parameter set is $E \times K$.

Definition 2.13[2]
Let (X, τ_X, E) and (Y, τ_Y, K) be two soft topological spaces. The soft product topology $\tau_X \otimes \tau_Y$ over X×Y with respect to E×K is the soft topology having the collection $\{F_E \otimes G_K / F_E \in \tau_X, G_K \in \tau_Y\}$ as the basis.

Definition 2.14 [9]
A soft topological space (X, τ, E) is said to be soft W-Hausdorff space of type 1 denoted by $(SW - H)_1$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist F_A ,$G_B \in \tau$, such that $F_A(e_1) = X, G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Definition 2.15 [9]
Let (X, τ, E) be a soft topological space and $H \subseteq E$. Then (X, τ_H, H) is called soft p-subspace of (X, τ, E) relative to the parameter set H where $\tau_H = \{(F_A)/H: H \subseteq A \subseteq E, F_A \in \tau\}$ and $(F_A)/H$ is the restriction map on H.

Proposition 2.16 [9]
(1) Soft subspace of a $(SW - H)_1$space is $(SW - H)_1$.
(2) Soft p-subspace of a $(SW - H)_1$ space is $(SW - H)_1$.
(3) Product of two $(SW - H)_1$spaces is$(SW - H)_1$.

Definition 2.17[4]
Let X be a non-empty set and τ_1 and τ_2 be two different topologies on X. Then (X, τ_1, τ_2) is called a bitopological space.

Definition 2.18 [3]
Let (X, τ_{1X}, E) and (X, τ_{2X}, E) be the two different soft topological spaces on X. Then $(X, \tau_{1X}, \tau_{2X}, E)$ is called a Soft bi topological space if the two soft topologies τ_{1X} and τ_{2X} independently satisfy the axioms of soft topology. The members of τ_{1X} are called τ_{1X} soft open sets and the complements of τ_{1X} soft open sets are called τ_{1X} soft closed sets. Similarly, The members of τ_{2X} are called τ_{2X} soft open sets and the complements of τ_{2X} soft open sets are called τ_{2X} soft closed sets.

Definition 2.19 [3] Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a soft bi topological space over X and Y be a non-empty subset of X. Then $\tau_{1Y} = \{(F_Y, E) : (F, E) \in \tau_{1X}\}$ and $\tau_{2Y} = \{(G_Y, E) : (G, E) \in \tau_{2X}\}$ are said to be the relative topologies on Y and $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ is called a Soft subspace of $(X, \tau_{1X}, \tau_{2X}, E)$.

III. W-HAUSDORFFNESS IN SOFT BITOPOLOGICAL SPACES

Definition 3.1
A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be Soft W-Hausdorff space of type 1 or soft W- T_2 space of type 1 denoted by $(SBW - H)_1$ if it is $(SW - H)_1$ with respect to τ_{1X} or $(SW - H)_1$ with respect to τ_{2X}.

Definition 3.2
A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be Soft W- Hausdorff space of type 2 denoted by $(SBW - H)_2$ if for every $e_1, e_2 \in E, e_1 \neq e_2$ there exist $F_A \in \tau_{1X}$, $G_B \in \tau_{2X}$ such that $F_A(e_1) = X, G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 3.3
Soft subspace of a $(SBW - H)_1$space is $(SBW - H)_1$.

Proof

Let (X, τ_{1X}, τ_{2X}, E) be a $(SBW-H)_1$ space. Then it is $(SW\text{-}H)_1$ with respect to τ_{1X} or $(SW\text{-}H)_1$ with respect to τ_{2X} . Let Y be a non null subset of X. Let {Y, τ_{1Y} , τ_{2Y} , E} be a soft subspace of (X, τ_{1X}, τ_{2X}, E) . From the proposition 2.16(1), a soft subspace of $(SW\text{-}H)_1$ space is $(SW\text{-}H)_1$. Therefore, {Y, τ_{1Y} , τ_{2Y} , E} is $(SW\text{-}H)_1$ with respect to τ_{1Y} or $(SW\text{-}H)_1$ with respect to τ_{2Y}. Hence {Y, τ_{1Y} , τ_{2Y} , E} is $(SBW-H)_1$.

Theorem 3.4

Soft subspace of a $(SBW-H)_2$ space is $(SBW-H)_2$.

Proof

Let (X, τ_{1X}, τ_{2X}, E) be a $(SBW-H)_2$ space. Let Y be a non null subset of X. Let {Y, τ_{1Y} , τ_{2Y} , E} be a soft subspace of (X, τ_{1X}, τ_{2X}, E) where

τ_{1Y} = {(F_Y, E) : (F, E) ∈ τ_{1X}} and

τ_{2Y} = {(G_Y, E) : (G, E) ∈ τ_{2X}} are said to be the relative topologies on Y . Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist $F_A \in \tau_{1X}$, $G_B \in \tau_{2X}$ such that $F_A(e_1) = X, G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Therefore $((F_A)_Y, E) \in \tau_{1Y}, ((G_B)_Y, E) \in \tau_{2Y}$

$$
\begin{aligned}
\text{Also } (F_A)_Y(e_1) &= Y \cap F_A(e_1)\\
&= Y \cap X\\
&= Y\\
(G_B)_Y(e_2) &= Y \cap G_B(e_2)\\
&= Y \cap X\\
&= Y\\
((F_A)_Y \cap G_B)_Y)(e) &= ((F_A \cap G_B)_Y)(e)\\
&= Y \cap (F_A \cap G_B)(e)\\
&= Y \cap \tilde{\phi}(e)\\
&= Y \cap \phi\\
&= \phi\\
(F_A)_Y \cap (G_B)_Y &= \tilde{\phi}
\end{aligned}
$$

Hence {Y, τ_{1Y} , τ_{2Y} , E}is $(SBW-H)_2$.

Definition 3.5

Let (X, τ_{1X}, τ_{2X}, E) be a soft bitopological space over X and H ⊆ E. Then {X, τ_{1H} , τ_{2H} , H} is called Soft p-subspace of (X, τ_{1X}, τ_{2X}, E) relative to the parameter set H where

$\tau_{1H} = \{(F_A)/H\text{: } H \subseteq A \subseteq E,\ F_A \in \tau_{1X}\}$,

$\tau_{2H} = \{(G_B)/H\text{: } H \subseteq B \subseteq E,\ G_B \in \tau_{2X}\}$ and $(F_A)/H$, $(G_B)/H$ are the restriction maps on H.

Theorem 3.6

Soft p-subspace of a $(SBW-H)_1$ space is $(SBW-H)_1$.

Proof

Let (X, τ_{1X}, τ_{2X}, E) be a $(SBW-H)_1$ space. Then it is $(SW\text{-}H)_1$ with respect to τ_{1X} or $(SW\text{-}H)_1$ with respect to τ_{2X} . Let H ⊆ E. Let (X,τ_{1H},τ_{2H}, H) be a soft p-subspace of (X, τ_{1X}, τ_{2X}, E) relative to the parameter set H. From the proposition 2.16(2), the soft p-subspace of $(SW\text{-}H)_1$ space is $(SW\text{-}H)_1$. Therefore, the soft p-subspace of $(SBW-H)_1$ is $(SW\text{-}H)_1$ with respect to τ_{1H} or with respect to τ_{2H}. Hence (X,τ_{1H},τ_{2H}, H) is $(SBW-H)_1$.

Theorem 3.7

Soft p-subspace of a $(SBW-H)_2$ space is $(SBW-H)_2$.

Proof

Let (X, τ_{1X}, τ_{2X}, E) be a $(SBW-H)_2$ space. Let H ⊆ E. Let (X,τ_{1H}, τ_{2H}, H) be a soft p-subspace of (X, τ_{1X}, τ_{2X}, E) relative to the parameter set H where

$\tau_{1H} = \{(F_A)/H\text{: } H \subseteq A \subseteq E,\ F_A \in \tau_{1X}\}$,

$\tau_{2H} = \{(G_B)/H\text{: } H \subseteq B \subseteq E,\ G_B \in \tau_{2X}\}$.

Consider $h_1, h_2 \in H$, $h_1 \neq h_2$.Then $h_1, h_2 \in E$. Therefore, there exist $F_A \in \tau_{1X}$, $G_B \in \tau_{2X}$ such that $F_A(e_1) = X, G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Therefore $(F_A)/H \in \tau_{1H}$, , $(G_B)/H \in \tau_{2H}$

$$
\begin{aligned}
\text{Also } ((F_A)/H)(h_1) &= F_A(h_1) = X\\
((G_B)/H)(h_2) &= G_B(h_2) = X \text{ and}\\
((F_A)/H) \cap ((G_B)/H) &= (F_A \cap G_B)/H\\
&= \tilde{\phi}/H\\
&= \tilde{\phi}
\end{aligned}
$$

Hence (X,τ_{1H},τ_{2H}, H) is $(SBW-H)_2$.

Theorem 3.8

Product of two $(SBW-H)_1$spaces is$(SBW-H)_1$.

Proof

Let (X, τ_{1X}, τ_{2X}, E) and {Y, τ_{1Y} , τ_{2Y} , K} be two $(SBW-H)_1$ spaces. Then (X, τ_{1X}, τ_{2X}, E) is $(SW\text{-}H)_1$ with respect to τ_{1X} or $(SW\text{-}H)_1$ with respect to τ_{2X} and {Y, τ_{1Y} , τ_{2Y} , K} is $(SW\text{-}H)_1$ with respect to τ_{1Y} or $(SW\text{-}H)_1$ with respect to τ_{2Y} . From proposition 2.16(3), the product of two $(SW\text{-}H)_1$ spaces is $(SW\text{-}H)_1$. Hence the product of two $(SBW-H)_1$ spaces is $(SBW-H)_1$.

Theorem 3.9

Product of two $(SBW-H)_2$ spaces is$(SBW-H)_2$.

Proof

Let (X, τ_{1X}, τ_{2X}, E) and {Y, τ_{1Y} , τ_{2Y} , K} be two $(SBW-H)_2$ spaces. Consider two distinct points (e_1, k_1), $(e_2, k_2) \in E \times K$. Either $e_1 \neq e_2$ or $k_1 \neq k_2$.Assume $e_1 \neq e_2$. Since (X, τ_{1X}, τ_{2X}, E) is $(SBW-H)_2$, there exist $F_A \in \tau_{1X}$, $G_B \in \tau_{2X}$ such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Therefore $F_A \otimes Y_K \in \tau_{1X} \otimes \tau_{1Y}$, $G_B \otimes Y_K \in \tau_{2X} \otimes \tau_{2Y}$

$$
\begin{aligned}
(F_A \otimes Y_K)\,(e_1, k_1) &= F_A(e_1) \times Y_K(k_1)\\
&= X \times Y
\end{aligned}
$$

$(G_B \otimes Y_K)\ (e_2, k_2) = G_B(e_2) \times Y_K(k_2)$
$= X \times Y$

If for any (e,k)$\in$ $(E \times K)$, $(F_A \otimes Y_K)\ (e,k) \neq \phi$
$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$
$\Rightarrow F_A(e) \times Y \neq \phi$
$\Rightarrow F_A(e) \neq \phi$
$\Rightarrow G_B(e) = \phi$
$(\text{since} F_A \cap G_B = \tilde{\phi} \Rightarrow F_A(e) \cap G_B(e) = \phi)$
$\Rightarrow G_B(e) \times Y_K(k) = \phi$
$\Rightarrow (G_B \otimes Y_K)(e,k) = \phi$
$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$

Assume $k_1 \neq k_2$. Since $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ is $(SBW-H)_2$, there exist $F_A \in \tau_{1Y}$, $G_B \in \tau_{2Y}$, such that $F_A(k_1) = Y, G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$.
Therefore $X_E \otimes F_A \in \tau_{1X} \otimes \tau_{1Y}, X_E \otimes G_B \in \tau_{2X} \otimes \tau_{2Y}$

$(X_E \otimes F_A)\ (e_1, k_1) = X_E(e_1) \times F_A(k_1)$
$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$
$= X \times Y$

If for any (e,k)$\in$ $E \times K$, $(X_E \otimes F_A)(e,k) \neq \phi$
$\Rightarrow X_E(e) \times F_A(k) \neq \phi$
$\Rightarrow X \times F_A(k) \neq \phi$
$\Rightarrow F_A(k) \neq \phi$
$\Rightarrow G_B(k) = \phi$
$(\text{Since} F_A \cap G_B = \tilde{\phi} \Rightarrow F_A(k) \cap G_B(k) = \phi)$
$\Rightarrow X_E(e) \times G_B(k) = \phi$
$\Rightarrow (X_E \otimes G_B)(e,k) = \phi$
$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_{1X} \otimes \tau_{1Y}, \tau_{2X} \otimes \tau_{2Y}, E \times K)$ is $(SBW-H)_2$.

IV. CONCLUSION

In this paper the concept of W-Hausdorffness in soft bitopological spaces is introduced and some basic properties regarding this concept are proved.

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