

A Study of General First-order Partial Differential Equations Using Homotopy Perturbation Method

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Abstract

In this work, we have studied a general class of linear first-order partial differential equations which is used as mathematical models in many physically significant fields and applied science. The homotopy perturbation method (HPM) has been used for solving generalized linear first-order partial differential equation. Also, we have tested the HPM on the solving of different implementations which show the efficiency and accuracy of the method. The approximated solutions are agree well with analytical solutions for the tested problems Moreover, the approximated solutions proved that the proposed method to be efficient and high accurate.

Keyword: First order; Homotopy; HPM; Partial Differential Equation ;PDE.

1 Introduction

The most important mathematical models for physical phenomena is the differential equation. Motion of objects, Fluid and heat flow, bending and cracking of materials, vibrations, chemical reactions and nuclear reactions are all modeled by systems of differential equations. Moreover, Numerous mathematical models in science and engineering are expressed in terms of unknown quantities and their derivatives. Many applications of differential equations (DEs), particularly ODEs of different orders, can be found in the mathematical modeling of real life problems(Mechee et al. (2014)).

The homotopy perturbation method (HPM), which is a well-known, is efficient technique to find the approximate solutions for ordinary and partial differential equations which describe different fields of science, physical phenomena, engineering, mechanics, and so on. HPM was proposed by Ji-Huan He in 1999 for solving linear and nonlinear differential equations and integral equations. Many researchers used HPM to approximate the solutions of differential equations and integral equations(Yıldırım (2010), Jalaal et al. (2010) & Ma et al. (2008)).

Many researchers published some papers in solving some classes differential equations using HPM. For example, Chun & Sakthivel (2010) used HPM for solving a linear and nonlinear second-order two-point boundary value problems while Gülkaç (2010) was solved the Black-Scholes equation for a simple European option in this method to obtain a new efficient recurrent relation to solve Black-Scholes equation. Moreover, numerous researches used HPM for solving nonlinear differential equations, Vahidi et al. (2011) was solved nonlinear DEs, which yields the Maclaurin series of the exact solution, Chang &

Liou (2006) developed a third-order explicit approximation to find the roots of the dispersion relation for water waves that propagate over dissipative media, Zhou & Wu (2012) solved the nonlinear PB equation describing spherical and planar colloidal particles immersed in an arbitrary valence and mixed electrolyte solution, Öziş & Akçı (2011) solved certain non-linear, non-smooth oscillators, Yazdi (2013) solved nonlinear vibration analysis of functionally graded plate while He (2004) applied HPM for solving nonlinear oscillators with discontinuities, nonlinear Duffing equation and some nonlinear ODEs. For class of linear partial differential equations, Al-Saif & Abood (2011) solved the Korteweg-de Vries (KdV) equation and convergence study of HPM, Babolian et al. (2009) used HPM to solve time-dependent differential equations, Aswhad & Jaddoa (2016) solved advection Problem, vibrating beam equation linear and nonlinear PDEs and the system of nonlinear PDEs and Babolian et al. (2009) used the homotopy perturbation method to solve time-dependent differential. Also, many researchers used HPM for solving the class of non-linear PDEs, Yazdi (2013) was approximated solution for free nonlinear vibration of thin rectangular laminated FGM plates, Liao (2004) has solved nonlinear PDEs, Yildirim (2009) was used to implement the nonlinear Korteweg-de Vries equation, Md Nasrudin et al. (2014) combined HPM-Pad [U+FFFD] proximate to acquire the approximate analytical solution of the KdV equation, Taghipour (2010) solved Parabolic equations and Periodic equation linear and nonlinear PDEs, Janalizadeh et al. (2008) obtained the solution of a second-order non-linear wave equation, Fereidoon et al. (2011) utilized to derive approximate explicit analytical solution for the nonlinear foam drainage equation, Momani & Odibat (2007) modified the algorithm which provides approximate solutions in the form of convergent series with easily computable components, Babolian et al. (2009) solved time-dependent differential equations while He (2000), solved non-linear problems using the homotopy technique. However, for the system of DEs, Bataineh et al. (2009) solved systems of second-order BVPs, Javidi (2009) solved SEIR model, Wang & Song (2007) solution of a model for HIV infection of $CD4 + T$ cells, Rafei et al. (2007) solution of the system of nonlinear ordinary differential equations governing on the problem, Noor et al. (2013) solved the system of linear equations. Noor & Mohyud-Din (2008) solution of linear and non-linear sixth-order boundary value problems and system of differential equations, Javidi (2009) solved system of linear Fredholm integral equations (LFIEs). Yusufoglu (2009) has solved a linear Fredholm type integro-differential equations with separable kernel. Javidi (2009) solved non-linear Fredholm integral equations, Saberi-Nadjafi & Tamamgar (2008) used modified HPM for solving the system linear and nonlinear of Volterra integral equations, Kumar et al. (2011) solved generalized Abel integral equation. Lastly, for the differential equations of fractional type, Odibat & Momani (2008) solved nonlinear differential equations of fractional order, Jafari et al. (2010) solved nonlinear problems of fractional Riccati differential equation & Yildirim & Agirseven (2009) solved the space-time fractional advection-dispersion equation.

Recently, we have studied a wide class of linear first-order partial differential equations which are used as mathematical models in many physically significant fields and applied science. The approximated solutions of this class of partial differential equations have studied using homotopy perturbation method (HPM). The proposed method applied for solving different examples for this class of partial differential equations. The approximated solutions for different tested problems show that the HPM is more efficient in the iterations complexity and high accurate in the absolute errors. It has been highlighted that the use of HPM is more suitable to approximate the solutions of general partial differential equations.

2 Preliminary

3 Preliminary

3.1 Homotopy Perturbation Method(HPM)

In this section, we present a brief description of the HPM, to illustrate the basic ideas of the homotopy perturbation method, we consider the following differential equation (Neamaty & Darzi (2010), Chun & Sakhivel (2010), Batiha (2015) & Abbasbandy (2006)):

$$A(u) - f(\tau) = 0, \quad \tau \in \Omega \tag{1}$$

with boundary conditions:

$$B(u, \frac{\partial u}{\partial \tau}) = 0, \quad \tau \in \partial\Omega \tag{2}$$

where A is general differential operator, B is a boundary operator, $f(\tau)$ a known analytic function and $\partial\Omega$ is the boundary of the domain Ω . The operator A can be generally divided into two parts of L and N where L is linear part, while N is the nonlinear part in the DE, Therefore Equation (1) can be rewritten as follows (He (1999)):

$$L(u) + N(u) - f(\tau) = 0. \tag{3}$$

By using homotopy technique, One can construct a homotopy

$$V(\tau, p) : \Omega \times [0, 1] \mapsto R$$

which satisfies:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[L(v) + N(v) - f(\tau)] = 0, \tag{4}$$

or

$$H(v, p) = L(v) - L(u_0) + pL(u_0 + p[N(v) - f(\tau)]) = 0, \tag{5}$$

where $p \in [0, 1]$, $\tau \in \Omega$ & p is called homotopy parameter and u_0 is an initial approximation for the solution of equation (1) which satisfies the boundary conditions obviously, Using equation (4) or (5), we have the following equation:

$$H(v, 0) = L(v) - L(u_0) = 0, \tag{6}$$

and

$$H(v, 1) = L(v) + N(v) - f(\tau) = 0. \tag{7}$$

Assume that the solution of (4) or (5) can be expressed as a series in p as follows:

$$V = v_0 + pv_1 + p^2v_2 + p^3v_3 + \dots = \sum_{i=0}^{\infty} p^i v_i, \tag{8}$$

set $p \rightarrow 1$ results in the approximate solution of (1).

Consequently,

$$u(\tau) = \lim_{p \rightarrow 1} V = v_0 + v_2 + v_3 + \dots = \sum_{i=0}^{\infty} v_i. \tag{9}$$

It is worth to note that the major advantage of He's homotopy perturbation method is that the perturbation equation can be freely constructed in many ways and approximation can also be freely selected

4 Analysis of HPM for Solving First-order Partial Differential Equation

In this section, we study the general first-order partial differential equation and introduce a review of the homotopy perturbation method for solving first-order partial differential equation.

The general form of first-order partial differential equation as follows:

$$\sum_{i=1}^n A_i(T_n) \frac{\partial u(T_n)}{\partial t_i} + C(T_n)u(T_n) = G(T_n), \tag{10}$$

subject to the initial condition

$$u(T_{n-1}, 0) = f(T_{n-1})$$

such that

$$T_n = (t_1, t_2, t_3, \dots, t_n), \quad T_{n-1} = (t_1, t_2, t_3, \dots, t_{n-1}),$$

and $A_n(T_n) \neq 0$ where $A_i(T_n)$, $C(T_n)$, $G(T_n)$ and $f(T_{n-1})$ are the given functions.

We describe a general technique for solving first-order partial differential equations in which the solution $u : \mathfrak{R}^n \rightarrow \mathfrak{R}$ is a function of n variables according the following algorithm:

4.1 The Proposed method

Firstly, we start with the initial approximation $u(T_{n-1}, 0) = f(T_n)$,

Secondly, we can construct a homotopy for the general PDE (10) as follow:

$$\begin{aligned} H(u, p) &= (1-p) \left(\frac{\partial u(T_n)}{\partial t_n} - \frac{\partial u_0(T_n)}{\partial t_n} \right) + p \left(\sum_{i=1}^n A_i(T_n) \frac{\partial u(T_n)}{\partial t_i} + C(T_n)u(T_n) \right. \\ &\quad \left. - G(T_n) \right) = 0. \end{aligned} \tag{11}$$

Thirdly, suppose that the solution of the Equation (11) is in the form

$$x(t) = x_0 + px_1 + p^2x_2 + p^3x_3 + \dots \tag{12}$$

Therefore,

$$\begin{aligned} H(u, p) &= (1-p) \left(\sum_{j=0}^{\infty} p^j \frac{\partial u_j(T_n)}{\partial t_n} - \frac{\partial u_0(T_n)}{\partial t_n} \right) + p \left(\sum_{i=1}^n A_i(T_n) \sum_{j=0}^{\infty} p^j \frac{\partial u_j(T_n)}{\partial t_i} \right. \\ &\quad \left. + C(T_n) \sum_{j=0}^{\infty} p^j u_j(T_n) - G(T_n) \right) = 0. \end{aligned} \tag{13}$$

Fourthly, collecting terms of the same power of p gives, as show in the following equations:

By collecting the terms of the same powers of p we obtain the following equation:

$$p^0 : \frac{\partial u_0(T_n)}{\partial t_n} - \frac{\partial u_0(T_n)}{\partial t_n} = 0, \tag{14}$$

$$p^1 : \frac{\partial u_1(T_n)}{\partial t_n} + \sum_{i=1}^n A_i(T_n) \frac{\partial u_0(T_n)}{\partial t_i} + C(T_n)u_0(T_n) - G(T_n) = 0, \tag{15}$$

$$p^2 : \frac{\partial u_2(T_n)}{\partial t_n} + \sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_1(T_n)}{\partial t_i} + C(T_n)u_1(T_n) = 0, \tag{16}$$

$$p^3 : \frac{\partial u_3(T_n)}{\partial t_n} + \sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_2(T_n)}{\partial t_i} + C(T_n)u_2(T_n) = 0, \tag{17}$$

$$p^4 : \frac{\partial u_4(T_n)}{\partial t_n} + \sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_3(T_n)}{\partial t_i} + C(T_n)u_3(T_n) = 0, \tag{18}$$

$$p^5 : \frac{\partial u_5(T_n)}{\partial t_n} + \sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_4(T_n)}{\partial t_i} + C(T_n)u_4(T_n) = 0, \tag{19}$$

...

Hence, for $n = 2, 3, 4, \dots$ we have,

$$p^m : \frac{\partial u_m(T_n)}{\partial t_n} + \sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_{m-1}(T_n)}{\partial t_i} + C(T_n)u_{m-1}(T_n) = 0 \tag{20}$$

...

Finally, using the Equations (14-20) with some simplifications, then we get the following sequence of the solutions:

$$\begin{aligned} u_0(T_n) &= f(T_n), \\ u_1(T_n) &= - \int \left(\sum_{i=1}^n A_i(T_n) \frac{\partial u_0(T_n)}{\partial t_i} + C(T_n)u_0(T_n) - G(T_n) \right) dt_n, \\ u_2(T_n) &= - \int \left(\sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_1(T_n)}{\partial t_i} + C(T_n)u_1(T_n) \right) dt_n \\ u_3(T_n) &= - \int \left(\sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_2(T_n)}{\partial t_i} + C(T_n)u_2(T_n) \right) dt_n \\ u_4(T_n) &= - \int \left(\sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_3(T_n)}{\partial t_i} + C(T_n)u_3(T_n) \right) dt_n \end{aligned}$$

and

$$u_5(T_n) = - \int \left(\sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_4(T_n)}{\partial t_i} + C(T_n)u_4(T_n) \right) dt_n$$

...

Hence, the general term has the following form:

$$u_n(T_n) = \int \left(\sum_{i=1}^{n-1} A_i(T_n) \frac{\partial u_{m-1}(T_n)}{\partial t_i} + C(T_n)u_{m-1}(T_n) \right) dt_n \quad n = 2, 3, 4, \dots$$

...

Then the solution of the Equation (10) is

$$u(T_n) = u_0(T_n) + u_1(T_n) + u_2(T_n) + u_3(T_n) + u_4(T_n) + u_5(T_n) + \dots \tag{21}$$

5 Implementations

In order to assess the accuracy of the solving first-order PDE using homotopy perturbation method (HPM) of some problems, we have introduced different examples to compare the approximated solutions with the exact solutions for tested problems, we will consider the following problems.

5.1 Problem1

Consider the following partial differential equation:

$$xu_x(x,t) + u_t(x,t) = 0, \quad x \in R, \quad t > 0, \quad (22)$$

subject to the initial condition

$$u(x,0) = ax.$$

Comparing Equation(22) we have

$$n = 2, \quad A_1 = X, \quad A_2 = 1, \quad G = 0 \quad \& \quad C = 0.$$

The initial approximation has the form $u_0(x,t) = ax$.

Substituting the Equation(21) into the Equation(22), we have

$$u_1(x,t) = - \int (x \frac{\partial u_0(x,t)}{\partial x} + \frac{\partial u_0(x,t)}{\partial t}) dt. \quad (23)$$

Accordingly,

$$u_n(x,t) = \int (-1)^n x \frac{\partial u_{n-1}}{\partial x}(x,t) dt; \text{ for } k = 2, 3, 4, \dots \quad (24)$$

Making some simplification of the Equation (24), the sequence of the solutions to can be identified as follow:

$$\begin{aligned} u_1(x,t) &= -axt, \\ u_2(x,t) &= ax \frac{t^2}{2!}, \\ u_3(x,t) &= -ax \frac{t^3}{3!}, \\ u_4(x,t) &= ax \frac{t^4}{4!}, \\ u_5(x,t) &= -ax \frac{t^5}{5!}, \\ &\dots \end{aligned}$$

Accordingly, the general solution of the Equation (22) is given as follow:

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t) + u_5(x,t) + \dots \\ &= axe^{-t}. \end{aligned}$$

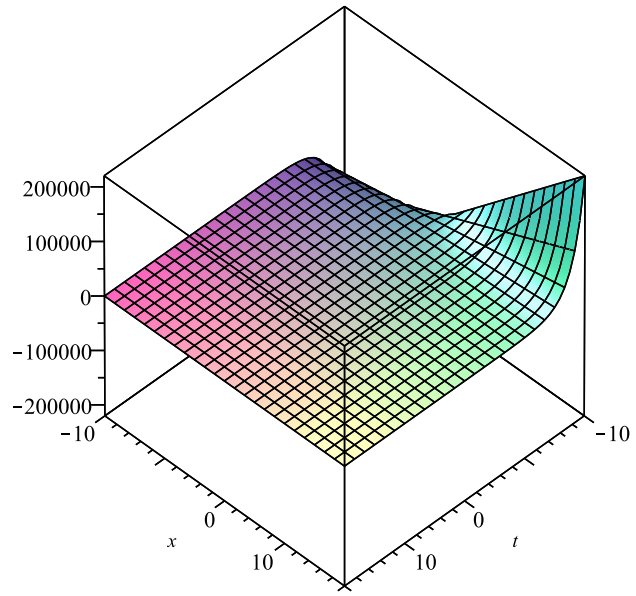


Figure 1: Approximated solution of problem1 using HPM at $a = 1$

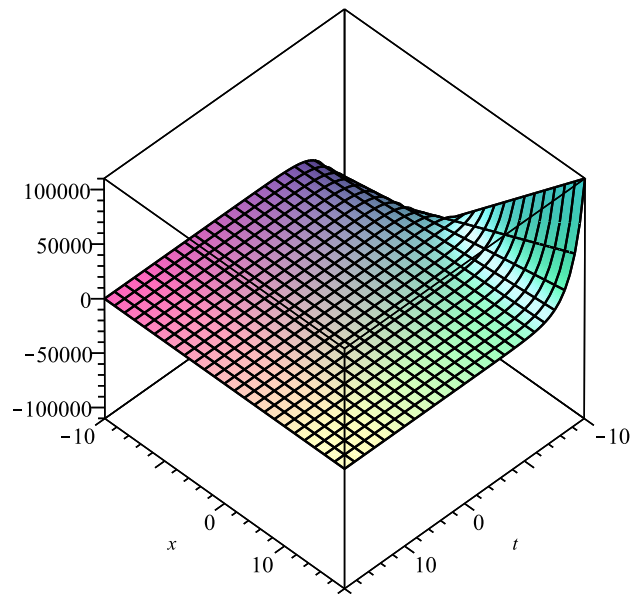


Figure 2: Approximated solution of problem1 using HPM at $a = 0.5$

5.2 Problem2

Consider the Cauchy problem:

$$\lambda u_x(x,t) + u_t(x,t) = 0, \quad x \in R, \quad t > 0, \quad (25)$$

subject to the initial condition

$$u(x,0) = h(x).$$

Comparing the Equation(25) we have

$$n = 2, \quad A_1 = \lambda, \quad A_2 = 1, \quad G = 0 \quad \text{and} \quad C = 0$$

Consider the initial approximation has the following form

$$u(x,0) = h(x)$$

Substituting the Equation(21) into the Equation(25), we have

$$\begin{aligned} u_1(x,t) &= - \int (\lambda \frac{\partial u_0(x,t)}{\partial x} + \frac{\partial u_0(x,t)}{\partial t}) dt \\ &= -\lambda t h^{(1)}(x), \\ u_2(x,t) &= - \int \lambda \frac{\partial u_1(x,t)}{\partial x} dt \\ &= \lambda^2 \frac{t^2}{2!} h^{(2)}(x), \\ u_3(x,t) &= - \int \lambda \frac{\partial u_2(x,t)}{\partial x} dt \\ &= -\lambda^3 \frac{t^3}{3!} h^{(3)}(x), \\ u_4(x,t) &= - \int \lambda \frac{\partial u_3(x,t)}{\partial x} dt \\ &= \lambda^4 \frac{t^4}{4!} h^{(4)}(x), \\ u_5(x,t) &= - \int \lambda \frac{\partial u_4(x,t)}{\partial x} dt \\ &= -\lambda^5 \frac{t^5}{5!} h^{(5)}(x), \\ &\dots \end{aligned}$$

and,

$$\begin{aligned} u_m(x,t) &= - \int \lambda \frac{\partial u_{m-1}(x,t)}{\partial x} dt \\ &= (-1)^m \lambda^m \frac{t^m}{m!} h^{(m)}(x). \end{aligned}$$

Then, the general solution of the Equation (25) is written as follow:

$$\begin{aligned}
u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t) + u_5(x,t) + \dots, \\
&= h(x) - \lambda t h^{(1)}(x) + \lambda^2 \frac{t^2}{2!} h^{(2)}(x) - \lambda^3 \frac{t^3}{3!} h^{(3)}(x) + \lambda^4 \frac{t^4}{4!} h^{(4)}(x) - \lambda^5 \frac{t^5}{5!} h^{(5)}(x) \\
&+ \dots + (-1)^m \lambda^m \frac{t^m}{m!} h^{(m)}(x) + \dots \\
&= \sum_{m=0}^{\infty} h^{(m)} (-1)^m \lambda^m \frac{t^m}{m!} h^{(m)}(x).
\end{aligned}$$

Now, we have study some special cases:

1. Case1: $h(x) = x$, approximated solution of PDE is written as follow:

$$u(x,t) = x - \lambda t$$

2. Case2: $h(x) = e^x$, approximated solution of PDE is written as follow:

$$u(x,t) = e^{x-\lambda t},$$

3. Case3: $h(x) = \sin(x)$, approximated the solution of equation is written as follow:

$$u(x,t) = \sin(x) \cos(\lambda t) - \cos(x) \sin(\lambda t).$$

5.3 Problem3

Consider the following PDE:

$$u_x(x,y,z) + yu_y(x,y,z) + 2x^2zu_z(x,y,z) = 0, \quad x,y,z \in R, \quad (26)$$

subject to the initial condition

$$u(x,y,z) = y.$$

Comparing the Equation(26),we have

$$n = 3, \quad A_1 = y, \quad A_2 = 2x^2z, \quad A_3 = 1, \quad G = 0 \quad \& \quad C = 0.$$

Consider the initial approximation has the form $u(x,y,z) = y$ substituting the Equation (21)

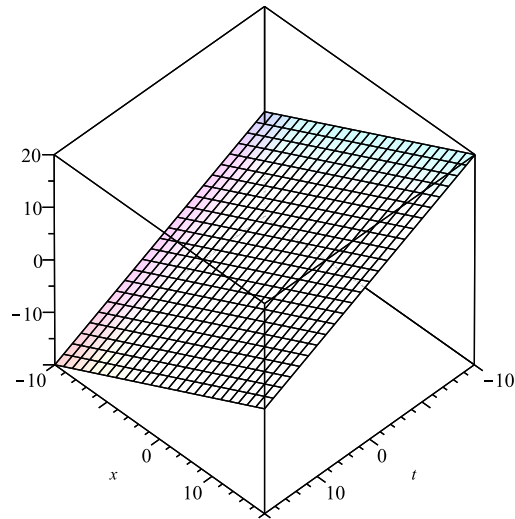


Figure 3: Approximated solution of problem2 using HPM at $\lambda = 1$ in case 1

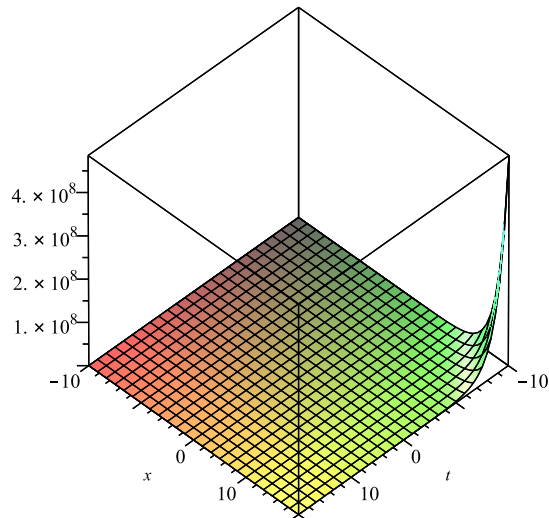


Figure 4: Approximated solution of problem2 using HPM at $\lambda = 1$ in case 2

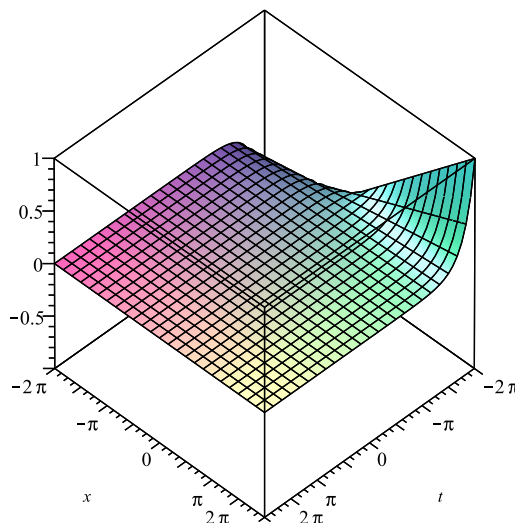


Figure 5: Approximated solution of problem2 using HPM at $\lambda = 1$ in case 3

into the Equation(26),we have

$$\begin{aligned}
u_1(x, y, z) &= - \int (y \frac{\partial u_0(x, y, z)}{\partial y} + 2x^2 z \frac{\partial u_0(x, y, z)}{\partial z} + \frac{\partial u_0(x, y, z)}{\partial x}) dx \\
&= -yx, \\
u_2(x, y, z) &= - \int (y \frac{\partial u_1(x, y, z)}{\partial y} + 2x^2 z \frac{\partial u_1(x, y, z)}{\partial z}) dx \\
&= y \frac{x^2}{2!}, \\
u_3(x, y, z) &= - \int (y \frac{\partial u_2(x, y, z)}{\partial y} + 2x^2 z \frac{\partial u_2(x, y, z)}{\partial z}) dx \\
&= -y \frac{x^3}{3!}, \\
u_4(x, y, z) &= - \int (y \frac{\partial u_3(x, y, z)}{\partial y} + 2x^2 z \frac{\partial u_3(x, y, z)}{\partial z}) dx \\
&= y \frac{x^4}{4!}, \\
u_5(x, y, z) &= - \int (y \frac{\partial u_4(x, y, z)}{\partial y} + 2x^2 z \frac{\partial u_4(x, y, z)}{\partial z}) dx \\
&= -y \frac{x^5}{5!},
\end{aligned}$$

and,

$$\begin{aligned}
u_m(x, y, z) &= - \int (y \frac{\partial u_{m-1}(x, y, z)}{\partial y} + 2x^2 z \frac{\partial u_{m-1}(x, y, z)}{\partial z}) dx \\
&= (-1)^m y \frac{x^m}{m!}.
\end{aligned}$$

Hence, the general solution of the Equation (26) is written as follow:

$$\begin{aligned}
u(x, y, z) &= u_0(x, y, z) + u_1(x, y, z) + u_2(x, y, z) + u_3(x, y, z) + u_4(x, y, z) + u_5(x, y, z) + \dots \\
&= ye^{-x}.
\end{aligned}$$

5.4 Problem4

Consider the following PDE:

$$u_x(x, y, z) + yu_y(x, y, z) + zu_z(x, y, z) = 0, \quad x, y, z \in R \quad (27)$$

subject to the initial condition

$$u(x, y, z) = zy.$$

Comparing the Equation(27)we have

$$n = 3, \quad A_1 = y, \quad A_2 = z, \quad A_3 = 1, \quad G = 0 \quad \text{and} \quad C = 0$$

Consider the initial approximation has the form $u(x, y, z) = zy$ substituting the Equation(21)

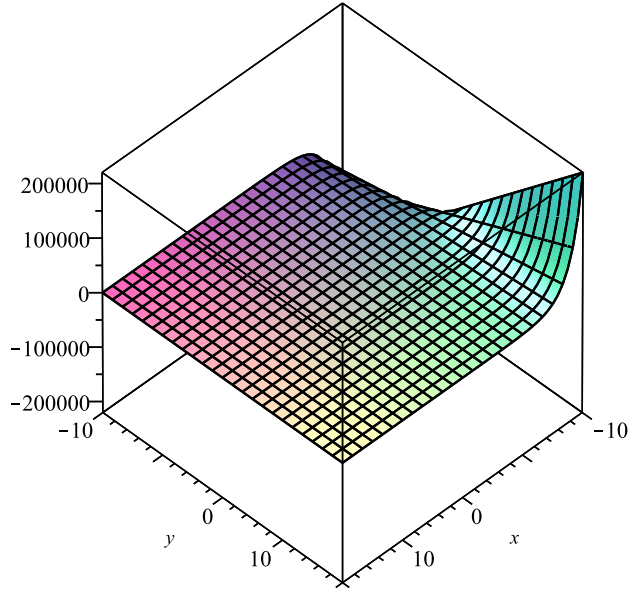


Figure 6: Approximated solution of problem3 using HPM

into the Equation(27),we have

$$\begin{aligned}
 u_1(x,y,z) &= - \int (y \frac{\partial u_0(x,y,z)}{\partial y} + z \frac{\partial u_0(x,y,z)}{\partial z} + \frac{\partial u_0(x,y,z)}{\partial x}) dx \\
 &= -2xyz, \\
 u_2(x,y,z) &= - \int (y \frac{\partial u_1(x,y,z)}{\partial y} + z \frac{\partial u_1(x,y,z)}{\partial z}) dx \\
 &= yz \frac{2x^2}{2!}, \\
 u_3(x,y,z) &= - \int (y \frac{\partial u_2(x,y,z)}{\partial y} + z \frac{\partial u_2(x,y,z)}{\partial z}) dx \\
 &= -yz \frac{2x^3}{3!}, \\
 u_4(x,y,z) &= - \int (y \frac{\partial u_3(x,y,z)}{\partial y} + z \frac{\partial u_3(x,y,z)}{\partial z}) dx \\
 &= yz \frac{2x^4}{4!}, \\
 u_5(x,y,z) &= - \int (y \frac{\partial u_4(x,y,z)}{\partial y} + z \frac{\partial u_4(x,y,z)}{\partial z}) dx \\
 &= -yz \frac{2x^5}{5!},
 \end{aligned}$$

and,

$$\begin{aligned}
 u_m(x,y,z) &= - \int (y \frac{\partial u_{m-1}(x,y,z)}{\partial y} + z \frac{\partial u_{m-1}(x,y,z)}{\partial z}) dx \\
 &= (-1)^m yz \frac{2x^m}{m!}.
 \end{aligned}$$

However, the general solution of Equation (27) is written as follow:

$$\begin{aligned}u(x, y, z) &= u_0(x, y, z) + u_1(x, y, z) + u_2(x, y, z) + u_3(x, y, z) + u_4(x, y, z) + u_5(x, y, z) + \dots \\ &= yze^{-2x}.\end{aligned}$$

6 Discussion and Conclusion

In this paper, the homotopy perturbation method (HPM) has been studied for solving generalized linear first-order partial differential equations. The approximated solutions of this class of PDEs have been studied. Also, we have tested the HPM on the solving of different implementations which are show the efficiency and accuracy of the proposed method. The approximated solutions are agree well with analytical solutions for the tested problems. Moreover, the approximated solutions proved that the proposed method to be efficient and high accurate.

7 Acknowledgements

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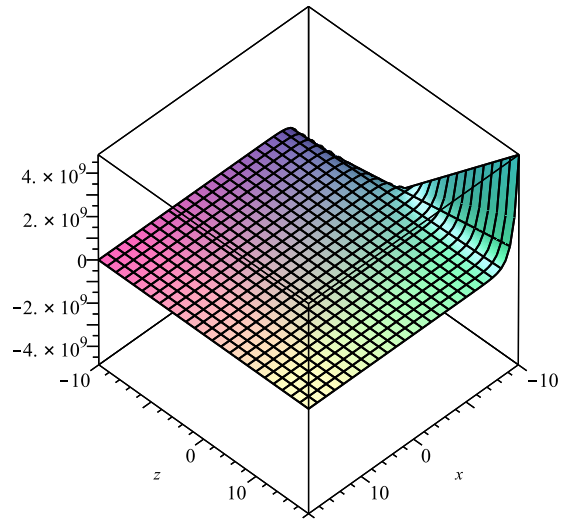


Figure 7: Approximated solution of problem4 using HPM at $y = 1$

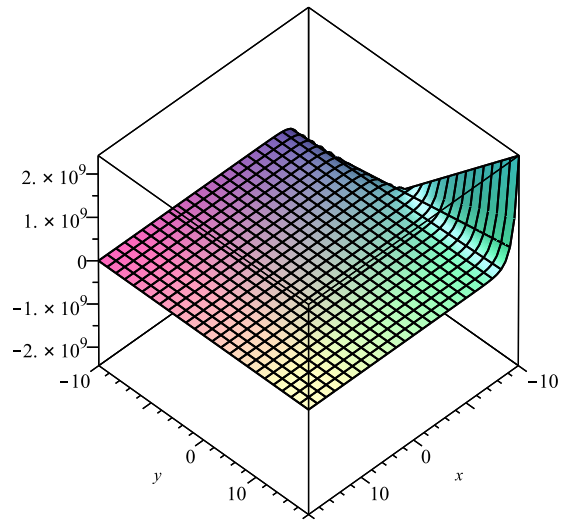


Figure 8: Approximated solution of problem4 using HPM at $z = 0.5$

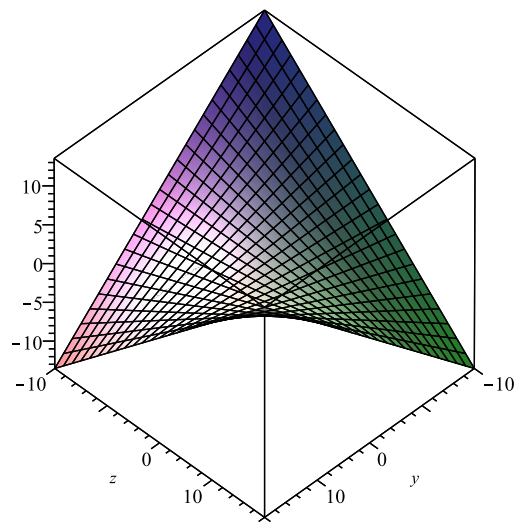


Figure 9: Approximated solution of problem4 using HPM at $x = 1$

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