On Semigroup Ideals and (θ, θ) -3-Derivations in Near-Rings

Ikram A. Saed

Department of Applied Science, University of Technology, Baghdad, Iraq.

Abstract

Let N be a near –ring and Θ is a mapping on N. In this paper we introduce the concept of (Θ, Θ) -3-derivation in near-ring N and we show that prime near-ring N satisfying some identities involving (Θ, Θ) -3-derivation and semigroup ideals is a commutative ring.

Keywords : *near-ring*, *prime near-ring*, *semigroup ideal*, *3-derivation*; (Θ, Θ) -*3-derivation*.

I. INTRODUCTION

Let N be a near –ring and θ is a mapping on N. This paper consists of two sections. In section one, we recall some basic definitions and other concepts, which be used in our paper, we explain these concepts by examples and remarks. In section two, we introduce the notion of (θ, θ) -3-derivation in near-ring N and we determine some conditions of (θ, θ) -3-derivation and semigroup ideals which make prime near-ring commutative ring.

II. BASIC CONCEPTS

Definition 2.1:[1] A ring R is called a prime ring if for any $a,b \in \mathbb{R}$, $aRb = \{0\}$ implies that either a=0 or b=0.

Example 2.2:[1] The ring of real numbers with the usual operation of addition and multiplication is prime ring .

Definition 2.3:[1] A ring R is said to be n-torsion free whenever na=0 with $a \in \mathbb{R}$, then a=0.

Definition 2.4:[1] Let R be a ring. Define a Lie product [,] on R as follows [x,y] = xy - yx, for all x, $y \in \mathbb{R}$.

Properties 2.5:[1] Let R be a ring , then for all x,y,z ∈R, we have :

 $\begin{aligned} 1-[x,yz] &= y[x,z] + [x,y]z \\ 2-[xy,z] &= x[y,z] + [x,z]y \\ 3-[x+y,z] &= [x,z] + [y,z] \\ 4-[x,y+z] &= [x,y] + [x,z] \end{aligned}$

Definition 2.6:[2] A right near-ring (resp. a left near-ring) is a nonempty set N equipped with two binary operations + and . such that (i)(N, +) is a group (not necessarily abelian) (ii)(N, .) is a semigroup . (iii) For all x,y,z \in N, we have (x+y)z = xz + yz (resp. z(x+y) = zx + zy)

Example 2.7:[2] Let G be a group (not necessarily abelian) then all mapping of G into itself from a right near-ring M(G) with regard to point wise addition and multiplication by composite.

Lemma 2.8:[2] Let N be left (resp. right) near-ring, then (i)x0 = 0 (resp. 0x = 0) for all $x \in N$. (ii) – (xy) = x(-y) (resp. – (xy) = (-x)y) for all x, y $\in N$. **Definition 2.9:[2]** A right near-ring (resp. left near-ring) is called zero symmetric right near-ring (resp. zero symmetric left near-ring) if x0 = 0 (resp. 0x = 0), for all $x \in N$.

Definition 2.10:[2] Let $\{N_i\}$ be a family of near-rings (i \in I, I is an indexing set). $N = N_1 \times N_2 \times ... \times N_n$ with regard to component wise addition and multiplication, N is called the direct product of near-rings N_i .

Definition 2.11:[2] A nonempty subset U of N will be called a semigroup right ideal (resp. semigroup left ideal) if $UN \subseteq U$ (resp. $NU \subseteq U$) and if U is both semigroup right ideal and semigroup left ideal, it be called a semigroup ideal.

Example 2.12:[2] Let S be a zero symmetric left near-ring . Suppose that

$$N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} : x_{y} y_{y} z_{y} 0 \in S \right\}, \quad U_{1} = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} : z_{y} 0 \in S \right\}$$
$$U_{2} = \left\{ \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : x_{y} 0 \in S \right\} \quad \text{and} \quad U_{3} = \left\{ \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} : y_{y} z_{y} 0 \in S \right\}$$

 U_1 is a semigroup right ideal of N but not semigroup left ideal of N, U_2 is a semigroup left ideal of N but not semigroup right ideal of N while U_3 is a semigroup ideal.

Remark 2.13:[3] Let N be a near-ring

(i)N x N x \dots x N forms a near-ring with regard to component wise addition and component wise multiplication .

(ii)If U_1 , U_2 , ..., U_n be nonzero semigroup right ideals (resp. semigroup left ideals) of N, then $U_1 \times U_2 \times \ldots \times U_n$ forms a nonzero semigroup right ideals (resp. semigroup left ideas) of N x N x ... x N.

Definition 2.14:[3] A near-ring N is called a prime near-ring if $aNb = \{0\}$, where $a, b \in \mathbb{N}$, implies that either a = 0 or b = 0.

Definition 2.15:[3] Let N be a near-ring. The symbol Z will denote the multiplicative center of N, that is $Z = \{x \in N | xy = yx \text{ for all } y \in N\}$.

Definition 2.16:[3] Let R be a ring. Define a Jordan product on R as follows : a = b = ab + ba, for all $a, b \in \mathbb{R}$.

Lemma 2.17:[3] Let N be a prime near-ring. If $z \in \mathbb{Z} \setminus \{0\}$ and x is an element of N such that $xz \in \mathbb{Z}$ or $zx \in \mathbb{Z}$, then $x \in \mathbb{Z}$.

Lemma 2.18:[4] Let N be a prime near-ring and U be a nonzero semigroup right ideal (resp. semigroup left ideal) and x is an element of N such that $Ux = \{0\}$ (resp. $xU = \{0\}$), then x = 0. **Lemma 2.19:**[4] Let N be a prime near-ring and U be a nonzero semigroup ideal of N. If $x,y \in N$ and $xUy = \{0\}$, then x = 0 or y = 0.

Lemma 2.20:[4] Let N be a prime near-ring and Z contains a nonzero semigroup left ideal or nonzero semigroup right ideal , then N is a commutative ring .

Definition 2.21:[4] Let N be a near-ring .An 3-additive mapping d: N x N x N \rightarrow N is said to be 3-derivation if the relations

 $\begin{array}{l} d(x_1x_1^{'},x_2,x_3) = d(x_1,x_2,x_3) \; x_1^{'} + x_1 \; d(x_1^{'},x_2,x_3) \\ d(x_1,x_2x_2^{'},x_3) = d(x_1,x_2,x_3) \; x_2^{'} + x_2 \; d(x_1,x_2^{'},x_3) \\ d(x_1,x_2,x_3x_3^{'}) = d(x_1,x_2,x_3) \; x_3^{'} + x_3 \; d(x_1,x_2,x_3^{'}) \\ hold \; for \; x_1,x_1^{'},x_2,x_2^{'},x_3,x_3^{'} \in \mathbb{N} \; . \end{array}$

Definition 2.22:[4] Let N be a near-ring and n be a fixed positive integer . An n- additive mapping $d : N \times N \times \dots \times N \rightarrow N$ is said to be n-derivation if the relations

 $\begin{aligned} &d(x_1x_1^{\prime}, x_2, \dots, x_n) = d(x_1, x_2, \dots, x_n) x_1^{\prime} + x_1 d(x_1^{\prime}, x_2, \dots, x_n) \\ &d(x_1, x_2x_2^{\prime}, \dots, x_n) = d(x_1, x_2, \dots, x_n) x_2^{\prime} + x_2 d(x_1, x_2^{\prime}, \dots, x_n) \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & &$

hold for $x_1, x_1, x_2, x_2, \dots, x_n, x_n \in \mathbb{N}$. Lemma 2.23:[4] Let N be a prime near-ring, then d is n-derivation of N if and only if

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Lemma 2.24:[4] Let N be a near-ring and d be n-derivation of N, then for every

 $\begin{array}{l} x_{1},x_{1}^{'},x_{2},...,x_{n},y \in N, \\ (i)(x_{1},d(x_{1}^{'},x_{2},...,x_{n}) + d(x_{1},x_{2},...,x_{n}) x_{1}^{'})y = x_{1} d(x_{1}^{'},x_{2},...,x_{n})y + d(x_{1},x_{2},...,x_{n}) x_{1}^{'}y \\ (ii)(d(x_{1},x_{2},...,x_{n}) x_{1}^{'} + x_{1} d(x_{1}^{'},x_{2},...,x_{n}))y = d(x_{1},x_{2},...,x_{n}) x_{1}^{'}y + x_{1} d(x_{1}^{'},x_{2},...,x_{n})y \end{array}$

Lemma 2.25:[4] Let d be n-derivation of a near-ring N , then d(Z,N,....,N) \sqsubseteq Z .

Lemma 2.26:[4] Let N be a prime near-ring and d be a nonzero n-derivation of N. Let U_1 , U_2 ,, U_n be nonzero semigroup right ideals (resp. semigroup left ideals) of N, if $d(U_1, U_2, ..., U_n) \subseteq Z$ then N is a commutative ring.

III. (θ, θ) -3- DERIVATIONS IN PRIME NEAR-RINGS

First we introduce the basic definition in this paper

Definition 3.1: Let N be a near-ring and θ is a mapping on N. An 3-additive mapping d: N x N x N \rightarrow N is said to be (θ, θ) - 3-derivation if the relations

$$\begin{split} & \mathsf{d}(x_1x_1^{\,\prime}, x_2, x_3) = \mathsf{d}(x_1, x_2, x_3) \; \boldsymbol{\theta}(x_1^{\,\prime}) + \; \boldsymbol{\theta}(x_1) \; \mathsf{d}(x_1^{\,\prime}, x_2, x_3) \\ & \mathsf{d}(x_1, x_2x_2^{\,\prime}, x_3) = \mathsf{d}(x_1, x_2, x_3) \; \boldsymbol{\theta}(x_2^{\,\prime}) + \; \boldsymbol{\theta}(x_2) \; \mathsf{d}(x_1, x_2^{\,\prime}, x_3) \\ & \mathsf{d}(x_1, x_2, x_3x_3^{\,\prime}) = \mathsf{d}(x_1, x_2, x_3) \; \boldsymbol{\theta}(x_3^{\,\prime}) + \; \boldsymbol{\theta}(x_3) \; \mathsf{d}(x_1, x_2, x_3^{\,\prime}) \\ & \mathsf{hold} \; \mathrm{for} \; x_1, x_1^{\,\prime}, x_2^{\,\prime}, x_2^{\,\prime}, x_3, \; x_3^{\,\prime} \; \boldsymbol{\in} \mathsf{N} \; . \end{split}$$

we now explain this definition by the following example

Example 3.2 : Let S be a commutative near-ring . Let us define

$$N = \left\{ \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} x, y, 0 \in S \right\}.$$

It can easily see that N is a non commutative zero symmetric left near-ring with respect to matrix addition and matrix multiplication .

Define d: N x N x N \rightarrow N such that

$$d\left(\begin{pmatrix} x_1 & y_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} x_2 & y_2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} x_3 & y_3 \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x_1 x_2 x_3 \\ 0 & 0 \end{pmatrix}$$

And θ : N \rightarrow N such that

$$\theta\left(\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}$$

It is easy to see that d is (θ, θ) - 3-derivation of N.

Theorem 3.3: Let N be a prime near-ring which admits a nonzero (θ, θ) - 3-derivation d, where θ is an automorphism on N, if U_1 , U_2 , U_3 are nonzero semigroup ideals of N. If $d([x, y], u_2, u_3) =$ $[d(x, u_2, u_3), \theta(y)]$ for all x, $y \in U_1, u_2 \in U_2, u_3 \in U_3$, then N is a commutative ring. **Proof :** Assume that $d([x, y], u_2, u_3) = [d(x, u_2, u_3), \theta(y)]$ for all $x, y \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$. (3.1)If we take y = x in (3.1) we get $[d(x, u_2, u_3), \theta(x)] = 0$ $d(x, u_2, u_3) \theta(x) = \theta(x) d(x, u_2, u_3)$ for all $x \in U_1, u_2 \in U_2, u_3 \in U_3$. (3.2) Replacing y by xy in (3.1) we get d([x, xy], u₂, u₃) = [d (x, u₂, u₃), θ (xy)] for all x, y \in U₁, u₂ \in U₂, u₃ \in U₃ Therefore $d(x [x, y], u_2, u_3) = [d (x, u_2, u_3), \theta(xy)]$ for all $x, y \in U_1$ $u_2 \in U_2$, $u_3 \in U_3$. By defining property of d we get $d(x, u_2, u_3) [\boldsymbol{\theta}(x), \boldsymbol{\theta}(y)] + \boldsymbol{\theta}(x) d([x, y], u_2, u_3) = [d(x, u_2, u_3), \boldsymbol{\theta}(xy)]$ for all x, y $\in U_1$, $u_2 \in U_2$, u_3 EU₂. Using (3.1) again we get $d(x, u_2, u_3) [\boldsymbol{\theta}(x), \boldsymbol{\theta}(y)] + \boldsymbol{\theta}(x) [d(x, u_2, u_3), \boldsymbol{\theta}(y)] = [d(x, u_2, u_3), \boldsymbol{\theta}(xy)]$ for all x, y $\in U_1$ u₂ $\in U_2$, $u_2 \in U_2$. Previous equation can be reduced to $\boldsymbol{\theta}(\mathbf{x}) d(\mathbf{x}, \mathbf{u}_2, \mathbf{u}_3) \boldsymbol{\theta}(\mathbf{y}) = d(\mathbf{x}, \mathbf{u}_2, \mathbf{u}_3) \boldsymbol{\theta}(\mathbf{y}\mathbf{x})$ for all $\mathbf{x}, \mathbf{y} \in \mathbf{U}_1, \mathbf{u}_2 \in \mathbf{U}_2, \mathbf{u}_3 \in \mathbf{U}_3$. Using (3.2) in previous equation yields $d(x, u_2, u_3) \theta(xy) = d(x, u_2, u_3) \theta(yx)$ for all x,y $\in U_1$, $u_2 \in U_2$, $u_3 \in U_3$. (3.3) If we replace y by yr, where $r \in N$, in (3.3) and using it again, we get $d(x, u_2, u_3) U_1[\theta(x), \theta(r)] = \{0\}$ for all $x \in U_1, u_2 \in U_2, u_3 \in U_3, r \in \mathbb{N}$. (3.4) By Lemma 2.19, we conclude that for each $x \in U_1$ either $\theta(x) \in Z$ or d (x, u_2 , u_3) = 0 for all $u_2 \in U_2$, $u_3 \in U_3$, but using Lemma 2.20 lastly we get $d(x, u_2, u_3) \in \mathbb{Z}$ for all $x \in U_1, u_2 \in U_2, u_3 \in U_3, d(U_1, U_2, U_3) \subseteq \mathbb{Z}$. Now by using Lemma 2.26 we find that N is a commutative ring.

Theorem 3.4: Let N be a prime near-ring which admits a nonzero (θ, θ) - 3-derivation d, where θ is an automorphism on N, if U_1 , U_2 , U_3 are nonzero semigroup ideals of N. If $[d(x, u_2, u_3), \theta(y)] =$ $[\theta(x), \theta(y)]$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$, then N is a commutative ring. **Proof** : Suppose that $[\mathbf{d}(\mathbf{x}, \mathbf{u}_2, \mathbf{u}_3), \boldsymbol{\theta}(\mathbf{y})] = [\boldsymbol{\theta}(\mathbf{x}), \boldsymbol{\theta}(\mathbf{y})]$ for all $\mathbf{x}, \mathbf{y} \in \mathbf{U}_1, \mathbf{u}_2 \in \mathbf{U}_2, \mathbf{u}_3 \in \mathbf{U}_3$. (3.5)If we take y = x in (3.5), we get $d(x, u_2, u_3) \theta(x) = \theta(x) d(x, u_2, u_3)$ for all $x \in U_1, u_2 \in U_2, u_3 \in U_3$. (3.6) Replacing x by yx in (3.5) and using it again, we get $[d(yx, u_2, u_3), \boldsymbol{\theta}(y)] = [\boldsymbol{\theta}(yx), \boldsymbol{\theta}(y)] = \boldsymbol{\theta}(y) [\boldsymbol{\theta}(x), \boldsymbol{\theta}(y)] = \boldsymbol{\theta}(y) [d(x, u_2, u_3), \boldsymbol{\theta}(y)] \text{ for all } x, y \in U_1, u_2$ $\in U_2$, $u_3 \in U_3$. Therefore $d(yx, u_2, u_3) \theta(y) - \theta(y) d(yx, u_2, u_3) = \theta(y) (d(x, u_2, u_3) \theta(y) - \theta(y) d(x, u_2, u_3))$ for all $x, y \in U_1, u_2$ $\in U_2$, $u_3 \in U_3$. In viem of Lemma 2.23 and 2.24 the last equation can be rewritten as $d(y, u_2, u_3) \theta(xy) = \theta(y) d(y, u_2, u_3) \theta(x)$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$. Using (3.6) in previous equation, we have $d(y, u_2, u_3) \theta(xy) = d(y, u_2, u_3) \theta(yx)$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$. (3.7)

Since equation (3.7) is the same as equation (3.3) in theorem 3.3, arguing as in the proof of Theorem 3.3, we find that N is a commutative ring .

Theorem 3.5 : Let N be a prime near-ring admitting a nonzero (θ, θ) - 3-derivation d of N, where θ is an automorphism on N. Let U_1, U_2, U_3 be nonzero semigroup ideals of N. If $[d(x, u_2, u_3), \theta(y)] \in \mathbb{Z}$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$, then N is a commutative ring. **Proof**: Suppose that $[d(x, u_2, u_3), \theta(y)] \in \mathbb{Z}$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$. (3.8)Replacing $\theta(y)$ by $d(x, u_2, u_3) \theta(y)$ in (3.8), we get $[d(x, u_2, u_3), d(x, u_2, u_3) \quad \theta(y)] \in \mathbb{Z}$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$. Which means that $[[d(x, u_2, u_3), d(x, u_2, u_3) \theta(y)], \theta(t)] = 0$, for all $x, y \in U_1, u_2 \in U_2$, $u_2 \in U_2$, $t \in \mathbb{N}$. Therefore $[d(x, u_2, u_3) [d(x, u_2, u_3), \theta(y)], \theta(t)] = 0$, for all $x, y \in U_1, u_2 \in U_2$, $u_3 \in U_3$, $t \in N$. Using (3.8) in previous equation implies that $[d(x, u_2, u_3), \theta(y)] [d(x, u_2, u_3), \theta(t)] = 0$, for all $x, y \in U_1, u_2 \in U_2$, $u_3 \in U_3$, $t \in N$. (3.9)In view of (3.8), equation (3.9) assures that $[d(x, u_2, u_3), \theta(y)] N [d(x, u_2, u_3), \theta(y)] = \{0\}$ for all $x, y \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$. (3.10)Primeness of N shows that $[d(x, u_2, u_3), \theta(y)] = 0$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$. $d(x, u_2, u_3) \theta(y) = \theta(y) d(x, u_2, u_3)$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$. Replacing y by yr, where $r \in \mathbb{N}$, in previous equation and using it again implies that $\theta(y) [d(x, u_2, u_3), u_3)$. $\theta(\mathbf{r}) = 0$, for all $\mathbf{x}, \mathbf{y} \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $\mathbf{r} \in \mathbf{N}$. $U_1[d(x, u_2, u_3), \theta(r)] = \{0\}$, for all $x \in U_1, u_2 \in U_2, u_3 \in U_3, r \in \mathbb{N}$, and using Lemma 2.18 lastly implies that $d(U_1, U_2, U_3) \subseteq Z$.

By Lemma 2.26 we conclude that N is a commutative ring.

Theorem 3.6: Let N be a 2-torsion free prime near-ring and d be a nonzero (θ, θ) - 3-derivation of N, where θ is an automorphism on N. Let U₁, U₂, U₃ be nonzero semigroup ideals of N. If d(x, u₂, u₃) $\circ \theta$ (y) $\in \mathbb{Z}$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$, then N is a commutative ring. **Proof**: Suppose that $d(x, u_2, u_3) \circ \theta(y) \in Z$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$. (3.11)Replacing $\theta(y)$ by $d(x, u_2, u_3) \theta(y)$ in (3.11), we get $d(x, u_2, u_3) (d(x, u_2, u_3) \circ \theta(y)) \in Z$ for all $x, y \in U_1$ $u_2 \in U_2$, $u_3 \in U_3$. (3.12)Using Lemma 2.17 in (3.12) implies $d(x, u_2, u_3) \circ \theta(y) = 0$ or $d(x, u_2, u_3) \in \mathbb{Z}$ for all $x, y \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$. (3.13)If there exists $x \in U_1$ such that $d(x, u_2, u_3) \in \mathbb{Z} \setminus \{0\}$, then using Lemma 2.17, (3.11) implies θ (y) + θ (y) \in Z for all y \in U₁ (3.14)Thus we get, $\theta(ry) + \theta(ry) = \theta(r) (\theta(y) + \theta(y)) \in \mathbb{Z}$ for all $y \in U_1$ and $r \in \mathbb{N}$. Since N is 2-torsion free, using (3.14) and Lemma 2.17 in previous equation implies that N = Z, and hence N is a commutative ring by Lemma 2.20. Now, in view of (3.13), we may suppose that $d(x, u_2, u_3) \circ \theta(y) = 0$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3$. $\boldsymbol{\theta}(\mathbf{y}) \mathbf{d}(\mathbf{x}, \mathbf{u}_2, \mathbf{u}_3) = -\mathbf{d}(\mathbf{x}, \mathbf{u}_2, \mathbf{u}_3) \boldsymbol{\theta}(\mathbf{y})$ for all $x, y \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$. (3.15)

Replacing y by yz, where $z \in N$, in (3.15) and using it again, we obtain $\theta(yz) d(x, u_2, u_3) = -d(x, u_2, u_3) \theta(yz) = d(x, u_2, u_3) \theta(y) (-\theta(z))$ $= \theta(y) d(-x, u_2, u_3) (-\theta(z))$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3, z \in N$. Thus we get $\theta(y) (-\theta(z) d(-x, u_2, u_3) + d(-x, u_2, u_3) \theta(z)) = 0$ for all $x, y \in U_1, u_2 \in U_2, u_3 \in U_3, z \in N$. Hence $U_1(-\theta(z) d(-x, u_2, u_3) + d(-x, u_2, u_3) \theta(z)) = \{0\}$ for all $x \in U_1, u_2 \in U_2, u_3 \in U_3, z \in N$.

By Lemma 2.18, we conclude that $d(-U_1, U_2, U_3) \subseteq Z$ and since $-U_1$ is a nonzero semigroup left ideal, by Lemma 2.26 it follows that N is a commutative ring.

IV.CONCLUSIONS

In present paper we introduce the notion of (θ, θ) - 3-derivation in near-ring and we see that a near-ring can be make commutative with the help of (θ, θ) - 3-derivation and other conditions.

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