A Review on Relation between Dominating and Total Dominating Color Transversal Number of Graph and Monotonicity

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ABSTRACT – *Domination is a very fast developing area in Graph Theory. This paper deals with combination of domination, total domination, proper colouring and transversal sets. Also demonstrate the domination transversal number for different types of graphs. We also extend some relation between domination and total domination colour transversal number of graphs. We also provide some examples to justify our results. We also obtain an upper bound of this number which increases monotonically.*

Keywords: *Domination, Total domination, Transversal sets, Proper colouring.*

1. INTRODUCTION

Graphs are mathematical structures used to model pair-wise relations between objects from a certain collection. Graph can be defined a set V of vertices and set of edges. Where, V is collection of |V| = n abstract data types. Vertices can be any abstract data types and can be presented with the points in the plane. These abstract data types are also called nodes. A line (line segment) connecting these nodes is called an edge. Again, more abstractly saying, edge can be an abstract data type that shows relation between the nodes (which again can be an abstract data types).

Euler proposed that any given graph can be traversed with each edge traversed exactly once if and only if it had, zero or exactly two nodes with odd degrees. The graph following this condition is called, Eulerian circuit or path. We can easily infer this theorem. Exactly two nodes are, (and must be) starting and end of your trip. If it has even nodes than we can easily come and leave the node without repeating the edge twice or more. In actual case of seven bridges of Konigsberg, once the situation was presented in terms of graph, the case was simplified as the graph had just 4 nodes, with each node having odd degree. So, Euler concluded that these bridges cannot be traversed exactly once.

2. DOMINATING COLOUR TRANSVERSAL NUMBER

An std-set D is minimal if and only if for every $u \in D$ any one of the following holds:

(i) u is an isolate of D

(ii) There exists a vertex $v \in V-D$ such that $N(v) \cap D = \{u\}$

(iii) For every χ-partition, $\Pi = \{V_1, V_2, \ldots, V_\chi\}$, there exists one V_i such that $V_i \cap D = \{u\}$ or $\emptyset$.

Proof

Let D be an std-set. If D is minimal, then $D - \{u\}$ is not an std-set for every $u \in D$.

This implies that either $D - \{u\}$ is not a dominating set or not a transversal of every χ-partition of G.

Case 1:

Suppose $D - \{u\}$is not a dominating set. Then there exists a vertex $v \in (V-D) \cup \{u\}$ that is not adjacent to any vertex of $D - \{u\}$.

If u = v, then u is an isolate of D. If $u \neq v$, then v is adjacent to u but not to any other vertex of D.

Hence $N(v) \cap D = \{u\}$.

Case 2:

Suppose $D - \{u\}$ is not a transversal for every χ-partition $\{V_1, V_2, \ldots, V_\chi\}$. This implies that $D - \{u\} \cap V_i = \emptyset$ for some i. That is $V_i \cap D = \{u\}$ or $\emptyset$ for some i.

Hence (iii) is satisfied.

Conversely assume any one of the three conditions. We prove that D is a minimal std-set. Suppose not. Then D is an std-set but not minimal. This implies that D and $D - \{u\}$ are std-sets for some $u \in D$.

Let $\{V_1,V_2,...,V_\chi\}$ be a χ-partition of V for which $D-\{u\}$ and D are transversals. Then $D-\{u\}\cap V_i \neq \emptyset$ and $D \cap V_i \neq \emptyset$ for every i.

This implies that $D \cap V_i \neq \{u\}$ or $\emptyset$ contradicting condition (iii).

Theorem: For any graph G, $\gamma \leq \gamma_g \leq \gamma_{st}$.

Proof :

Let D be any std-set of G. So there exists a χ-partition $\Pi=\{V_1,V_2,...,V_\chi\}$ such that $D \cap V_i \neq \emptyset$ for every i. Each V_i is an independent set and so is a clique in G. Since $D\cap V_i \neq\emptyset$ for every i, D is a dominating set for G. Hence D is a global dominating set and $\gamma_g \leq \gamma_{st}$.

Result : If G has k-components say $G_1,G_2,...,G_k$ such that $\chi(G_1) \geq \chi(G_i)$ for i = 2, 3, ..., k, then $\gamma_{st}(G) \leq \gamma_{st}(G_1) + \sum_{i=2}^{k} \gamma(G_i)$.

Proof: It is given that $G = \sum_{i=1}^{k} G_i$. As $\chi(G_1) \geq \chi(G_i)$, for every i, any γ_{st}-set of G_1 is a transversal of every χ-partition of G. Hence the union of a γ_{st}-set of G_1 and γ-sets of G_i, i = 2,3, ..., k is an std-set of G. So $\gamma_{st}(G) \leq \gamma_{st}(G_1) + \sum_{i=2}^{k} \gamma(G_i)$.

3. TOTAL DOMINATING COLOR TRANSVERSALS IN GRAPH

Let G = (V, E) be a graph with chromatic number χ and minimum total dominating set S. If $< S >$ contains a complete sub graph of order χ, then $\gamma_{tstd}(G) = \gamma_t(G)$.

Proof: Suppose S is a minimum total dominating set of G and $< S >$ contains complete sub graph H of order χ. All the vertices of H must be assigned distinct χ colors and hence S is a transversal of every χ –partition of G. Hence S is a total dominating color transversal set of G.

Therefore $\gamma_{tstd}(G) \leq \gamma_t(G)$. As $\gamma_t(G) \leq \gamma_{tstd}(G)$, Hence $\gamma_{tstd}(G) = \gamma_t(G)$.

Theorem: If $\chi(G) = 2$ then $\gamma_{tstd}(G) = \gamma_t(G)$.

Proof : Given that $\gamma(G) = 2$. We know that if S is a minimum total dominating set of G then $< S >$ contains complete sub graph of order 2. Hence by the above theorem, $\gamma_{tstd}(G) = \gamma_t(G)$.

Corollary:

For $n \geq 2$, $\gamma_{tstd}(P_n) = \gamma_t(P_n)$ and $\gamma_{tstd}(T) = \gamma_t(T)$

Proof : Let P_n and T are bipartite graph. We know that if S is a minimum total dominating set of G then $< S >$ contains complete sub graph of order 2. Hence $\gamma_{tstd}(P_n) = \gamma_t(P_n)$ and $\gamma_{tstd}(T) = \gamma_t(T)$.

Result: For $n \geq 2$, $\gamma_t(P_n) = \frac{n}{2}$, if $n\equiv 0 \bmod(4)$

$= \frac{n+2}{2}$, if $n\equiv 2 \bmod(4)$

$= \frac{n+1}{2}$, otherwise.

Result: For $n \geq 3$, $\gamma_t(C_n) = \gamma_t(P_n)$.

Theorem:

For $n \geq 4$, $\gamma_{tstd}(C_n) = \gamma_t(C_n) = \frac{n}{2}$, if $n\equiv 0 \bmod(4)$

$= \frac{n+2}{2}$, if $n\equiv 2 \bmod(4)$

$= \frac{n+1}{2}$, otherwise.

Proof: We first note that cycle with even vertices is bipartite and otherwise it is tripartite. Divide vertices of C_n into groups of four like $\{v_1,v_2,v_3,v_4\}$, $\{v_5,v_6,v_7,v_8\}$,…..where the last group may contain one, two, three or four vertices.

Case 1: n≡ 0 mod(4) or n≡ 2 mod(4)

In such case Last group has four vertices or two vertices. So cycle C_n will have even number of vertices and hence C_n will be bipartite. Hence by theorem 4.2 and by $\gamma_t(C_n) = \gamma_t(P_n)$, we have $n \geq 4$, $\gamma_{tstd}(C_n) = \gamma_t(C_n) = \frac{n}{2}$, if $n\equiv 0 \bmod(4)$

$= \frac{n+2}{2}$, if $n\equiv 2 \bmod(4)$.

Case 2: n≡ 1mod(4)

Here we first note that $\gamma_{tstd}(C_n) \geq \gamma_t(C_n) = \frac{n+2}{2}$ .In this case last group has one vertex. So cycle C_n will have odd number of vertices and hence C_n will be tripartite. Select middle two vertices from each group of four vertices except second last group and from second last group select three vertices. The resultant set S, will be a total dominating set with cardinality is $\frac{n-5}{2} + 3 = \frac{n+1}{2}$.

Consider the χ- coloring of vertices of each group by using 1,2, and 3 colors as $\{1,2,1,3\},\{2,1,2,3\},\{2,1,2,3\},\{2,1,2,3\}$. Note that the set S will be a transversal of χ-partition of G formed by such χ-coloring of G is a minimum total dominating color transversal set of G with cardinality $\frac{n+1}{2}$. Hence $\gamma_{tstd}(C_n) = \frac{n+1}{2}$, if $n\equiv 1 \bmod(4)$.

Case3: n≡ 3 mod(4)

Here we first that $\gamma_{tstd}(C_n) \geq \gamma_t(C_n) = \frac{n+1}{2}$.

In this case cycle C_n will be tripartite. Select middle two vertices from each group except from last group and from last group select last two vertices. The resultant set S, will be a total dominating set with cardinality $\frac{n-3}{2}+2=\frac{n+1}{2}$.

Consider the χ-coloring of vertices of each group by using 1,2 and 3 colors as $\{1,2,1,3\},\{2,1,2,3\},\{2,1,2,3\},\ldots\ldots,\{1,2,3\}$. Note that the S will be a transversal of χ-partition of G formed by such χ-coloring of G and this set S is a minimum total dominating color transversal set of G with cardinality $\frac{n+1}{2}$. Hence $\gamma_{tstd}(C_n) = \frac{n+1}{2}$, if $n \equiv 3 \bmod (4)$. Hence the theorem.

4. RELATION BETWEEN DOMINATING AND TOTAL DOMINATING COLOR TRANSVERSAL NUMBER OF GRAPH.

Theorem: For any graph G, $\gamma_{std}(G) \leq \gamma_{tstd}(G) \leq 2\gamma_{std}(G)$

Proof: $\gamma_{std}(G) \leq \gamma_{tstd}(G)$ as total dominating set is always a dominating set. If dominating color transversal set S is not a total dominating color transversal set then there exists isolates in S.

At most |S| number of vertices in S can be isolates. As G is a graph without isolated vertices, each vertex in S has adjacent vertex in G and hence by adding at most |S| vertices to S from V\S, we obtain a total dominating color transversal set. Hence $\gamma_{tstd}(G) \leq 2\gamma_{std}(G)$.

Example: Consider a disconnected graph G

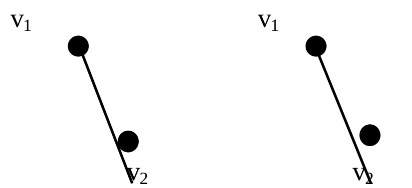

Figure-1: Disconnected Graph G

$\gamma_{tstd}(G) = 4$ and $\gamma_{std}(G) = 2$.

Example: Consider a graph G

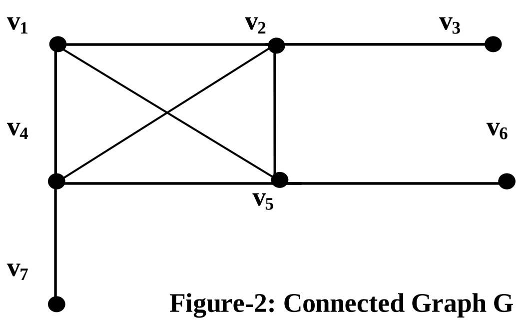

Figure-2: Connected Graph G

$\gamma = \{v_2, v_4, v_5\}$, $\gamma(G) = 3$

$\gamma_t = \{v_2, v_4, v_5\}$, $\gamma_t(G) = 3$

$\chi(G) = 4$, $\gamma_{std} = \gamma_{tstd} = 3$

Example: Consider a graph G

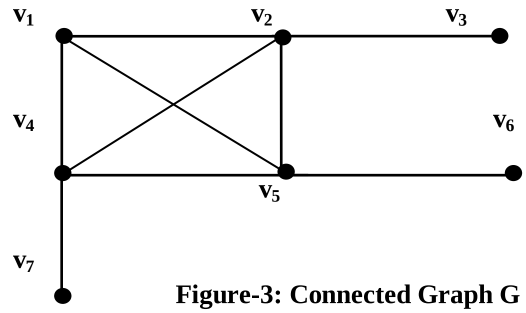

Figure-3: Connected Graph G

$\gamma_{tstd}(G) = \gamma_{std}(G) = \chi(G) = 4$ but $\gamma(G) = \gamma_t(G) = 3$.

Example: Consider a graph G ($\neq K_n$) for which $\gamma_{tstd}(G) = \gamma_{std}(G) \neq \chi(G)$ and $\gamma(G) \neq \gamma_t(G)$.

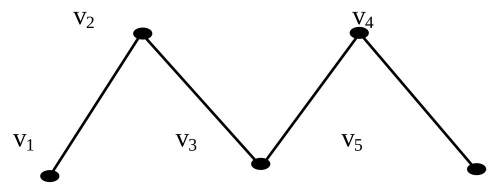

Figure-4: Bipartite Graph G

$\gamma_{tstd}(G) = \gamma_{std}(G) = 2 \neq \chi(G) = 3$ and $\gamma(G) = 2 \neq \gamma_t(G) = 3$.

Example:

Consider a graph G is connected.

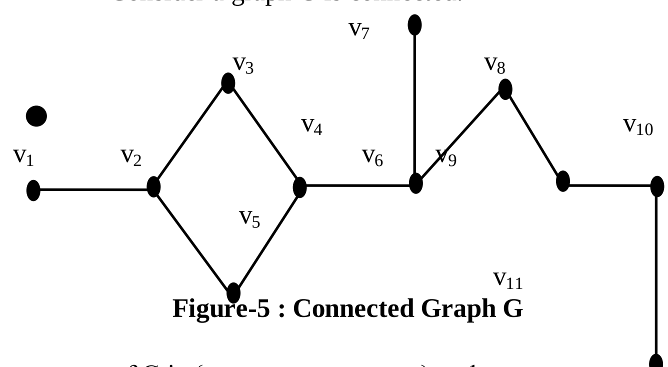

Figure-5 : Connected Graph G

γ_{tstd} – set of G is $\{v_2, v_3, v_6, v_8, v_9, v_{10}\}$ and

γ_{std} – set of G is $\{v_2, v_6, v_{10}\}$

So, $\gamma_{tstd}(G) = 6$ and $\gamma_{std}(G) = 3$.

Hence $\gamma_{tstd}(G) = 2\gamma_{std}(G)$.

5. TOTAL DOMINATING COLOR TRANSVERSAL NUMBER OF GRAPHS AND MONOTONICITY

Theorem: Let G = (V, E) be a graph with k – components say $G_1, G_2, \ldots, G_k$ and each component is without isolated vertex. If $\chi(G) \leq 2k$ then $\gamma_{tstd}(G) = \gamma_t(G)$.

Proof : Let $\chi(G) = 2k$. Let $\gamma_t(G_i)$ denote the Total Domination number of G_i (i =1, 2, 3,...., k). Each γ_t - set has at least two adjacent vertices. So assign two distinct colors to two adjacent vertices in γ_t - set of component G_1. Again, assign two arbitrary colors, not used early, to two different vertices in γ_t - set of component G_2.

Likewise continue assigning two arbitrary distinct colors, not used early, to two distinct vertices of γ_t - set of components of G till we reach upto γ_t - set to of component $G_{\frac{x}{2}}$ (= G_k) of G. Hence $\bigcup_{i=1}^{k} \gamma_t(G_i) = \gamma_t(G)$ is a transversal of such χ – Partition of G. Therefore $\gamma_{tstd}(G) = \gamma_t(G)$. For $\chi(G) < 2k$, the result is obvious by applying the above method of coloring the γ_t – set of components of G.

Theorem: Let G = (V, E) be a graph with k – components, say $G_1, G_2, \ldots, G_k$, such that $\chi(G_1) \geq \chi(G_i), \forall\ i \in \{2, \ldots, k\}$. Then $\gamma_{tstd}(G) \leq \gamma_{tstd}(G_1) + \sum_{i=2}^{k} \gamma_t(G_i)$.

Proof : Trivially as $\chi(G_1) \geq \chi(G_i)$, $\forall$ $i \in \{2, \ldots, k\}$ we have $\chi(G) = \chi(G_1)$**.** Then any γ_{tstd} -set of G_1 is a transversal of every χ– Partition of G. So union of γ_{tstd} -set of G_1 and γ_t -Sets of each G_i (i = 2, 3,....., k) yields Total Dominating Color Transversal set of G**.** Hence $\gamma_{tstd}(G) \leq \gamma_{tstd}(G_1) + \sum_{i=2}^{k} \gamma_t(G_i)$.

6. DOMINATION GRAPH WITH APPLICATION

Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. The last 30 years have witnessed spectacular growth of Graph theory due to its wide applications to discrete optimization problems, combinatorial problems and classical algebraic problems. It has a very wide range of applications to many fields like engineering, physical, social and biological sciences; linguistics etc., the theory of domination has been the nucleus of research activity in graph theory in recent times. This is largely due to a variety of new parameters that can be developed from the basic definition of domination. The NP-completeness other basic domination problems and its close relationship to other NP-completeness problems have contributed to the enormous growth of research activity in domination theory.

Applications of Domination in Graph

Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a fality is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced. Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor must stand in order to take height measurements for an entire region).

6.1 School Bus Routing

Most school in the country provide school buses for transporting children to and from school Most also operate under certain rules, one of which usually states that no child shall have to walk farther than, say one quarter km to a bus pickup point. Thus, they must construct a route for each bus that gets within one quarter km of every child in its assigned area. No bus ride can take more than some specified number of minutes, and Limits on the number of children that a bus can carry at any one time. Let us say that the following figure represents a street map of part of a city, where each edge represents one pick up block. The school is located at the large vertex. Let us assume that the school has decided that no child shall have to walk more than two blocks in order to be picked up by a school bus. Construct a route for a school bus that leaves the school, gets within two blocks of every child and returns to the school.

6.2 Computer Communication Networks

Consider a computer network modeled by a graph G = (V,E) for which vertices represents computers and edges represent direct links between pairs of computers. Let the vertices in following figure represent an array, or network, of 16 computers, or processors. Each processor to which it is directly connected. Assume that from time to time we need to collect information from all processors. We do this by having each processor route its information to one of a small set of collecting processors (a dominating set). Since this must be done relatively fast, we cannot route this information over too long a path. Thus we identify a small set of processors which are close to all other processors. Let us say that we will tolerate at most a two unit delay between the time a processor sends its information and the time it arrives at a nearby collector. In this case we seek a distance-2 dominating set among the set of all processors.

6.3 Radio Stations

Suppose that we have a collection of small villages in a remote part of the world. We would like to locate radio stations in some of these villages so that messages can be broadcast to all of the villages in the region. Since each radio station has a limited broadcasting range, we must use several stations to reach all villages. But since radio stations are costly, we want to locate as few as possible which can reach all other villages. Let each village be represented by a vertex. An edge between two villages is labeled with the distance, say in kilometers, between the two villages.

6.4 Locating Radar Stations Problem

The problem was discussed by Berge. A number of strategic locations are to be kept under surveillance. The goal is to locate a radar for the surveillance at as few of these locations as possible. How a set of locations in which the radar stations are to be placed can be determined.

7. CONCLUSION

This paper "A Review on relation between dominating and total dominating color transversal number of graph and monotonicity" is discussed about new domination parameters and characterize the graphs that attain some bounds. Every total dominating set is dominating set. A graph G is bipartite graph then total dominating color transversal number and total dominating number of graph are equal.

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