A Review on Domination Block Subdivision Graphs

[1] M.Karthiga [2] V.Revathi

[1]Research Scholar, Sakthi College of Arts and Science For Women, Oddanchatram.
[2]Assistant Professor, Department of Mathematics, Sakthi College of Arts and Science For Women, Oddanchatram.

ABSTRACT –*This paper, some result ϒ[BS(G)] were obtained in terms of vertices, edges and other different parameters of G. But not in terms of the numbers of BS(G). In addition, we establish the relationship of ϒ"[BS(G)] with other domination parameters of G. Also its relationship with other domination parameters were established.*

Keywords: *Domination sets, Block Subdivision, Edge Domination number, Connected Domination number.*

1. INTRODUCTION

Graph theory is study of points and lines. In particular, it involves the ways in which sets of points, called vertices, can be connected by lines or arcs, called edges. Graphs in this context differ from the more familier coordinate plots that potray mathematical relations and functions. Graph theory concerns the relationship among lines and points and some lines between them. No attention is paid to the position of points and the length of the lines. Dominating set other related subjects and the corresponding graph parameters form an important research area of graph theory, which is rich in history, application, interesting results and unsolved research questions. This is one of the fastest growing areas in Graph Theory.

2. DOMINATION SUBDIVISION NUMBERS OF GRAPHS

Theorem: For any connected graph G and edge uv, where $deg(u) \geq 2$ and $deg(v) \geq 2$, $sd_\gamma(G) \leq deg(u) + deg(v) - 1$.

Proof: $sd_\gamma(G)$ is defined for every connected graph *G* of order $n \geq 3$. Every such graph G either has an edge *uv*, where $deg(u) \geq 2$ and $deg(v) \geq 2$, or it does not. If G has such an edge *uv*, To shows that the domination number of G must increase if every edge incident to either *u* or *v* is subdivided. If G does not have such an edge, then for every edge *uv*, either $deg(u) = 1$ or $deg(v) = 1$. But this implies that G is a star $K_{1,n}$. But for $G = K_{1,n}$, since $n \geq 3$, it is easy to see that the domination number is increased by subdividing any edge, that is, $sd_\gamma(G) = 1$.

Therefore, $sd_\gamma(G)$ is defined for every connected graph of order $n \geq 3$. Although the upper bound in this theorem for the subdivision number of an arbitrary graph is not a constant, it can be used to obtain a constant upper bound for the domination subdivision number of all graphs in some classes of graphs.

Results : 1. For any r x s grid graph $G_{r,s}$, $1 \leq sd_\gamma(G_{r,s}) \leq 4$.
2.For any k-regular graph G, where $k \geq 2$, $1 \leq sd_\gamma(G) \leq 2k-1$.

Theorem: For any cubic graph G, $1 \leq sd_\gamma(G) \leq 5$.
Proof: A vertex which is adjacent to only one other vertex is called a leaf, and its neighbor is called a support vertex. A vertex which is adjacent to two or more leaves is called a strong support vertex.

Theorem: If G has a strong support vertex, then $sd_\gamma(G) = 1$.
Proof : Let *w* be adjacent to leaves *u* and *v*. Subdividing either edge *wu* or *wv* will increase the domination number. Thus, $sd_\gamma(G) = 1$.

Theorem: If G has adjacent support vertices, then $sd_\gamma(G) \leq 3$.
Proof : Let *w* and *x* be adjacent support vertices, and let *u* and *y* be leaves adjacent to *w* and *x*, respectively. Subdividing edges *wu*, *wx*, and *xy* will increase the domination number. Thus, $sd_\gamma(G) \leq 3$.
We show that $sd_\gamma(G) = 1$ for any graph G having $\gamma(G) = 1$.

Theorem: If G is a graph of order $n \geq 3$ and $\gamma(G) = 1$, then $sd_\gamma(G) = 1$.
Proof : If you subdivide any edge in a graph of order *n* whose domination number equals one, the resulting graph cannot have domination number equal to one. We are able to

determine an upper bound on $sd_\gamma(G)$ in terms of $\gamma(G)$ for graphs G with no isolated vertices. For this purpose we need the following result which establishes a connection between the matching number of a graph and its domination subdivision number. A matching in a graph G is a set M of edges having the property that no two edges in M have a vertex in common. The maximum cardinality of a matching in G is called the matching number of G and is denoted $\beta_1(G)$.

3. EDGE DOMINATION IN BLOCK SUBDIVISION GRAPHS

Theorem: For any connected graph G, $\Upsilon'[BS(G)] \leq n(G)$ where n(G) is the number of blocks of G. Equality holds if G is isomorphic to K_2.

Proof : The result can be proved by induction on number of blocks n of G. If n(G) = 2, then $\Upsilon'[BS(G)] = 1 < n(G)$. Assume the result is true for all connected graphs G with n -1 blocks. hat is, $\Upsilon'[BS(G)] \leq n(G) - 1$

Let G_1 be a connected graph with n block With this n^{th} block of G only one edge will be added in $BS(G_1)$. Then by the definition of edge dominating set, $\Upsilon'[BS(G)] \leq (n(G) -1) + 1$ $\Upsilon'[BS(G)] \leq n(G)$. Hence, by induction on n(G), $\Upsilon'[BS(G)] \leq n(G)$ For an equality, if G is isomorphic to K_2, then $\Upsilon'[BS(G)] = 1 = n(G)$. …….(∵ for K_2, n(G) = 1)

Theorem: For any separable connected (p, q) graph $\Upsilon'[BS(G)] \leq q(G)$. Equality holds if G is isomorphic to K_2.

Proof : Let *G* be a graph with n blocks. For any connected graph G, $n(G) \leq p(G)$ - and $p(G)-1 \leq q(G)$ This implies $n(G) \leq q(G)$. For any connected graph G, $\Upsilon'[BS(G)] \leq n(G)$. $\Upsilon'[BS(G)] \leq q(G)$. Hence, $\Upsilon'[BS(G)] \leq q(G)$. If G is isomorphic to K_2, then $\Upsilon'[BS(G)] = 1 = q(G)$ (∵ for K_2, q(G) = 1)

Theorem : For any connected graph G, $\Upsilon'(G) + \Upsilon'[BS(G)] \leq 2q(G)$. Equality holds if $G \cong K_2$.

Proof : For any separable connected (p,q) graph *G*, $\Upsilon'[BS(G)] \leq q(G)$. For any nontrivial connected graph G, $\Upsilon'(G) \leq q(G)$. Hence , $\Upsilon'(G) + \Upsilon'BS(G) \leq 2q(G)$. For an equality if G is isomorphic to K_2, then $\Upsilon'[BS(G)] = 1$, $\Upsilon'(G) = 1$ and q(G) = 1. Hence, $\Upsilon'(G) + \Upsilon'[BS(G)] = 2$. Therefore, $\Upsilon'(G) + \Upsilon'[BS(G)] = 2\,q(G)$ (∵ q(G) = 1)

Theorem : For any connected graph G, $\Upsilon'[BS(G)] < p(G)$.

Proof : Suppose $H = \{ v_1 , v_2 , v_3, \ldots v_s \}$, $D \subseteq V[BS(G)]$ be the set of vertices in BS(G) corresponding to the blocks $\{ B_1 , B_2 , B_3 \ldots. B_S \}$ in S(G). Then $D \subseteq E[BS(G)]$ forms an edge dominating set of BS(G) with $\Upsilon'[BS(G)] = | D |$. Suppose P′ be the number of vertices in BS(G). Since for any connected graph G, $\Upsilon'[BS(G)] = | D | \leq \frac{P}{2} < p(G)$. Therefore, $\Upsilon'[BS(G)] < p(G)$.

4. CONNECTED DOMINATION IN BLOCK SUBDIVISION GRAPHS

Theorem: For any connected (p,q) graph G, $\Upsilon_c[BS(G)] < 2p - 2$

Proof :Case (i)

Suppose G is a tree, then q = p - 1 and |E(G)| = q Therefore, |E[S(G)]| = 2q

That is, V[BS(G)] = 2q

V[BS(G)] = 2(p-1)

V[BS(G)] = 2p – 2

Let $D \subseteq V[BS(G)]$ is a connected dominating set in BS(G) such that , $\Upsilon_c[BS(G)] = |D|$. Since total number of vertices in BS(G) is 2p - 2, from the definition of connected dominating set in BS(G), $\Upsilon_c[BS(G)] = | D | < 2p - 2$. Hence the theorem is true in this case.

Case (ii): Suppose *G* is not a tree and at least one block contains maximum number of vertices. Let D denote the connected dominating set in BS(G) such that $\Upsilon_c[BS(G)] = | D |$ and $|D| < 2p – 2$ Hence, $\Upsilon_c[BS(G)] < 2p - 2$. Therefore, the theorem is true in this case.

Theorem: For any connected (p,q) graph G, $\Upsilon_c[BS(G)] < p(G) + s(G)$ where S(G) is number of cut vertices of G.

Proof : If G has no cut vertices, then G is non separable, $\Upsilon_c[BS(G)] = 1 < p(G) + s(G)$.

For any separable graph G consider the following two cases.

Case(i): Let the graph G be separable and c be the cut vertices of G. Let G be a tree.

Since S is number of cut vertices of G, $C \subseteq V(G)$ and | C | = S(G). Suppose $C' \subseteq V[BS(G)]$ be the set of cut vertices of BS(G) such that $\Upsilon[BS(G)] = | D |$. Now $F = \{ u_i \in N(D)\} - \{ v_j \}$ where $\{ u_i\}$ is a set of elements in neighbourhood of D and $\{ v_j \}$ is a set of end vertices of BS(G) such that $< D \cup F >$ forms a connected dominating set of BS(G). Hence, $\Upsilon_c[BS(G)] = | < D \cup F > | = | D | + | F | = \Upsilon[BS(G)] + | F |$ Since $\Upsilon[BS(G)] < P$, then $\Upsilon_c[BS(G)] < P(G) + S(G)$.

Case(ii) : Suppose G is not a tree and at least one block contains maximum number of vertices. Then, $\Upsilon_c[BS(G)] < P(G) + S(G)$ From the above two cases, $\Upsilon_c[BS(G)] < P(G) + S(G)$.

5. SPLIT BLOCK SUBDIVISION DOMINATION IN GRAPHS

All graphs considered here are simple, finite, nontrivial, undirected and connected. As usual p,q and n denote the number of vertices, edges and blocks of a graph G respectively. The minimum degree and maximum degree of a graph G are denoted by $\delta(G)$ and $\Delta(G)$ respectively. A vertex cover of a graph G is a set of vertices that covers all the edges of G .

The vertex covering number $\alpha_o(G)$is a minimum cardinality of a vertex cover in G. The vertex independence number $\beta_o(G)$ is the maximum cardinality of an independent set of vertices. A edge cover of G is a set of edges that covers all the vertices. The edge covering number $\alpha_1(G)$of G is minimum cardinality of a edge cover. The edge independence number $\beta_1(G)$ of a graph G is the minimum cardinality of an independent set of edges. A set of vertices $D \subseteq V(G)$ is a dominating set. If every vertex in V-D is adjacent to some vertex in D .The Domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set in G. A dominating set D of a graph G is a split dominating set if the induced subgraph <V-D> is disconnected. The split domination number $\gamma_S(G)$ of G a graph is the minimum cardinality of a split dominating set.

Theorem: For any graph G n-blocks and $n \geq 2$, then $\gamma_{ssb}(G) \leq n-1$.

Proof : For any graph G with n=1 block, a split domination does not exists. Hence we required $n \geq 2$ blocks. Let $S=\{B_1,B_2,B_3,\ldots,B_n\}$ be the number of G and $M=\{ b_1,b_2,b_3,\ldots,b_n \}$ be the vertices in B(G) with corresponding to the blocks of S. Also $V=\{ v_1,v_2,v_3,\ldots, v_n \}$ be the set of vertices in[SB(G)].

Let, $V_1=\{ v_1,v_2,v_3,\ldots, v_i \}$, $1\leq i\leq n$, $V_1\subseteq V$ be a set of cut vertices. Again consider a subset $V_1^1$of V such that$\forall$ $v_i\in N(V) \cap N(V_1^1)$ and $V_1=V-V_1^1$. Let $V_1=\{ v_1,v_2,v_3,\ldots, v_s \}$, $1 \leq s \leq n$, $\forall v_s\in V$ which are not cut vertices such that $N(V_1)\cap N(V_2)=\phi$,then $\{V_1\cup V_2\}$is a dominating set. Clearly $V[SB(G)-\{V_1\cup V_2\}] =H$ is disconnected graph. Then $(V_1\cup V_2)$ is a γ_{ssb}-set of G.

Hence $|V_1\cup V_2|= \gamma_{ssb}(G)$ which gives $\gamma_{ssb}(G) \leq n-1$. In the following Theorem, we obtain an upper bound for in terms of vertices added to B(G).

Theorem: For any connected (p, q) graph with $n\geq 2$ blocks, then $\gamma_{ssb}(G) \leq R$ where R is the number of vertices added to B(G).

Proof: For any nontrivial connected graph G. If the graph G has n=1 block .Then by the definition, split domination set does not exists. Hence $n \geq 2$ blocks.

Let $S=\{B_1,B_2,B_3,\ldots,B_n\}$ be the blocks of G and $M=\{ b_1,b_2,b_3,\ldots,b_n \}$ be the vertices in B(G) which corresponds to the blocks of S. Now we consider the following cases.

Case 1: Suppose each block of B(G) is an edge. Then $R=q=E[B(G)]$. Let $V_1=\{ v_1,v_2,v_3,\ldots, v_i \}$ be the set of vertices of [SB(G)]. Now consider, $V_1=\{ v_1,v_2,v_3,\ldots, v_i \}$, $1 \leq i \leq n$ is a set of cut vertices in [SB(G)]. Let $V_1 \subseteq V_2$, $\forall v_i \in V_2$ are adjacent to end vertices of [SB(G)]. Again there exists a subset $V_3$of V_1 with the property $V [SB(G)]-\{V_2\cup V_3\}=H$. where $\forall v_n \in H$ is adjacent to atleast one vertex of $(V_2\cup V_3)$ and H is a disconnected graph. Hence $V_2\cup V_3$ is as γ_{ssb} set of G.

$$|V_2\cup V_3| \leq R.$$

Case 2 : Suppose each block of B(G) is a complete graph with $p\geq 3$ vertices. Again we consider the sub cases of case 2.

Subcase 2.1: Assume $B(G)=K_p$,$p\geq 3$. Then $V[SB(G)]= V[B(G)]+ q[B(G)]$ and $V[SB(G)]- V[B(G)]= q[B(G)]$ where is an isolates.

Hence $|q[B(G)]\geq V[B(G)]|$ which gives $\gamma_{ssb}(G) \leq R$.

Subcase 2.2: Assume every block of B(G) is K_p, $p\geq 3$. Let $B(G)=\{K_{p1},K_{p2},K_{p3},\ldots\ldots,K_{pm}\}$ then $V\{S[B_1(G) \cup B_2(G)\cup B_3(G)\cup\ldots\ldots \cup B_m(G)]\} = V[B1, B_2, B_3, B_4 ,\ldots\ldots,B_m]+ q_1[B (G)] \cup q_2[B (G)] \cup q_3[B (G)]\ldots\ldots. \cup q_m[B (G)]$ and $V[S[B_1(G) \cup B_2(G)\cup B_3(G)\cup\ldots\ldots \cup B_n(G)] -V[B1, B_2, B_3, B_4 ,\ldots\ldots,B_n] = q_1[B (G)] \cup q_2[B (G)] \cup q_3[B (G)]\ldots\ldots$

∪ $q_m[B\ (G)]$ where $v_i \in q_1[B\ (G)] \cup q_2[B\ (G)] \cup q_3[B\ (G)] \ldots\ldots \cup q_m[B\ (G)]$ is an isolate.
Hence $|q_1[B(G)] \cup q_2[B(G)] \cup q_3[B(G)] \ldots \cup q_m[B(G)]| \geq |V[B1, B_2, B_3, B_4, \ldots, B_m]|$ which gives $\gamma_{ssb}(G) \leq R$.

We establish an upper bound involving the Maximum degree $\Delta(G)$ and the vertices of G for split block sub division domination in graphs.

6. CONCLUSION

A non-trivial connected graph G with at least one cut vertex is called separable graph, otherwise a non-separable graph. This paper deals "The Review on Domination Block Subdivision Graph", can make an in-depth study in domination subdivision and its related works.

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