ENERGY OF FUZZY LABELING IN COMPLETE GRAPH THROUGH HAMILTONIAN CIRCUIT

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***Abstract:** A Hamiltonian circuit includes each vertex of the graph once and only once. This paper generalizing the energy of fuzzy labeling graph of complete graphs through Hamiltonian circuit and studied properties of complete graph. Discussion of upper bound and lower bound of energy.*

***Keyword:** complete graph, fuzzy labeling graph, energy of fuzzy labeling graph, Hamiltonian circuit*

I. INTRODUCTION

Energy of graph is the sum of eigen values of adjacency matrix. This name suggests by energy in chemistry. The study of π – electron energy in chemistry dates back to 1940's but it is in 1978 that I.Gutman defined it mathematically for all graphs. Organic molecules can be represented by graphs called molecular graphs. In [2] R. Balakrishnan defined the energy of π electrons of the molecule is approximately the energy of its molecular graph. Energy of graphs has many mathematical properties which are being investigated. The nature of values discussed in energy of graph is never an odd integer [3] and energy of a graph is never the square root of an odd integer [14]. Study of Energy of different graphs such as regular, non-regular, circulate and random graphs, signed graphs, weighted graphs [4,5,6,8,16] by I.Gutman and Shao. In [1] Anjali Narayanan and Sunil Mathew introduced concept of the energy of fuzzy graphs.
In 1965 Fuzzy relations on a set was first defined by Zadeh in 1965[19].A fuzzy set is defined mathematically by assigning to each possible individual in the universe of discourse a value, representing its grade of membership, which

corresponds to the degree, to which that individual is similar or compatible with the concept represented by the fuzzy set. Fuzzy relation fuzzy graph introduced by Kaufman in 1973 . Fuzzy graphs have many more applications in modeling real time systems where the level of information inherent in the system varies with different level of precision. In 1975 Azriel Rosenfield developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts like bridge and tree [15], Complement fuzzy tree[17], fuzzy line graph[9], fuzzy cycle[10] , established some characterization of fuzzy tree using fuzzy bridges and fuzzy cut nodes [12]. In [11] introduced the concept of bipartite fuzzy graph. S.Vimala and A.Nagarani [13,18] introduced the concept of energy of fuzzy labeling graph. Our result consider for simple directed in degree graphs.

II. PRELIMINARIES

Hamilton path: [7] A Hamilton path in a graph is a path that includes each vertex of the graph once and only once.
Hamiltonian circuit: [7] A Hamiltonian circuit is a circuit that includes each vertex of the graph once and only once.
Hamiltonian graph: [7] If a graph has a Hamiltonian circuit, then the graph is called a Hamiltonian graph. Number of Hamiltonian circuit in K_N is (N-1)!
Complete graph: [7] A graph with N vertices in which every pair of distinct vertices is joined by an edge is called a Complete graph on N vertices and is denoted by the symbol K_N.
Fuzzy graph: [11] Let U and V be two sets. Then ρ is said to be a fuzzy relations from $U \times V$ to [0,1]. A

fuzzy graph G= (σ, μ) is a pair of function of vertices and edges consider $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$, where for all u,v $\in$ V,we have μ (u,v) $\leq \sigma$ (u) $\wedge$ σ (v).

Fuzzy Path: [11] A path P in a fuzzy graph is a sequence of distinct nodes v_1, v_2, …, v_n such that μ $(v_i,v_{i+1}) > 0$; $1 \leq i \leq n$: here n ≥ 1 is called the length of the path P.

Fuzzy labeling graph:[11] A graph G = (σ, μ) is said to be a fuzzy labeling graph, if $\sigma : V \to [0,1]$ and μ :V $\times$ V $\to [0,1]$ is bijective such that the membership value of edges and vertices are distinct and μ (u,v) $< \sigma(u) \wedge \sigma(v)$ for all u, v $\in$ V.

Proposition 1:[11] Every fuzzy labeling graph has at least one weakest arc.

Proposition 2:[12] A cycle graph G^* is said to be a fuzzy labeling cycle graph if it has fuzzy labeling.

Energy of fuzzy labeling graph:[13,18] The following three conditions are true if the graph is called a energy of fuzzy labeling graph.

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$
$$\mu (u, v) > 0$$
$$\mu (u, v) < \sigma(u) \wedge \sigma(v)$$

The bounds of energy are
$$\sqrt{2 \sum_{i=1}^{m} m_i^2 + n(n-1) |A|^{\frac{2}{n}}} > \sqrt{2(\sum_{i=1}^{m} m_i^2)n} > EF_l(G).$$

III. MAIN RESULTS

Theorem 1. Let $K_N[F_l(G)]$ be a complete fuzzy labeling graph $|V| = N$ vertices and $\mu = \frac{N(N-1)}{2}$ edges. If N=3, then the number of distinct Hamiltonian circuit has (N -1)! and eigen values and energies are constant.

Proof:

Let K_N be a directed simple complete graph. If fuzzy graph G = K_N , then sum of the degrees of all vertices is N (N-1). By the Euler's sum of degrees theorem that the number of edges is $\frac{N(N-1)}{2}$ .If every vertex is connected to every other vertex then Hamilton circuits form in different ways (with respect to in degree) to rearrange (N-1) ! vertices . So K_N has (N-1) ! Hamiltonian circuit. Form different adjacency matrix A_1 (G).Find eigen values by determinant of adjacency matrix λ_1 ,λ_2,…,λ_n (say).The energy of graph E (G) $= \sum_{i=1}^{n} |\lambda_i|$.Eigen value and energy of Hamiltonian circuit is constant.

Preposition

The lower bound of energy is greater than the upper bound energy of Hamiltonian circuit.

Proof:

[2,1] derived lower and upper bound of the energy graph and fuzzy energy graph. By above statement proved eigen values based on that fuzzy energy. Fuzzy energy calculated by [13,18].So fuzzy energy resulted by

Example

Let G be directed fuzzy graph. Consider in degree values form circular graph.

If N =3, Number of distinct Hamiltonian circuit has (3 -1)! = 2.(i.e) 2 Hamiltonian circuit Namely ABCA, ACBA.
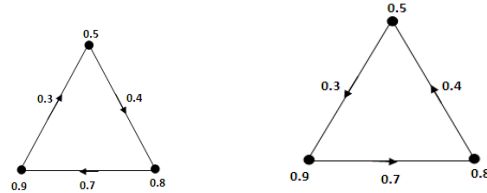
$$A_1F_l(G)=\begin{bmatrix} 0 & 0.4 & 0 \\ 0 & 0 & 0.7 \\ 0.3 & 0 & 0 \end{bmatrix} \text{ and } A_2F_l(G)=\begin{bmatrix} 0 & 0 & 0.3 \\ 0.4 & 0 & 0 \\ 0 & 0.7 & 0 \end{bmatrix}$$

The eigen values are 0.4380, 0.4380, 0.4380, Energy = 1.3140, Lower Bound = 2.1071, Upper Bound = 1.6225 in the circuits ABCA, ACBA.

Hence Hamiltonian circuit has same eigen value and same energies in K_3.

Theorem 2. Let $K_N[F_l(G)]$ be a complete fuzzy labeling graph $|V| = N$ vertices and $\mu = \frac{N(N-1)}{2}$ N= 4, then the pair of Hamiltonian circuit has same energies and same eigen values in (N -1)!.

Proof:

If N = 4, Number of distinct Hamiltonian circuit has (N -1)! (i.e.) 6 Hamiltonian circuits, Namely ABCDA, ADCBA, ABDCA, ADBCA, ACBDA, ACDBA .

$$A_1F_1(G)=\begin{bmatrix} 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.14 \\ 0.2 & 0 & 0 & 0 \end{bmatrix} A_2F_1(G)=\begin{bmatrix} 0 & 0 & 0 & 0.2 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.14 & 0 \end{bmatrix}$$

$$A_3F_1(G)=\begin{bmatrix} 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0.9 \\ 0.3 & 0.8 & 0 & 0 \\ 0 & 0 & 0.14 & 0 \end{bmatrix} A_4F_1(G)=\begin{bmatrix} 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0.8 & 0 \\ 0.3 & 0 & 0 & 0 \\ 0 & 0.9 & 0 & 0 \end{bmatrix}$$

$$A_5F_1(G)=\begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0.9 \\ 0 & 0.8 & 0 & 0 \\ 0.2 & 0 & 0 & 0 \end{bmatrix} A_6F_1(G)=\begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 \\ 0 & 0.9 & 0 & 0 \end{bmatrix}$$

(i) The eigen values are 0.3077,0.3077,-0.3077, 0.3077Energy =1.2308, Lower Bound =2.6223,Upper Bound = 2.6223 in the circuits ABCDA, ADCBA.

(ii) The eigen values are -0.3507, 0.3507, 0.3507, 0.3507, Energy = 1.4028, Lower Bound = 2.9388, Upper Bound = 0.8274 in the circuits ABDCA, ADBCA.

(iii) The eigen values are -0.4559, 0.4559, 0.4559, 0.4559, Energy = 1.8236, Lower Bound = 3.5552, Upper Bound = 0.8159 in the circuits ACBDA, ACDBA.

Hence every pair of Hamiltonian circuit has same energy and same eigen value in K_5.

Theorem 3. Let $K_N[F_1(G)]$ be a complete fuzzy labeling graph $|V| = N$ vertices and $\mu = \frac{N(N-1)}{2}$ edges and $5 \leq n \leq 7$ then the Hamiltonian circuit has either same eigen values or difference in $(N-1)!$

Proof:
Case(i): If N = 5,it has 24 Hamiltonian circuit the first five circuits are ABCDEA, ABCEDA, ABDCEA, ABDECA, ABECDA.

$$A_1F_1(G)=\begin{bmatrix} 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.19 \\ 0.1 & 0 & 0 & 0 & 0 \end{bmatrix} A_2F_1(G)=\begin{bmatrix} 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.13 \\ 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.19 & 0 \end{bmatrix}$$

(i) The eigen values are 0.3000, 0.3000, 0.3001, 0.3001, 0.3000 Energy =1.5002 in the circuite ABCDEA.

(ii) The eigen values are 0.2753, 0.2753, 0.2753, 0.2753, 0.2753, Energy =1.3765 in the circuite ABCEDA.

Hence the Eigen value difference is 0.001 for K_5

Case(ii): If N=6. It has 120 Hamiltonian circuit. The first five circuits are ABCDEFA, ABCDFEA, ABDCEFA, ABDEFCA, ABFCDEA .

$$A_1F_1(G)=\begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.14 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.19 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.24 \\ 0.4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} A_2F_1(G)=\begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.14 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.19 & 0 \\ 0.3 & 0 & 0 & 0 & 0 & 0.24 \\ 0 & 0 & 0 & 0 & 0 & 0.24 \end{bmatrix}$$

$$A_3F_1(G)=\begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.12 \\ 0 & 0 & 0.14 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.24 & 0 \\ 0.4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} A_4F_1(G)=\begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & 0.19 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.24 \\ 0 & 0 & 0.13 & 0 & 0 & 0 \end{bmatrix}$$

(i) The eigen values are -0.3236, 0.3236, 0.3236, 0.3236, 0.3236, 0.3236, Energy =1.9416 in the circuite ABCDEFA

(ii) The eigen values are 0,0.3244, 0.3244, 03244, 0.3244, 0.3244, Energy =1.9464 in the circuite ABCDFEA,

(ii) The eigen values are 0, 0.3061, 0.3061, 0.3061, 0.3061, 0.3061, Energy =1.8366 in the circuite ABDCEFA.

(ii) The eigen values are -0.2488, 0.2488, 0.2488, 0.2488, 0.2488, 0.2488 Energy =1.4928 in the circuite ABDCEFA

Case (iii): If N=7. It has 720 Hamiltonian circuit. The first five circuits are ABCDEFGA, ABDCEGFA, ACBEDFGA, ADBCEGFA, AEDBCFGA

$$A_1F_1(G)=\begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.14 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.19 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.24 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.28 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.32 \\ 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2F_1(G)=\begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.18 & 0 & 0 \\ 0 & 0 & 0.19 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.27 \\ 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.32 & 0 \end{bmatrix}$$

$$A_3F_l(G) = \begin{bmatrix} 0 & 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.12 & 0 & 0 \\ 0 & 0.14 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.23 & 0 \\ 0 & 0 & 0 & 0.24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.32 \\ 0.8 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_4F_l(G) = \begin{bmatrix} 0 & 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0.14 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.18 & 0 & 0 \\ 0 & 0.13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.29 \\ 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.82 & 0 \end{bmatrix}$$

(i) The eigen values are 0.3019, 0.3019, 0.3019, 0.3020,0.3020,0.3020, 0.3020 Energy =2.1137 in the circuite ABCDEFGA.

(ii) The eigen values are 0.2827, 0.2827, 0.2827, 0.2827, 0.2827, 0.2827, 0.2827, Energy =1.9789 in the circuite ABDCEGFA.

(iii) The eigen values are 0.2412, 0.2412, 0.2412, 0.2412, 0.2412, 0.2412, 0.2412, Energy =1.6884 in the circuite ACBEDFGA.

(iv) The eigen values are 0.2516, 0.2516, 0.2516 0.2516 , 0.2516, 0.2516 , 0.2516 Energy =1.7612 in the circuite ADBCEGFA,

IV. CONCLUSION

Continuing the process K_N has (N-1)!,Hamiltonian circuit of $5 \le N \le 7$ has either a same eigen values or eigen values difference from 0.0001.

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