A Review on Relationship between Domination, Independent Transversal Domination and Equitable Domination in Graphs

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ABSTRACT – A set S □ V of vertices in a graph G = (V,E) is called a dominating set if every vertex in V − S is adjacent to a vertex in S. A dominating set which intersects every maximum independent set in G is called an independent transversal dominating set. In this paper we begin an investigation of relationship between domination, independent transversal domination and equitable domination in graphs.

Keywords: Dominating set, independent set, independent transversal dominating set, equitable dominating set.

1. INTRODUCTION

One of the fastest growing areas within graph theory is the study of domination, independent transversal dominating set, equitable dominating set and related subset problems such as independence, covering and matching. An independent dominating set S is a dominating set such that S is an independent set. The independent domination number i(G) is the minimum cardinality of an independent dominating set. The maximum cardinality of an independent set is called the independence number and is denoted by β_0(G). A subset D of $V(G)$ is called an equitable dominating set of a graph G if for every $u \in \langle V - D\rangle$, there exists a vertex $v \in D$such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of such a dominating set is denoted by $\gamma_e(G)$ and is called equitable domination number of G. An equitable dominating set which intersects every maximum independent set in G is called an independent transversal equitable dominating set. The minimum cardinality of an independent transversal equitable dominating set is called the independent transversal equitable domination number of G and is denoted by $\gamma_{ite}(G)$.

2. INDEPENDENT TRANSVERSAL DOMINATION IN GRAPHS

Theorem : For any graph G, we have γ(G) ≤ γ_{it}(G) ≤ γ(G) + δ(G).

Proof: Since an independent transversal dominating set of G is a dominating set, it follows that γ(G) ≤ γ_{it}(G).Now, let u be a vertex in G with deg u = δ(G) and let S be a γ-set in G. Then every maximum independent set of G contains a vertex of N[u] so that S □ N[u] is an independent transversal dominating set of G. Also, since S intersects N[u], it follows that |S □ N[u]| ≤ γ(G) + δ(G) and hence the right inequality follows.

Theorem: If G is a graph with diam G = 2, then γ_{it}(G) ≤ δ(G) + 1.

Proof: Let u be a vertex with deg u = δ(G). Then N[u] is a dominating set of G, because diam G = 2.
Now, it follows from the fact that every maximum independent set contains a vertex of N[u].This closed neighborhood itself is an independent transversal dominating set so that γ_{it}(G) ≤ δ(G) + 1.

Theorem: Let G be a bipartite graph with bipartition (X, Y) such that |X| ≤ |Y | and γ(G) = |X|. Thenγ_{it}(G) = γ(G) + 1 if and only if every vertex in X is adjacent to at least two pendant vertices.

Proof: We first claim that δ(G) = 1. Suppose δ(G) ≥ 2. Since γ(G) = |X|, X is a γ-set. Also, since γ_{it}(G) = γ(G) + 1 it follows that β_0(G) = |Y |. Now, let u □ X and v □ N(u). Since _(G) ≥ 2, it follows that S = (X − {u}) □ {v} is a dominating set of G. Now since β_0(G) = |Y | and δ(G) ≥ 2, everyβ_0- set contains either the vertex v or a vertex w ≠ u in X. Hence S intersects everyβ_0 - set so that γ_{it}(G) = |X| = γ(G), which is a contradiction. Thus δ(G) = 1.Further suppose there exists a vertex u in X such that N(u) contains at most one pendant vertex. Then S = (X−{u})∪{v}, where v∈ N(u), and v is chosen to be a pendant vertex if it exists, is a dominating set of G. Also since β_0(G) = |Y |, it follows that S intersects every β_0-set of G and hence γ_{it}(G) ≤ |X| = γ(G), which is a contradiction. Thus every vertex in X is adjacent to at least two pendant vertices.Conversely, if every vertex in X is adjacent to at least two pendant vertices, then X is the only γ-set of G so that γ_{it}(G) = γ(G) + 1.

3. INDEPENDENT TRANSVERSAL EQUITABLE DOMINATION IN GRAPHS

Theorem: For any graph G, γ(G) ≤ γ_e(G) ≤ γ_{ite}(G).

Proof: Let *G* be a graph with minimum independent transversal equitable dominating set *D*. Then *D* is an equitable dominating set of *G* and any equitable dominating set is also dominating set of *G*.From the definitions of the parameters, γ(G), γ_e(G) and γ_{ite}(G), we get for any γ(G) ≤ γ_e(G) ≤ γ_{ite}(G) graph *G*.

Theorem: For any graph G, γ(G) ≤ γ_{it}(G) ≤ γ_{ite}(G).

Proof : Let *G* be a graph with minimum independent transversal equitable dominating set *S*.
From the definition of the independent transversal dominating set of *G*, *S* is also independent transversal dominating set, and any independent transversal dominating set is dominating set of *G*.
Hence γ(G) ≤ γ_{it}(G) ≤ γ_{ite}(G).

Theorem : For any graph G which all of its edges are equitable, $\gamma_{it}(G)$ = γ_{ite}(G).

Proof: Let *G* be a graph *G* = (*V*,*E*) such that for any edge e ∈ E e is equitable edge. Then any dominating set of *G* will also be equitable dominating set of *G*. Suppose that *D* is an independent transversal dominating set of *G* with size |*D*| = $\gamma_{it}(G)$ .Then *D* is also an independent transversal equitable dominating set, that means $\gamma_{it}(G)$ = γ_{ite}(G).Hence $\gamma_{it}(G)$ = γ_{ite}(G).

Example

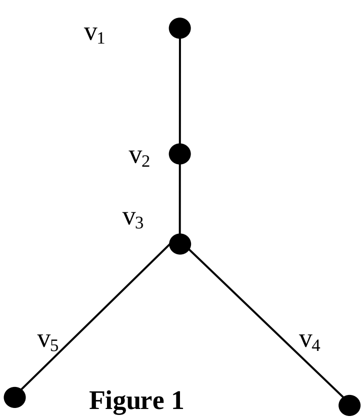

Figure 1

Let G be a graph as in Figure 5.2, we have $\gamma_{it}(G)$ = γ_{ite}(G) =3 the maximum independent sets in G are {v_1, v_4, v_5} and {v_2, v_4, v_5}.The minimum independent transversal dominating sets are {v_1, v_4, v_5}, {v_2, v_4, v_5} and {v_1, v_2 ,v_3} and γ_{it}(G) = 3.

The minimum independent transversal equitable dominating set of G is only {v_2, v_4, v_5}. So γ_{ite}(G) = 3.Hence $\gamma_{it}(G)$ = γ_{ite}(G), but the edge v_3v_4 and v_3v_5 are not equitable edges in G.

In Figure1 $\gamma_{it}(G)$ = γ_{ite}(G) but not all the edges are equitable edges.

4. CONCLUSION

This paper deals about "A Review on relationship between domination, independent transversal domination and equitable domination in graphs". we discuss the relation between domination and independent transversal domination also the relation between independent transversal and equitable domination. So that if we find the domination number we can find the independent transversal domination number. Also we can find the equitable domination number.

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