Closure of Some Bivariate Ageing Classes under Poisson Shock Models

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Abstract

In this paper, we determine the closure properties of the bivariate ageing classes of life distributions under Poisson Shock models.

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1 Introduction

Life time of systems or components are considered in many areas, for example, in the analysis of product or system reliability. In stochastic modeling, such life times are represented by random variables with respective distributions. Ageing properties plays an important role in the analysis of life time distributions. Positive ageing signifies the adverse effect on the age of the systems. It may be caused by wearout or fatigue. A positive ageing class contains life time distributions that show a characteristic behavior of positive ageing. Ageing properties are often expressed in terms of the residual life time of units of different ages. In particular, a probability distribution may belong to a certain positive ageing class if the random residual life time of the corresponding unit decreases with increasing age. Thus, it is necessary to compare probability distributions.

The best studied univariate ageing class is the Increasing Failure Rate (IFR) class. A life time distribution with survival function $\overline{F}$ belongs to this class, if the residual life time of a unit of less age dominates the residual life time of a unit of greater age with respect to the stochastic order. Besides the IFR class, there is a variety of different positive ageing classes introduced in the literature. In particular, the closure properties of these classes with respect to the formation of coherent systems, convolutions and mixtures are considered. For further details, one may refer to Barlow and Proschan (1981). The IFR class is not closed under the formation of coherent systems with independent components. The smallest univariate class that contains the exponential distributions and that is closed under the formation of coherent systems with independent components and under limits in distribution is the Iincreasing Failure Rate Average (IFRA) class. The exponential distribution is the only distribution with a constant hazard rate. Therefore, the exponential distributions play a vital role in the study of ageing classes. Note here that the exponential distribution belongs to the IFR class and to its dual class, the Decreasing Failure Rate (DFR) class, because of its memoryless property. The DFR class is a negative ageing class.

The treatment of ageing concepts is predominantly restricted to univariate properties. Nevertheless, there were different approaches to find suitable bivariate versions of existing univariate classes.

Shock model is one of the important research in the mathematical theory of reliability. Shock models are generally used to describe the system failure, maintenance and other life phenomena in random changes in environment. Esary and Marshal (1973) have studied the univariate life distribution of the system and obtained the nature of the Increasing Failure Rate (IFR), Increasing Failure Rate Average (IFRA) and New Better than Used (NBU) ageing classes of the survival function when the basic process is homogeneous Poisson process.

A-Hameed and Proschan (1975) have studied shock models with underlying birth process. They have presented the preservation of some univariate ageing classes, viz., IFR, IFRA, NBU and NBUE. Klefsjö (1981) has studied the preservation of Harmonic New Better than Used in Expectation class under some shock models.

In this paper, we focus on very strong versions of some Bivariate Ageing classes defined by Syed Tahir Hussainy (2015). We study the preservation of some bivariate ageing classes under Poisson shock models. The rest of the paper is organized as follows. In section 2, we give the preliminaries. Closure properties of the bivariate ageing classes BNBU, BNBUE and BHNBUE under Poisson Shock models are presented in Section 3. Finally conclusion is given in Section 4.

2 Preliminaries

In this section, we first give the definitions of some bivariate ageing classes.

A bivariate random variable (X,Y) or its distribution $\overline{F}(t,s)$ is said to have Bivariate New Better than Used (BNBU), if

$$\overline{F}(x+t,y+s)\leq\overline{F}(x,y)\cdot\overline{F}(t,s)$$

for $x,y,t,s\geq 0.$

Remark 1. The above definition is a very strong version of the BNBU ageing class. This class may be referred as BNBU-VS. Other versions of the BNBU ageing class are listed below.

- *strong Bivariate New Better than Used (BNBU-S)* :
  $$\overline{F}(x+t,y+t)\leq\overline{F}(x,y)\cdot\overline{F}(t,t)$$
  for $x,y,t\geq 0.$
- *weak Bivariate New Better than Used (BNBU-W)* :
  $$\overline{F}(x+t,x+s)\leq\overline{F}(x,x)\cdot\overline{F}(t,s)$$
  for $x,t,s\geq 0.$
- *very weak Bivariate New Better than Used (BNBU-VW)* :
  $$\overline{F}(x+t,x+t)\leq\overline{F}(x,x)\cdot\overline{F}(t,t)$$
  for $x,t\geq 0.$

A bivariate random variable (X,Y) or its distribution $\overline{F}(t,s)$ is said to have Bivariate New Better than Used in Expectation (BNBUE), if

$$\int_0^\infty\int_0^\infty\overline{F}(x+t,y+s)\,dtds\leq\overline{F}(x,y)\int_0^\infty\int_0^\infty\overline{F}(t,s)\,dtds,$$

for $x,y,t,s\geq 0.$

Remark 2. The above definition is a very strong version of the BNBUE ageing class. This class may be referred as BNBUE-VS. On similar lines to Remark 1, the other versions of the BNBUE ageing class may be defined.

A bivariate random variable (X,Y) or its distribution $\overline{F}(t,s)$ is said to have Bivariate Harmonic New Better than Used in Expectation (BHNBUE), if

$$\int_t^{\infty}\int_s^{\infty}\overline{F}(x,y)\,dydx \le \mu \exp\left[-\frac{t+s}{\mu}\right];\; t\ge 0, s\ge 0.$$

Remark 3. The above definition is a very strong version of the BHNBUE ageing class. This class may be referred as BHNBUE-VS. On similar lines to Remark 1, the other versions of the BHNBUE ageing class may be defined.

Since the discrete counterpart of to the exponential distribution is the geometric distribution, we have the following discrete analogue of the above definitions.

A bivariate discrete survival probability

$$\overline{P}(k,l) = \sum_{i=k+1}^{\infty}\sum_{j=l+1}^{\infty} p_{i,j}$$

with support on $\{0,1,2,\ldots\}$ and with frequency functions

$$p_{i,j} = \left[\overline{P}(i,j-1) - \overline{P}(i-1,j-1)\right]\left[\overline{P}(i-1,j) - \overline{P}(i-1,j-1)\right]$$

for $i = 1,2,\ldots$ and $j = 1,2,\ldots$ and $\overline{P}(0,0) = 1 - P(0,0)$ is said to have

• discrete Bivariate New Better than Used (discrete-BNBU), if

$$\overline{P}(i+k,\, j+l) \le \overline{P}(i,\, j)\, \overline{P}(k,l)$$

• discrete Bivariate New Better than Used in Expectation (discrete BNBUE), if

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\overline{P}(i+k,\, j+l) \le \overline{P}(k,l)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\overline{P}(i,\, j)$$

or, equivalently

$$\sum_{i=k}^{\infty}\sum_{j=l}^{\infty}\overline{P}_{i,j} \le \overline{P}(k,l)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\overline{P}(i,\, j).$$

• discrete Harmonic New Better than Used in Expectation (discrete-HNBUE), if

$$\sum_{i=k}^{\infty}\sum_{j=l}^{\infty}\overline{P}(i,\, j) \le \mu\left(1-\frac{1}{\mu}\right)^{\sqrt{k^2+l^2}},$$

for $k = 0,1,2,\ldots;\; l = 0,1,2,\ldots$ where $\mu = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\overline{P}(i,\, j)$ is finite.

Suppose that a device with two components is subjected to shocks occurring randomly as events in Poisson process with constant intensity λ. Suppose further that the device has the probability $\overline{P}(i,\, j)$ of surviving k shocks on the first component and l shocks on the second component. The survival function of the device is then given by

$$\overline{H}(t_1,t_2) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} e^{-\lambda t_1}\frac{(\lambda t_1)^k}{k!} e^{-\lambda(t_2-t_1)}\frac{[\lambda(t_2-t_1)]^l}{l!}\overline{P}(k,l), \tag{1}$$

for $0 \le t_1 \le t_2$.

3 Poisson Shock Model

In this section, we will prove that, if $\{\overline{P}(i, j)\}_{i,j=0}^{\infty}$ has the discrete BNBU, BNBUE and BHNBUE property, then this property will be reflected to $\overline{H}(t_1,t_2)$, given by the equation (1).

Theorem 3.1 *The survival function* $\overline{H}(t_1,t_2)$ *in equation (1) is BNBU, if* $\{\overline{P}(i, j)\}_{i,j=0}^{\infty}$ *has the discrete-BNBU property.*

Proof. Consider

$$\overline{H}(x+t, y+s) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} e^{-\lambda(x+t)} \frac{[\lambda(x+t)]^k}{k!} e^{-\lambda(y+s-x-t)} \frac{[\lambda(y+s-x+t)]^l}{l!}$$

$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} e^{-\lambda(x+t)} \frac{[\lambda(x+t)]^k}{k!} e^{-\lambda(y+s-x-t)} \frac{[\lambda(y+s-x+t)]^l}{l!} \overline{P}(k,l)$$

$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \overline{P}(k,l) \left(\sum_{\alpha=0}^{k} \binom{k}{\alpha} (\lambda x)(\lambda t)^{k-\alpha} \frac{e^{-\lambda(x+t)}}{k!} \right) \left(\sum_{\beta=0}^{l} \binom{l}{\beta} (\lambda(y-x))^{\beta} (\lambda(s-t))^{l-\beta} \frac{e^{-\lambda(y+s-x-t)}}{l!} \right)$$

$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \overline{P}(k,l) \left[\sum_{\alpha=0}^{k} \frac{(\lambda x)^{\alpha} (\lambda t)^{k-\alpha}}{\alpha!(k-\alpha)!} e^{-\lambda(x)} e^{-\lambda t} \right] \left[\sum_{\beta=0}^{l} \frac{[\lambda(y-x)]^{\beta} [\lambda(s-t)]^{l-\beta}}{\beta!(l-\beta)!} e^{-\lambda y} e^{-\lambda s} e^{\lambda x} e^{\lambda t} \right]$$

$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} e^{-\lambda(y+s)} \sum_{\alpha=0}^{k} \frac{(\lambda x)^{\alpha} (\lambda t)^{k-\alpha}}{\alpha!(k-\alpha)!} \sum_{\beta=0}^{l} \frac{[\lambda(y-x)]^{\beta} [\lambda(s-t)]^{l-\beta}}{\beta!(l-\beta)!} \overline{P}(k,l)$$

$$= \sum_{k=0}^{\infty}\sum_{\alpha=0}^{k} \frac{(\lambda x)^{\alpha} (\lambda t)^{k-\alpha}}{\alpha!(k-\alpha)!} \sum_{l=0}^{\alpha}\sum_{\beta=0}^{l} \frac{[\lambda(y-x)]^{\beta} [\lambda(s-t)]^{l-\beta}}{\beta!(l-\beta)!} e^{-\lambda(y+s)} \overline{P}(k,l)$$

$$= \sum_{\alpha=0}^{\infty}\sum_{k=\alpha}^{\infty} \frac{(\lambda x)^{\alpha} (\lambda t)^{k-\alpha}}{\alpha!(k-\alpha)!} \sum_{\beta=0}^{\infty}\sum_{l=\beta}^{\infty} \frac{[\lambda(y-x)]^{\beta} [\lambda(s-t)]^{l-\beta}}{\beta!(l-\beta)!} e^{-\lambda(y+s)} \overline{P}(k,l)$$

$$= \sum_{\alpha=0}^{\infty}\sum_{\gamma=0}^{\infty} \frac{(\lambda x)^{\alpha} (\lambda t)^{\gamma}}{\alpha!(\gamma)!} \sum_{\beta=0}^{\infty}\sum_{\delta=0}^{\infty} \frac{[\lambda(y-x)]^{\beta} [\lambda(s-t)]^{\delta}}{\beta!\,\delta!} e^{-\lambda(y+s)} \overline{P}(\alpha+\gamma, \beta+\delta)$$

$$=\sum_{\alpha=0}^{\infty}\frac{(\lambda x)^{\alpha}}{\alpha!}\sum_{\beta=0}^{\infty}\frac{[\lambda(y-x)]^{\beta}}{\beta!}\sum_{\gamma=0}^{\infty}\frac{(\lambda t)^{\gamma}}{\gamma!}\sum_{\delta=0}^{\infty}\frac{[\lambda(s-t)]^{\delta}}{\delta!}e^{-\lambda(y+s)}\overline{P}(\alpha+\gamma,\beta+\delta)$$

$$\leq\sum_{\alpha=0}^{\infty}\frac{(\lambda x)^{\alpha}}{\alpha!}\sum_{\beta=0}^{\infty}\frac{[\lambda(y-x)]^{\beta}}{\beta!}\sum_{\gamma=0}^{\infty}\frac{(\lambda t)^{\gamma}}{\gamma!}\sum_{\delta=0}^{\infty}\frac{[\lambda(s-t)]^{\delta}}{\delta!}e^{-\lambda(y+s)}\overline{P}(\alpha,\beta)\overline{P}(\gamma,\delta)$$

$$=\left[\sum_{\alpha=0}^{\infty}\frac{(\lambda x)^{\alpha}}{\alpha!}\sum_{\beta=0}^{\infty}\frac{\lambda(y-x)^{\beta}}{\beta!}e^{-\lambda x}e^{-\lambda(y-x)}\overline{P}(\alpha,\beta)\right]$$
$$\left[\sum_{\gamma=0}^{\infty}\frac{(\lambda t)^{\gamma}}{\gamma!}\sum_{\delta=0}^{\infty}\frac{[\lambda(s-t)]^{\delta}}{\delta!}e^{-\lambda t}e^{-\lambda(s-t)}\overline{P}(\gamma,\delta)\right]$$

$$=\left[\sum_{\alpha=0}^{\infty}\sum_{\beta=0}^{\infty}e^{-\lambda x}\frac{(\lambda x)^{\alpha}}{\alpha!}e^{-\lambda(y-x)}\frac{\lambda(y-x)^{\beta}}{\beta!}\overline{P}(\alpha,\beta)\right]$$
$$\left[\sum_{\gamma=0}^{\infty}\sum_{\beta=0}^{\infty}e^{-\lambda t}\frac{(\lambda t)^{\gamma}}{\gamma!}e^{-\lambda(s-t)}\frac{[\lambda(s-t)]^{\delta}}{\delta!}\overline{P}(\gamma,\delta)\right]$$

$$=\overline{H}(x,y)\overline{H}(t,s)$$

The inequality is because $\{\overline{P}(i,j)\}_{i,j=0}^{\infty}$ is discrete-BNBU. It follows that $\overline{H}$ is BNBU and this completes the proof.

Theorem 3.2 *The survival function* $\overline{H}(t_1,t_2)$ *in equation (1) is BNBUE, if* $\{\overline{P}(i,j)\}_{i,j=0}^{\infty}$ *has the discrete-BNBUE property.*

Proof. Consider

$$\overline{H}(t,s)\mu-\int_t^{\infty}\int_s^{\infty}\overline{H}(x,y)dydx$$

$$=\overline{H}(t,s)\int_0^{\infty}\int_0^{\infty}\overline{H}(x,y)dydx-\int_t^{\infty}\int_s^{\infty}\overline{H}(x,y)dydx$$

$$=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}e^{-\lambda t}\frac{(\lambda t)^{i}}{i!}e^{-\lambda(s-t)}\frac{[\lambda(s-t)]^{j}}{j!}\overline{P}(i,j)$$
$$\int_0^{\infty}\int_0^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}e^{-\lambda x}\frac{(\lambda x)^{k}}{k!}e^{-\lambda(y-x)}\frac{[\lambda(y-x)]^{l}}{l!}\overline{P}(k,l)dydx$$
$$\int_t^{\infty}\int_s^{\infty}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}e^{-\lambda x}\frac{(\lambda x)^{i}}{i!}e^{-\lambda(y-x)}\frac{[\lambda(y-x)]^{j}}{j!}\overline{P}(i,j)dydx$$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-\lambda t}\frac{(\lambda t)^i}{i!} e^{-\lambda(s-t)}\frac{[\lambda(s-t)]^j}{j!}\overline{P}(i,j)\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\overline{P}(k,l)$$
$$\int_0^{\infty}\int_0^{\infty} e^{-\lambda x}\frac{(\lambda x)^k}{k!} e^{-\lambda(y-x)}\frac{[\lambda(y-x)]^l}{l!}dydx - \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\overline{P}(i,j)$$
$$\int_t^{\infty}\int_s^{\infty} e^{-\lambda x}\frac{(\lambda x)^i}{i!} e^{-\lambda(y-x)}\frac{[\lambda(y-x)]^j}{j!}dydx$$
$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-\lambda t}\frac{(\lambda t)^i}{i!} e^{-\lambda(s-t)}\frac{[\lambda(s-t)]^j}{j!}\overline{P}(i,j)\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\overline{P}(k,l).\frac{1}{\lambda^2}$$
$$-\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\overline{P}(i,j)\frac{1}{\lambda}\sum_{k=0}^{i}\sum_{l=0}^{j} e^{-\lambda t}\frac{(\lambda t)^i}{i!} e^{-\lambda(s-t)}\frac{[\lambda(s-t)]^j}{j!}$$

$$= \frac{1}{\lambda^2}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-\lambda t}\frac{(\lambda t)^i}{i!} e^{-\lambda(s-t)}\frac{[\lambda(s-t)]^j}{j!}\overline{P}(i,j)\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\overline{P}(k,l)$$
$$-\frac{1}{\lambda^2}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=i}^{\infty}\sum_{l=j}^{\infty}\overline{P}(k,l)e^{-\lambda t}\frac{(\lambda t)^i}{i!} e^{-\lambda(s-t)}\frac{[\lambda(s-t)]^j}{j!}$$

$$= \frac{1}{\lambda^2}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-\lambda t}\frac{(\lambda t)^i}{i!} e^{-\lambda(s-t)}\frac{[\lambda(s-t)]^j}{j!}\left[\overline{P}(i,j)\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\overline{P}(k,l) - \sum_{k=i}^{\infty}\sum_{l=j}^{\infty}\overline{P}(k,l)\right]$$
$$\geq 0$$

The third equality is obtained using Exercise 5, page 63 of statistical theory of Reliability and Life testing, Barlow & Proschan, 1981. This implies that

$$\overline{H}(t,s)\mu - \int_t^{\infty}\int_s^{\infty}\overline{H}(x,y)dydx \geq 0$$

That is, $\int_t^{\infty}\int_s^{\infty}\overline{H}(x,y)dydx \leq \mu\overline{H}(t,s)$

$$\int_t^{\infty}\int_s^{\infty}\overline{H}(x,y)dydx \leq \overline{H}(t,s)\int_0^{\infty}\int_0^{\infty}\overline{H}(x,y)dydx,$$

which inturn implies that $\overline{H}$ is BNBUE. This completes the proof of the theorem.

Theorem 3.3 *The survival function* $\overline{H}(t_1,t_2)$ *in equation (1) is BHNBUE, if* $\{\overline{P}(i,j)\}_{i,j=0}^{\infty}$ *has the discrete-BHNBUE property.*

Proof. Consider

$$\int_t^{\infty}\int_s^{\infty}\overline{H}(x,y)dxdy = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\int_t^{\infty}\int_s^{\infty} e^{-\lambda t_1}\frac{(\lambda t_1)^i}{i!} e^{-\lambda(t_2-t_1)}\frac{[\lambda(t_2-t_1)]^j}{j!}\overline{P}(i,j)\right]dxdy$$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\frac{1}{\lambda}\sum_{k=0}^{i}\sum_{l=0}^{j} e^{-\lambda t_1}\frac{(\lambda t_1)^k}{k!}e^{-\lambda(t_2-t_1)}\frac{[\lambda(t_2-t_1)]^l}{l!}\right]\overline{P}(i,j)$$

$$= \frac{1}{\lambda}\left[\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} e^{-\lambda t_1}\frac{(\lambda t_1)^k}{k!}e^{-\lambda(t_2-t_1)}\frac{[\lambda(t_2-t_1)]^l}{l!}\right]\sum_{i=k}^{\infty}\sum_{j=l}^{\infty}\overline{P}(i,j)$$

$$\le \frac{1}{\lambda}\left[\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} e^{-\lambda t_1}\frac{(\lambda t_1)^k}{k!}e^{-\lambda(t_2-t_1)}\frac{[\lambda(t_2-t_1)]^l}{l!}\mu\left(1-\frac{1}{\mu}\right)^{\sqrt{k^2+l^2}}\right]$$

$$= \frac{\mu}{\lambda}e^{-\lambda t_1}e^{-\lambda(t_2-t_1)}\left(\sum_{k=0}^{\infty}\frac{(\lambda t_1)^k}{k!}\sum_{l=0}^{\infty}\frac{[\lambda(t_2-t_1)]^l}{l!}\left(1-\frac{1}{\mu}\right)^{\sqrt{k^2+l^2}}\right)$$

$$\le \frac{\mu}{\lambda}e^{\frac{-\lambda(t_1+t_2)}{\mu}}$$

The first inequality is because $\overline{P}(k,l)$ is discrete BHNBUE and the last inequality is trivial. It follows that $\overline{H}(t_1,t_2)$ in equation (1) is BHNBUE and this completes the proof of the theorem.

4 Conclusion

In this paper, the discrete version of some bivariate ageing classes of life distribution have been introduced. The closure property of these bivariate ageing classes of life distributions under Poisson shock models have been studied.

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