On Heat Transfer in case of a Viscous Flow over a Plane Wall with Periodic Suction by Artificial Neural Network

U. K. Tripathy[#1], S. M. Patel[*2]
[#1] Retd. Professor & Head Dept of Mathematics
V.S.S.University, Burla-768017, Sambalpur, India.
[*2]Lecturer, Department of Mathematics
Sundargarh Engineering School, Sundargarh. 7700073, India

Abstract: *This paper shows that the problem of heat transfer in case of a viscous flow over a plane wall with periodic suction has been studied taking into account the effect of viscous dissipative terms. The solution of the equation of heat balance has been obtained approximately by perturbation method choosing ε, the amplitude of the periodic suction velocity to be perturbation parameter and the artificial neural networks method. The effect of the Prandtl number* (Pr), *Eckert number* (E) *and Reynold's number* (Re) *on the correction factor (F) to the rate of heat transfer from the wall has been compared for the both type of solutions. The effect of the parameters* E, Pr *and* Re *on* F *are exactly same as the numerical calculation of* $F(Re)$ *which is the correction to the quasi two-dimensional rate of heat transfer (Nusselt number* Nu*) from the wall. The results obtain by artificial neural network (ANN) gives a better approximation then that of the numerical technique and the most important part of this paper is ANN technique can easily be handle with a large number of data in a short time. Hence the ANN model which provides an exact, quick and reliable result than the conventional time consuming numerical method.*

Key words: *artificial neural network, back-error propagation, correction to the quasi two-dimensional rate of heat transfer.*

1. *INTRODUCTION*

The problem of a non-Newtonian fluid passing through a porous plate under the influence of a magnetic field has attached the interest of different research communities because of its various applications. In view of these applications, the problem of flow and heat transfer of a viscous incompressible liquid along a plan wall with periodic suction has been solved by [1]. A similar type of problem is solved by [2] taking into account the viscous dissipative terms. The problem described here is solved by following [3]. The boundary layer flow of heat and mass transfer over a stretching surface under different physical situations has been done by [4], [5], [6], [7]. Likewise, many others as [8], [9], [10], [11] have examined the MHD flow of non-Newtonian viscoelastic flow, heat and mass transfer under different physical situations. The numerical solution for the MHD flow problems of power law fluids has been widely reported in the literature [12], [13], [14]. These types of problems are intrinsically non linear like the most fluid mechanics problems and do not have an exact solution. Again related to this article variable viscosity and thermal conductivity on steady free convection heat transfer of micro-polar fluid flow over a porous hot vertical plate with constant heat flux by [15] and MHD and radiation effect on heat transfer in a non-Newtonian Maxwell fluid over an unsteady stretching sheet with heat source/sink by [16]. The solved paper Estimation of MHD boundary layer slip flow over a permeable stretching cylinder in the presence of chemical reaction through numerical and ANN modeling by [17]. The solved paper Estimation of the flow and heat transfer in MHD flow of a power law fluid over a porous plate ANNs by [18]. As described by [18] we have solved the present problem.

This paper has been arranged as follows: Section 2 the heat transfer of a viscous flow over a plane wall with periodic suction are presented. In section 3 formulations of the equations and solutions are presented. Section 4 contains the importance and the discussion of ANN method and the Back Propagation algorithm. Section 5, concluded the achievement of this study.

2. *THE HEAT TRANSFER OF A VISCOUS FLOW OVER A PLANE WALL WITH PERIODIC SUCTION*

The asymptotic solution for the flow of an incompressible viscous liquid past an infinite porous flat plate has been discussed by [19]. Here the heat transfer along a plane wall with periodic suction of a viscous flow is discussed whose solution is available in [2]. The same problem is solving by backpropagation neural networks and its result is compared with that of the solution avaible in the above reference.

Problems of this type are quite important in the theory of laminar flow control (LFC) system. Assuming the wall to be the $\bar{x}\bar{z}-plane$ and

$\bar{y}-axis$ to be normal to it, the suction velocity is assumed to be of the form

$$\bar{v}_w(z) = \bar{v}_0\left(1+\varepsilon\cos\pi\frac{\bar{z}}{l}\right) \qquad (2.1)$$

where $\bar{v}_0 \langle 0$, l is the wave length of the periodic suction velocity distribution and ε is the amplitude of suction velocity variation.

In the paper cited above the authors have solved the energy equation neglecting the viscous dissipation. Our aim in this note is to solve the energy equation including the viscous dissipative term and compare its results with that of the artificial neural networks solution following [17] and [18]. The ANN method has not been used or tested for heat transfer analysis of fluid flow with vicious dissipative term. Therefore this study primarily focuses on the applicability of ANN of the above said heat transfer analysis. In the present study, the effects of different parameters of the heat flow analysis are compared with the solution of the ANN method. Appropriate ANN is applied to the problem and training procedure for the ANN presented here. A back error propagation training algorithms is employed to train the network and find the best weight and the performance is examined.

3. *FORMULATION OF THE EUQATIONS AND SOLUTIONS*

Denoting $\bar{w},\bar{v},\bar{w}$ to be the components of the fluid velocity and any point $\bar{x},\bar{y},\bar{z}$, Gersten and Gross[1] have found out the expressions for these as follows:

$$u = (1-e^{-Rey}) + \varepsilon\frac{Re}{\pi-\lambda}\left[\left(\frac{\pi}{2\lambda}-\frac{\lambda}{\pi}\right)e^{-\lambda y}\right.$$
$$\left.-\frac{\pi}{2\lambda}e^{-(Re+\lambda)y}+\frac{\lambda}{\pi}e^{-(Re+\pi)y}\right]\cos\pi z \qquad (3.1)$$

$$v = 1+\varepsilon\frac{1}{\pi-\lambda}(\pi e^{-\lambda y}$$
$$-\lambda e^{-\pi y})\cos\pi z \qquad (3.2)$$

and

$$w = \frac{\lambda}{\pi-\lambda}(e^{-\lambda y}-e^{-\pi y})\sin\pi z \qquad (3.3)$$

In the above expression $y=\frac{\bar{y}}{l}$ and $z=\frac{\bar{z}}{l}$ are the normalized space variables, $u=\frac{\bar{u}}{U_\infty}, v=\frac{\bar{v}}{\bar{v}}$ and $w=\frac{\bar{w}}{\bar{v}_o}$ are the dimensionless velocity components, U_∞ is the free stream velocity in the x-direction, $\mathrm{Re}=\frac{-\bar{v}_0 l}{v}$ is the, Reynold's number and $\lambda=\frac{\mathrm{Re}}{2}+\left\{\left(\frac{\mathrm{Re}}{2}\right)^2+\pi^2\right\}^{\frac{1}{2}}$, where v is the co-efficient of kinematic viscosity.

The equation of heat balance in the dimensionless from including viscous dissipative terms given by

$$v\frac{\partial\theta}{\partial y}+w\frac{\partial\theta}{\partial z}=-\frac{1}{RePr}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)$$
$$-\frac{E}{Re}\left[2L\left(\frac{\partial v}{\partial y}\right)^2+2L\left(\frac{\partial w}{\partial y}\right)^2+L\left(\frac{\partial v}{\partial z}\right)^2\right.$$
$$+L\left(\frac{\partial w}{\partial y}\right)^2+2\frac{\partial v}{\partial z}.\frac{\partial w}{\partial y}+\left(\frac{\partial u}{\partial y}\right)^2$$
$$\left.+\left(\frac{\partial u}{\partial z}\right)^2\right] \qquad (3.4)$$

where

$\mathrm{Pr}=\frac{C\mu}{\lambda^1}$, $E=\frac{U_\infty{}^2}{C(T_\infty-T_w)}$ are the Prandtl number and Eckert number respectively. $L=\frac{\bar{v}_0}{U\infty}$ is a dimensionless quantity, $\theta=\frac{T-T_w}{T_\infty-T_w}$ is the dimensionless temperature function. The quantities c and λ^1 are the specific heat and the thermal conductivity respectively.

The boundary conditions to which equation (3.4) is simplified as

$$\begin{aligned} y=0 &: \theta=0 \\ y\to\infty &: \theta=1 \end{aligned} \qquad (3.5)$$

When $\varepsilon=0$ (the case of strong suction) equation (3.4) is simplified as

$$\frac{\partial\theta}{\partial y}=-\frac{1}{RePr}\frac{\partial^2\theta}{\partial y^2}-ERe\,e^{-2Rey} \qquad (3.6)$$

The solution of this equation when $\mathrm{Pr}\neq 2$ is

$$\theta = 1 + \frac{1}{2(2-\mathrm{Pr})}[\{\mathrm{Pr}\,(E+2)-4\}e^{-RePr\,y} - EPre^{-2Rey}] \qquad (3.7)$$

Since $\varepsilon \langle\langle 1$, we assume the solution for θ when $\mathrm{Pr} \neq 2$, in general as

$$\theta = 1 + \frac{1}{2(2-\mathrm{Pr})}[\{\mathrm{Pr}\,(E+2)-4\}e^{-RePr\,y} - EPre^{-2Rey}] + \varepsilon\theta_1 + \varepsilon^2\theta_2 + .. \qquad (3.8)$$

We further assume that

$$\theta_1 = \theta_{11}(y)\cos\pi z \qquad (3.9)$$

First of all we substitute equations (3.1), (3.2), (3.3) and (3.8) into equation (3.4) and collect terms of the same order and ε and then in the resulting equation for θ_1 we substitute the expression for θ_1 as suggested in equation (3.9) and obtain

$$\frac{d^2\theta_{11}}{dy^2} + RePr\frac{d\theta_{11}}{dy} - \pi^2\theta_{11} = \frac{Re^2Pr}{\pi-\lambda}\Big[\left(\pi e^{-\lambda y} - \lambda e^{-\pi y}\right)\Big\{2 - \frac{EPre^{-2Rey}}{2-Pr} + (4 - PrE - 2\mathrm{Pr})Pre^{-2RePr\,y}\Big\} - 2E\Big\{\frac{\lambda+Re}{2}e^{-(Re+\lambda)\,y}\left(\frac{\lambda}{\pi} + \frac{\pi}{\lambda}e^{-Rey}\right) - \frac{\lambda(Re+\pi)}{\pi}e^{-(2Re+\lambda)\,y}\Big\}\Big] \qquad (3.10)$$

From equations (3.5), (3.8) and (3.9) the derived boundary conditions for equation (3.10) are

$$\begin{aligned} y = 0 \quad &: \theta_{11} = 0 \\ y \to \infty \quad &: \theta_{11} = 0 \end{aligned} \qquad (3.11)$$

The solution of equation (3.10) when $\mathrm{Pr} \neq 2$ under the boundary conditions (3.11) is

$$\theta_{11} = A_1 e^{-(Re+\lambda)\,y} + e^{-2Re\,y}\left(A_2e^{-\lambda y} + A_3e^{-\pi y}\right) + e^{-2RePr\,y}\left(A_4e^{-\lambda y} + A_5e^{-\pi y}\right) + A_6e^{-\bar{\lambda}y} \qquad (3.12)$$

where,

$$\bar{\lambda} = \frac{RePr}{2} + \left\{\left(\frac{RePr}{2}\right)^2 \pi^2\right\}^{\frac{1}{2}}$$

$$A_1 = -\frac{RePrE\lambda(Re+\lambda)}{\pi(\pi-\lambda)(Re+3\lambda-RePr-Pr\lambda)}$$

$$A_2 = \frac{-RePrE\pi}{(\pi-\lambda)(4Re+5\lambda-2RePr-Pr\lambda)}\left(\frac{(Re+\lambda)}{\lambda} + \frac{Pr}{2-Pr}\right)$$

$$A_3 = \frac{RePrE\lambda}{(\pi-\lambda)(4Re+4\pi-2RePr-Pr\pi)}\left(\frac{2(Re+\pi)}{\pi} + \frac{Pr}{2-Pr}\right)$$

$$A_4 = -\frac{RePr^2\pi(4-PrE-2\mathrm{Pr})}{2\lambda(\pi-\lambda)(2-Pr)(\mathrm{Pr}+1)}$$

$$A_5 = \frac{RePr\lambda(4-PrE-2\mathrm{Pr})}{2\pi(\pi-\lambda)(2-Pr)}, \quad A_6 = -\sum_{i=1}^{5} A_i$$

When Pr = 2 proceeding in the same manner as above we can find out the solution for θ_{11} as

$$\theta_{11} = B_1e^{-(Re+\lambda)y} + e^{-2Re\,y}\left(B_2e^{-\lambda y} + B_3e^{-\pi y}\right) + ye^{-2Re\,y}\left(B_4e^{-\lambda y} + B_5e^{-\pi y}\right) + B_6e^{-\bar{\lambda}_1 y} \qquad (3.13)$$

where

$$\bar{\lambda}_1 = 2\left(Re + \sqrt{Re^2} + \pi^2\right), \quad B_1 = -\frac{\lambda ERe(Re+\lambda)}{\pi(\pi-\lambda)(\lambda-Re)}$$

$$B_2 = \frac{Re\pi\{E(Re+\lambda) - 3\lambda(4+E)\}}{9\lambda^2(\pi-\lambda)}, \quad B_3 = \frac{Re\lambda\pi(4+E)}{2\pi^2(\pi-\lambda)}$$

$$B_4 = \frac{2Re^2 E\pi}{3\lambda(\pi - \lambda)}, \quad B_5 = -\frac{Re^2 E\lambda}{\pi(\pi-\lambda)}, \quad B_6 = -\sum_{i=1}^{3} B_i$$

The expressions for the Nusselt number, Nu which is $\frac{d\theta}{dy}$ at $y = 0$ for the case $\Pr \neq 2$ and Pr=2 with the help of equations (3.8), (3.9), (3.12) and (3.13) are respectively

$$Nu = \frac{RePr}{2(2-\Pr)}[2E + 4 - \Pr(E+2)] + \varepsilon F(Re)\cos\pi z \qquad (3.14)$$

and

$$Nu = 2Re + \frac{ERe}{2} + \varepsilon F(Re)\cos\pi z \qquad (3.15)$$

The function F(Re) is given by

$$F(Re) = \begin{cases} -[A_1(Re+\lambda) + A_2(Re+\lambda) + A_3(2Re+\pi) + A_4(\lambda + RePr) + A_5(\pi + RePr) + A_6\bar{\lambda}]; & Pr \neq 2 \\ -[B_1(Re+\lambda) + B_2(Re+\lambda) + B_3(2Re+\pi) + B_4 + B_5 + B_6\bar{\lambda}_1]; & Pr = 2 \end{cases}$$

The function F(Re), signifies the correction to the quasi two-dimensional rate of heat transfer Nusselt number (Nu) from the wall.

4. *ANN METHOD*

The training of the neural network is accomplished by adjusting the weights and is carried out through a large number of training sets and training cycles (epochs). The purpose of the learning procedure is to find the optimal set of weights, which is an ideal case would produce the correct output for any relative input. The output of the network is compared with a desired response to determine an error. The performance of the multilayer perceptron (MLP) is measured in terms of the desired signal and the criterion for convergence. For training, validation, test and all samples, the root mean square error (RMSE) and the absolute fraction of variance (R^2) are determined from.

$$RSME = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(T_i - out_i)^2} \qquad (4.1)$$

and

$$R^2 = 1 - \frac{\sum_{i=1}^{M}(T_i - out_i)^2}{\sum_{i=1}^{M}(T_i)^2} \qquad (4.2)$$

where T_i and out_i are the desired (target) output and output of neural network values respectively for the ith output neuron and M is total number of data sets. The absolute fraction of variance ranges between 0 and 1. The values closer to 1 indicate a very good fit, while the values closer to 0 indicate a poor fit.

In this study, ANN structure has been designed and trained using the MATLAB Neural Network Toolbox. The Back-Error Propagation (BEP) training algorithm has been used in feed-forward with one hidden layer. In the structural network (shown in figure 1.), the inputs are E, Re, Pr and the output is $F(Re)$.

A sigmoid function has been used as the activation function of artificial neurons and training has been done using a large number of epochs. The total 30 numerical results were used to train, validated and test the artificial neural network (ANN) model. The 20 data set were used for the training set, 5 data set were used for validate and rest of the data were used for testing of the model. The performance of $F(Re)$ of the proposed ANN model is shown in figure 2. (a), (b), (c) and (d) respectively. The value of F_{Num} are considered as the x-axis and that of F_{ANN} on y-axis.

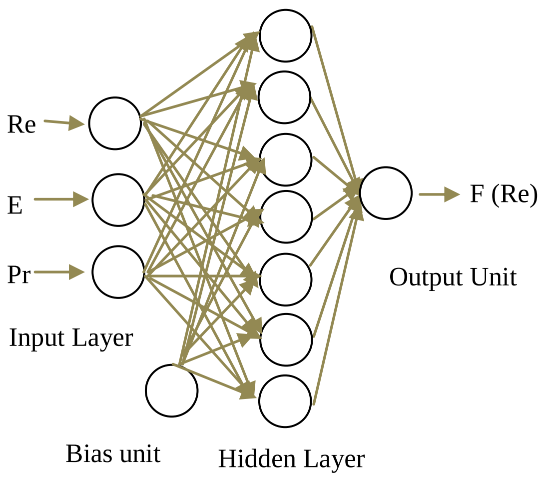
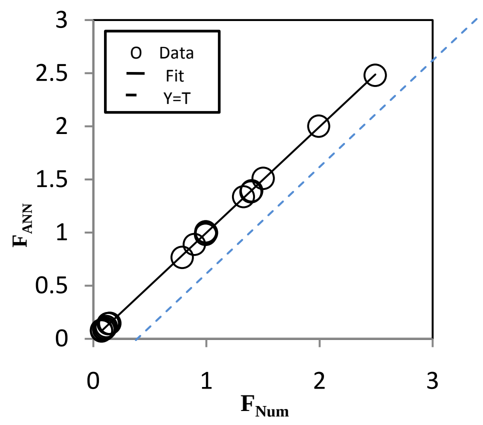


Figure 1. Schematic diagram of a multi layer ANN

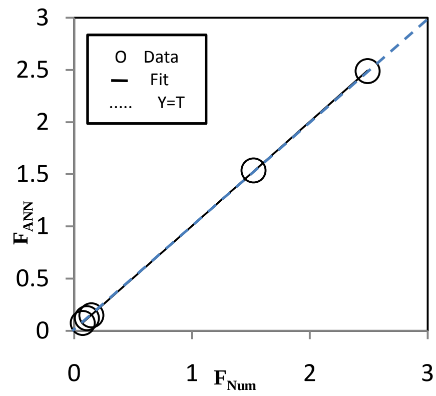
Table 1. ***Skin friction data of F(Re) for F_{Num} and F_{ANN} with error.***

Sl. No	Re	Pr	E	F_{Num}	F_{ANN}	% Error
1	20	0.7	0.001	0.07392	0.07318	1.001082251
2	20	1.0	0.001	0.11629	0.11648	0.163384642
3	20	2.0	0.001	0.13013	0.13022	0.069161608
4	20	3.0	0.001	0.89321	0.88895	0.476931517
5	30	0.7	0.001	0.99382	1.00792	1.418767986
6	30	0.7	0.300	1.29687	1.31721	1.568391589
7	30	0.7	0.000	1.32792	1.33816	0.771130791
8	30	0.7	1.500	1.40071	1.39287	0.559716144
9	40	1.0	0.001	0.14692	0.14686	0.040838552
10	40	1.0	0.010	0.04698	0.14700	212.8991060
11	40	1.0	0.300	0.14712	0.14812	0.679717238
12	40	2.0	1.000	0.78621	0.76583	2.592182750
13	50	2.0	1.000	0.99321	0.98356	0.971597145
14	60	2.0	1.000	1.00061	0.99983	0.077952449
15	70	2.0	0.001	1.39286	1.38265	0.733024137
16	70	2.0	0.300	1.50125	1.51186	0.706744380
17	70	2.0	1.000	1.52392	1.53481	0.714604441
18	40	0.7	0.001	1.99284	2.00137	0.428032356
19	50	0.7	0.300	2.49108	2.486921	0.166955698
20	60	0.7	1.000	2.49232	2.48198	0.414874494
21	5.0	0.7	0.000	0.07296	0.08329	14.15844298
22	5.0	0.7	0.010	0.07301	0.07386	1.164224079
23	7.5	0.7	0.000	0.09387	0.08875	5.454351763
24	7.5	0.7	0.010	0.09561	0.08823	7.718857860
25	10	0.7	0.000	0.09572	0.08881	7.218972002
26	10	0.7	0.010	0.09712	0.08761	9.792009885
27	5.0	3.0	0.000	0.10697	0.099872	6.635505282
28	5.0	3.0	0.010	0.10816	0.120312	11.23520710
29	10	3.0	0.000	0.13286	0.14492	9.077224146
30	10	3.0	0.010	0.13375	0.14360	7.364485981



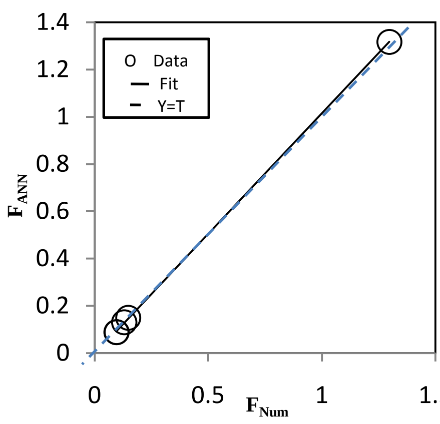
$y = 0.998x + 0.000$, $R^2 = 0.999$, $RSME = 0.00938$

(a). **Training**



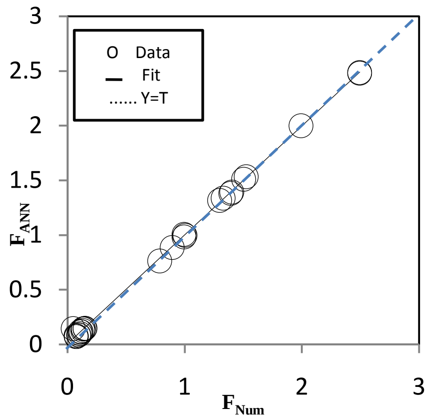
$y = 0.998x + 0.005$, $R^2 = 1$, $RSME = 0.00753$

(b). **Validation**



$y = 1.020x + 0.006$, $R^2 = 0.999$, $RSME = 0.01058$

(c). **Test**



$y = 0.996x + 0.006\,, R^2 = 0.999, RSME = 0.0205$

(d). **All**

Figure 2.(a),(b),(c),(d) Graphical representation of F(Re)

5. *CONCLUSION*

The function $F(Re)$, signifies the correction to the quasi two-dimensional rate of heat transfer (Nusselt number Nu) from the wall. The effect of E (the Eckert number), Pr (the Prandtl number) and Re (the Reynolds number) on F is discussed. It is seen that when Re increases for any value of E and Pr, F increases in its value. As E and Pr increases the quasi two-dimensional correction F increases. For large Reynolds number Re, as is seen from the table 1., the effect of E and Pr on F is appreciably significant.

In the present work the ANN approach is developed successfully to simulate the quasi two-dimensional error in a problem of heat transfer along a plane wall with periodic suction. This ANN structure has been trained, validated and tested using the MATLAB environment. Table 1.shows effect of the parameters E, Pr and Re on F are exactly same as the numerical calculation of $F(Re)$ which is the correction to the quasi two-dimensional rate of heat transfer (Nusselt number Nu) from the wall. The bias function $\theta_j = 0.56$, and the number of epochs (1296) have been used in this ANN training model. The prediction of $F(Re)$ through ANN model is in good agreement with a numerical data calculated in this chapter. The percentage of error between F_{Num} and F_{ANN} is varying within 7% barring few data. Hence the ANN model which provides an exact, quick and reliable result than the conventional time consuming numerical method.

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