Regarding Edge Domination in Hypergraph

D. K. Thakkar[1] and V. R. Dave[2]

[1]Department of Mathematics, Saurashtra University, Rajkot – 360 005, India
[2]Shree M. & N. Virani Science College, Kalavad Road, Rajkot – 360 005, India

Abstract: *In this paper we further prove more results about edge domination in hypergraphs. In particular we prove necessary & sufficient conditions under which the edge domination number of a hypergraph increases or decreases when an edge is added or removed from the hypergraph. We have proved thatif $\gamma_E(G - v) > \gamma_E(G)$ & if F is a minimum edge dominating set of G then there is an edge e containing v ∋ e ∈ F &Prn[e, F] contains two distinct edges also we have proved that if $\gamma_E(G + h) < \gamma_E(G)$ then there are at least two vertices x & y in h ∋ all the edges containing x or y except h are in the complement of F. Where F is any minimum edge dominating set of G + h.*

Keywords: *Hypergraph, Dominating Set in Hypergraph,Edge Dominating Set, Edge Domination Number, Minimal Edge Dominating Set, Minimum Edge Dominating Set, Edge Degree, Edge Neighbourhood, Sub Hypergraph, Partial Sub Hypergraph, Dual Hypergraph, edge addition, edge removal.*

AMS Subject Classification (2010): 05C15, 05C69, 05C65

1. Introduction

Edge dominating set & edge domination number have been explored by several authors [5, 6]. The concept of edge domination requires the adjacency relation among the edges of a graph. The same relation is also available in hypergraphs and therefore we have considered edge domination in hypergraphs [7].

The change in the edge domination number when an edge is added to the hypergraph or an edge is removed from the hypergraph has been studied here.

We have considered the operation of vertex removal from hypergraph in [8]. Here also the edge domination number may increase, decrease or remains unchanged when a vertex is removed from the hypergraph.

2. Preliminaries

Definition 2.1 Hypergraph: [4] A *hypergraph*G is an ordered pair $(V(G), E(G))$ where $V(G)$ is a non-empty finite set &$E(G)$ is a family of non-empty subsets of $V(G)$∋ their union $= V(G)$.The elements of $V(G)$ are called *vertices*& the members of $E(G)$ are called *edges of the hypergraph* G.

We make the following assumption about the hypergraph.

(1) Any two distinct edges intersect in at most one vertex.

(2) If e_1 and e_2 are distinct edges with $|e_1|, |e_2| > 1$ then $e_1 \not\subseteq e_2$ & $e_2 \not\subseteq e_1$

Definition 2.2 Edge Degree: [4] Let G be a hypergraph & v ∈V(G) then the *edge degree* of v = $d_E(v)$ = the number of edges containing the vertex v.The minimum edge degree among all the vertices of G is denoted as $\delta_E(G)$ and the maximum edge degree is denoted as $\Delta_E(G)$.

Definition 2.3 Dual Hypergraph: [4] Let G be a hypergraph. For every v ∈V(G) define $\bar{v}$ as follows.

$\bar{v} = \{e \in E(G) \,/\, v \in e\}$. Let E(G*) = $\{\bar{v}$ / v ∈ V(G)} and let V(G*) = E(G). Then the *dual hypergraph* of the given hypergraph G is the hypergraph G* whose vertex set is V(G*) & the edge set is E(G*). We will write G* = (V(G*), E(G*)).

Definition 2.4 Dominating Set in Hypergraph: [1] Let G be a hypergraph &$S \subseteq V(G)$ then S is said to be a *dominating set* of G if for every $v \in V(G) - S$ there is $u \in S$ ∋u and v are adjacent vertices.

A dominating set with minimum cardinality is called *minimum dominating set* and cardinality of such a set is called *domination number* of G and it is denoted as $\gamma(G)$.

Definition 2.5 Edge Dominating Set: [7] Let G be a hypergraph &$S \subseteq E(G)$ then S is said to be an *edge dominating set* of G if for every $e \in E(G) - S$ there is some f in S ∋ e and f are adjacent edges.

An edge dominating set with minimum cardinality is called a *minimum edge dominating set* and cardinality of such a set is called *edge domination number* of G and it is denoted as $\gamma_E(G)$.

Definition 2.6 Minimal Edge Dominating Set: [7] Let G be a hypergraph &$F \subseteq E(G)$ then F is said to be a *minimal edge dominating set* if

(1) F is an edge dominating set (2) No proper subset of F is an edge dominating set of G.

Definition 2.7 Sub hypergraph and Partial sub hypergraph: [3] Let G be a hypergraph &$v \in V(G)$. Consider the subset $V(G) - \{v\}$ *of* $V(G)$.This set will induce two types of hypergraphs from G.

(1)First type of hypergraph: Here the vertex set $= V(G) - \{v\}$and the edge set $= \{ e' / e' = e - \{v\}\ for\ some\ e \in E(G) \}$. This hypergraph is called the *sub hypergraph* of G & it is denoted as $G - \{v\}$.

(2) Second type of hypergraph: Here also the vertex set = $V(G) - \{v\}$ and edges in this hypergraph are those edges of G which do not contain the vertex v.This hypergraph is called the *partial sub hypergraph* of G.

Definition 2.8 Edge Neighbourhood: [3] Let G be a hypergraph & e be any edge of G then

Open edge neighbourhood of $e = N(e) = \{ f \in E(G) / f\ is\ adjacent\ to\ e \}$.

Close edge neighbourhood of $e = N[e] = N(e) \cup \{e\}$.

Definition 2.9 Private Neighbourhood of an edge: [3] Let G be a hypergraph. F be a set of edges & e ∈ F, then the *private neighbourhood of e with respect to set F* = Prn[e, F] = { f ∈ E(G) / N[f] ∩ F = {e}}

3. Edge Removal from the Hypergraph:

Definition 3.1Edge Removal in Hypergraph: Let G be a hypergraph & e be an edge of G then G – e will denote the partial hypergraph whose vertex set is V(G) & edge set is E(G) – {e}. (Now, we assume that in G – {e} there are no isolated vertices, which is the requirement of any hypergraph)

Now, we consider the effect of removing an edge from a hypergraph G. The following examples show that the edge domination number may increase, decrease or remain unchanged.

Example 3.2:

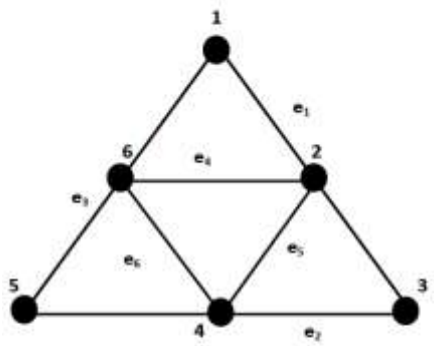

Fig.1

Here, $\gamma_E(G) = 2$, When any edge is removed from this graph $\gamma_E(G) = 1$ in resultant graph.

Thus $\gamma_E(G - e) < \gamma_E(G)$.

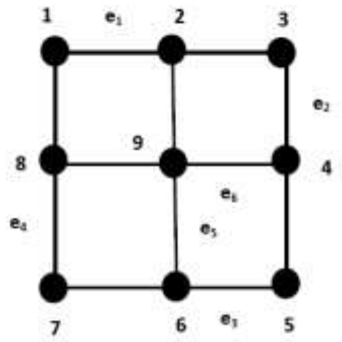

Fig.2

Here, $\gamma_E(G) = 2$, When any edge is removed from this graph $\gamma_E(G)$ will remain same in resultant graph.

Thus $\gamma_E(G - e) = \gamma_E(G)$.

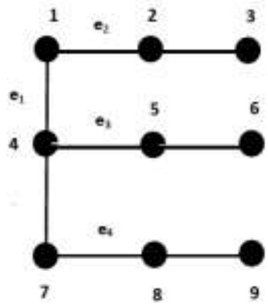

Fig.3

Here, $\gamma_E(G) = 1$, When e_1 is removed from this graph $\gamma_E(G - e_1) = 3$ in resultant graph.

Thus $\gamma_E(G - e) > \gamma_E(G)$.

Theorem 3.3: Let G be a hypergraph & e ∈ E(G) then $\gamma_E(G - e) < \gamma_E(G)$ iff there is a minimum edge dominating set F of G containing e ∋Prn[e, F] = {e}

Proof: Suppose $\gamma_E(G - e) < \gamma_E(G)$

Let F_1 be a minimum edge dominating set of G – e. Then F_1 cannot be an edge dominating set of G. Therefore, e cannot intersect any member of F_1. Let $F = F_1 \cup \{e\}$. It is obvious that F is a minimum edge dominating set of G & e ∈ F. Since, e is not adjacent with any other member of F, e ∈Prn[e, F]. Let f be any edge of G ∋ f ≠ e then f is an edge of G – e. Since F_1 is an edge dominating set of G – e, F intersect some member of F_1. Therefore, f ∉Prn[e, F]. Therefore, Prn[e, F] = {e}.

Conversely suppose there is a minimum edge dominating set F of G ∋ e ∈ F &Prn[e, F] = {e}. Let F_1 = F - {e} then F_1 is a set of edges of G – e. Let f be any edge of G – e ∋ f ∉ F_1 then f ∉F. Now, f is adjacent to some member of F. Suppose f is adjacent to e. Now by assumption $f \notin Prn[e, F]$. Therefore, f must be adjacent to some other member h of F, then $h \in F_1$. Thus f is adjacent to some member of F_1. Thus, F_1 is an edge dominating set of G – e.

$\therefore \gamma_E(G - e) \leq |F_1| < |F| = \gamma_E(G)$

$\therefore \gamma_E(G - v) < \gamma_E(G)$ ■

Corollary 3.4: Let G be a hypergraph & $e \in E(G)$ if $\gamma_E(G - e) < \gamma_E(G)$ then $\gamma_E(G - e) = \gamma_E(G) - 1$

Proof: Obvious

Remark 3.5: Let G be a hypergraph & e be an edge of G ∋ edge degree of $x \geq 2$ for every x in e. Now, consider hypergraph $(G - e)^*$ & G^* - e.

The vertices of $(G - e)^*$ are those edges of G which are different from e. The vertices of G^* - e are those edges of G which are different from e. Therefore, $V(G^* - e) = V((G - e)^*)$. Let x be a vertex of G ∋ $x \in e$. Let $\bar{x}_0 = \{f \in E(G) / x \in f \ \& \ f \neq e\}$ then it is obvious that $\bar{x}_0$ is an edge of $(G - e)^*$. Suppose $x \notin e$ & let $\bar{x}_0 = \{f \in E(G) / x \in f \ \& \ f \neq e\}$ then it is obvious that $\bar{x}_0 = \bar{x}$. Thus, $\bar{x}_0 = \bar{x} - \{e\}$ if $x \in e$ & $\bar{x}_0 = \bar{x}$ if $x \notin e$.

Thus, we have $\{\bar{x}_0 / x \in V(G)\}$ which is the edge set of $(G - e)^*$. On the other hand if we consider the sub hypergraph G^* - e then $\bar{x}' = \bar{x}$ if $e \notin \bar{x}$ (i.e. $x \notin e$) & $\bar{x}' = \bar{x} - \{e\}$ if $e \in \bar{x}$ (i.e. $x \in e$). Thus, we observed that $\bar{x}' = \bar{x}_0$ for every $x \in V(G)$.

∴The edge set of $(G - e)^*$ = The edge set of sub hypergraph G^* - e.

∴ The dual hypergraph $(G - e)^*$ = The sub hypergraph G^* - e of G^*.

Theorem 3.6: [8] Let G be a hypergraph & $v \in V(G)$ such that {v} is not an edge of G then $\gamma(G - v) > \gamma(G)$ iff the following two conditions are satisfied.

(1) $v \in S$ for every minimum dominating set S of G.

(2) There is no subset S of G – v ∋ $S \cap N[v] = \phi$, $|S| \leq \gamma(G)$ & S is a dominating set of G – v. (Here G – v is the sub hypergraph of G.)

Theorem 3.7: Let G be a hypergraph & $e \in E(G)$ ∋ edge degree of $x \geq 2$ for every x in e. Then $\gamma_E(G - e) > \gamma_E(G)$ iff the following conditions are satisfied

(1) $e \in F$ for every minimum edge dominating set F of G.

(2) There is no subset F of E(G) – {e} ∋ $|F| \leq \gamma_E(G)$, $F \cap N[e] = \phi$ & F is an edge dominating set of G – e.

Proof: Suppose $\gamma_E(G - e) > \gamma_E(G)$

(1) Suppose F be a minimum edge dominating set of G then F is a minimum dominating set of G^*. Since $\gamma_E(G - e) > \gamma_E(G)$ it follows that $(\gamma_E(G - e))^* > \gamma(G^*)$ but $(G - e)^* = G^*$ - e.

$\therefore \gamma_E(G^* - e) > \gamma(G^*)$. By above theorem $e \in F$. Thus, condition (1) is satisfied.

(2) Suppose there is an edge dominating set F of G – e ∋ $|F| \leq \gamma_E(G)$, $F \cap N[e] = \phi$. Then F is a dominating set of $(G - e)^* = G^*$ - e, $|F| \leq \gamma(G^*)$ (because $\gamma(G^*) = \gamma_E(G)$) & $F \cap N[e] = \phi$.

This implies that $\gamma_E(G^* - e) \leq \gamma(G^*)$. This is a contradiction. Thus, no such F exists.

Conversely suppose (1) & (2) hold

First suppose that $\gamma_E(G - e) = \gamma_E(G)$.

Let e is adjacent to some edge in F. Then F is an edge dominating set of G. Since $\gamma_E(G - e) = \gamma_E(G)$, F is a minimum edge dominating set of G not containing e. This violets condition (1).

Suppose that e is not adjacent to any edge in F then $|F| \leq \gamma_E(G)$, $F \cap N[e] = \phi$ & F is an edge dominating set of G – e. This contradicts (2).

$\therefore \gamma_E(G - e) = \gamma_E(G)$ is not possible.

Suppose that $\gamma_E(G - e) < \gamma_E(G)$.

Let F be a minimum edge dominating set of G – e. Since $|F| < \gamma_E(G)$, F cannot be an edge dominating set of G. Therefore, e is not adjacent to any member of F. Thus, $|F| \leq \gamma_E(G)$, $F \cap N[e] = \phi$ & F is an edge dominating set of G – e. This contradicts (2).

$\therefore \gamma_E(G - e) < \gamma_E(G)$ is also not possible.

Hence, $\gamma_E(G - e) > \gamma_E(G)$. ■

In the following theorem we consider the partial sub hypergraph.

Theorem 3.8: [8] Let G be a hypergraph & $v \in V(G)$ such that {v} is not an edge of G then $\gamma_E(G - v) < \gamma_E(G)$ iff there is a minimum edge dominating set F & edge e containing v of G ∋ $e \in F$ & the following two conditions are satisfied.

(1) $e \in Prn[e, F]$

(2) Prn[e, F] is a subset of $N_E(v)$

Corollary 3.9: Let G be a hypergraph & e be an edge of G. If $\gamma_E(G - e) < \gamma_E(G)$ then $\gamma_E(G - x) < \gamma_E(G)$ for all x in e. (We consider here partial sub hypergraph G – x).

Proof: Since $\gamma_E(G - e) < \gamma_E(G)$, there is a minimum edge dominating set F of G ∋ $e \in F$ & $Prn[e, F] = \{e\}$. Let $x \in e$. Then F is a minimum edge dominating set containing some edge (namely e) containing x ∋ Prn[e, F] contains e & Prn[e, F] contains only those edges which contain the vertex x (Which is empty set in this case).

Thus by above theorem $\gamma_E(G - x) < \gamma_E(G)$. ■

Theorem 3.10:[8] Let G be a hypergraph & $v \in V(G)$ such that $\{v\}$ is not an edge of G if $\gamma_E(G - v) > \gamma_E(G)$ & if F is a minimum edge dominating set of G then there is an edge e containing v $\ni e \in F$ &Prn[e, F] contains two distinct edges.

Corollary 3.11: Let G be a hypergraph & $v \in V(G)$ such that $\{v\}$ is not an edge of G. If $\gamma_E(G - v) > \gamma_E(G)$ then there is no edge e containing v $\ni \gamma_E(G - e) < \gamma_E(G)$.

Proof: Suppose there is an edge e containing v $\ni \gamma_E(G - e) < \gamma_E(G)$. Let F_1 be a minimum edge dominating set of G then by the above theorem there is an edge f containing v $\ni f \in F_1$&Prn[f, F_1] contains two distinct edges. Since $\gamma_E(G - e) < \gamma_E(G)$ there is a minimum edge dominating set $F_1 \ni e \in F_1$ &Prn[e, F_1] = {e}. Now, we may assume that $f \in F_1$. We may note that $f \neq e$ because Prn[e, F_1] = {e} &Prn[f, F_1] contains two distinct edges. Since e & f both contain v, e & f are adjacent edges. This contradicts the fact that Prn[e, F_1] = {e}.

$\therefore$ There is no edge e containing v $\ni \gamma_E(G - e) < \gamma_E(G)$. ■

4. Edge Addition in Hypergraph

Definition 4.1: Let G be a hypergraph & h be a non empty set of vertices such that $h \cap e$ is at most one vertex for every edge e of the hypergraph G & $|h| \geq 2$.

We define a new hypergraph G + h as follows.

(1) The vertex set of G + h is V(G).

(2) The edge set of G + h = $E(G) \cup \{h\}$

Example 4.2: Consider the hypergraph G mentioned below.

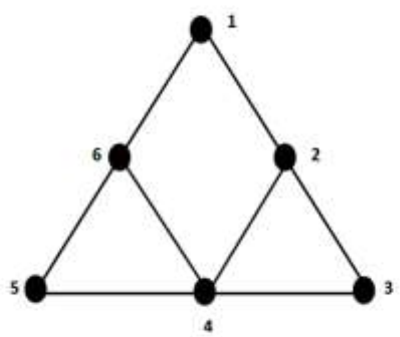

Fig. 4

Let h = {2, 6} then the hypergraph G + h is shown below.

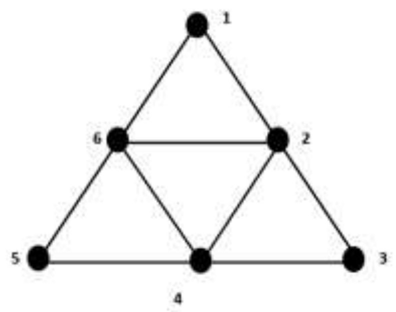

Fig. 5

Now, we stat & prove a necessary & sufficient condition under which the edge domination number of a hypergraph increases when an edge h is added to the hypergraph G.

Theorem 4.3: Let G be a hypergraph & h be a subset of V(G) $\ni h \cap e$ is at most one vertex for every edge e of G then the following statements are equivalent.

(1) $\gamma_E(G + h) > \gamma_E(G)$

(2) There is a minimum edge dominating set F_1 of G + h $\ni h \in F_1$&Prn[h, F_1] = {h}

(3) For every minimum edge dominating set F of G, $h \cap e = \phi$ for every $e \in F$.

(4) There is a minimum edge dominating set F_1 of G + h $\ni F_1 = F \cup \{h\}$ for some minimum edge dominating set F of G.

Proof: (1) $\Rightarrow$ (3)

Let F be a minimum edge dominating set of G. Since $|F| = \gamma_E(G) < \gamma_E(G + h)$, F cannot be an edge dominating set of G + h. Since every edge of G intersect some member of F, h does not intersect any member of F.

Thus (1) $\Rightarrow$ (3) is proved.

(3) $\Rightarrow$ (2)

Let F be a minimum edge dominating set of G $\ni$ h does not intersect any member of F.

Let $F_1 = F \cup \{h\}$. Obviously, F_1 is an edge dominating set of G + h & $h \in F_1$. Since h does not intersect any other member of F_1, Prn[h, F_1] = {h}.

Thus (3) $\Rightarrow$ (2) is proved.

(2) $\Rightarrow$ (1)

Let $F = F_1 - \{h\}$ then F is a set of edges of G & $|F| < |F_1|$. Let f be any edge of G $\ni f \notin F$ then $f \notin F_1$. Suppose $f \cap h \neq \phi$.

Now, $f \notin$ Prn[h, F_1], $f \cap g \neq \phi$ for some g in F_1 with $g \neq h$. Then $g \in F$ & $f \cap g \neq \phi$.

Suppose $f \cap h = \phi$.

■

Since F_1 is an edge dominating set of G + h, f must be adjacent with some other edge h′ of F_1. Thus, $h' \in F$ & $f \cap h' \neq \phi$.

Thus in either case f is adjacent to some member of F & therefore F is an edge dominating set of G.

$\therefore \gamma_E(G) \leq |F| < |F_1| = \gamma_E(G + h)$.

Thus, (2) $\Rightarrow$ (1) is proved.

(1) $\Rightarrow$ (4)

Let F be a minimum edge dominating set of G. Since $\gamma_E(G) < \gamma_E(G + h)$, F cannot be an edge dominating set of G + h. Therefore, h does not intersect any member of F.

Let $F_1 = F \cup \{h\}$. Obviously, F_1 is a minimum edge dominating set of G + h.

Thus, (1) $\Rightarrow$ (4) is proved.

(4) $\Rightarrow$ (1)

From the statement (4) it follows that $\gamma_E(G) = |F| < |F_1| = \gamma_E(G + h)$.

Thus, (4) $\Rightarrow$ (1) is proved.

■

Corollary 4.4: Let G be a hypergraph & h be a subset of V(G) $\ni h \cap e$ is at most one vertex for every edge e of G. If $\gamma_E(G + h) > \gamma_E(G)$ then $\gamma_E(G + h) = \gamma_E(G) + 1$.

Proof: Obvious

Remark 4.5: From the above theorem it is clear that there are minimum edge dominating set of G + h which contain the edge h.

Suppose there is a vertex x in h $\ni$ edge degree of x $\geq$ 2. Let f be any edge of G $\ni x \in f$. Let F_1 be any minimum edge dominating set of G then f intersects some member of F_1.

Now, let $F = F_1 \cup \{f\}$ then F is a minimum edge dominating set of G + h if $\gamma_E(G + h) > \gamma_E(G)$.

$\therefore$ It is possible that a minimum edge dominating set of G + h does not contain the edge h.

Counting the minimum edge dominating sets of G + h

Suppose there are m minimum edge dominating sets of G. By adding the edge h to each of them we get m minimum edge dominating sets of G + h.

Suppose |h| = k.

For each x in h & for each edge h_x of G containing x we get m new minimum edge dominating sets of G + h by adding h_x to each of them.

Thus, every vertex x in h gives rise to m $\times$ edge degree of x, minimum edge dominating set of G + h.

$\therefore$ All the vertices in h gives rise to $(\sum_{x \in h} m)$ $\times$edge degree of x.

Thus, there are at least m $+(\sum_{x \in h} m)$ $\times$edge degree of x minimum edge dominating sets of G + h.

Now, we consider the possibility that the edge domination number of a hypergraph decreases when an edge h is added to the hypergraph.

Theorem 4.6: Let G be a hypergraph & h be as above. If $\gamma_E(G + h) < \gamma_E(G)$ then $h \in F$ for every minimum edge dominating set Fof G + h.

Proof: Suppose there is a minimum edge dominating set Fof G + h $\ni h \notin F$. Then F consists of edges of G. Let f be any edge of G $\ni f \notin F$. Now, f is also an edge of G + h therefore it is adjacent to some member of F. Thus, F is an edge dominating set of G.

$\therefore \gamma_E(G) \leq |F| = \gamma_E(G + h)$.

Thus, $\gamma_E(G) \leq \gamma_E(G + h)$. This is a contradiction.

Thus, $h \in F$ for every minimum edge dominating set F of G + h.

■

Proposition 4.7: Let G be a hypergraph & h be as above. Let F be any minimum edge dominating set of G + h. If $\gamma_E(G + h) < \gamma_E(G)$ then there are at least two vertices x & y in h $\ni$ all the edges containing x or y except h are in the complement of F.

Proof: From above theorem $h \in F$. Let $F_1 = F - \{h\}$ then F_1 cannot be an edge dominating set of G because $|F_1| < |F| < \gamma_E(G)$. Therefore, there is an edge f of G $\ni$ f does not intersect any member of F_1 but F is an edge dominating set of G + h & f is an edge of G + h. Therefore, f intersects some unique member of F. This member of F must be h.

Now, suppose $f \cap h = \{x\}$. Suppose there is some edge e_x containing x $\ni e_x \neq h$ & $e_x \in F$. Then it means that f intersects some member of F_1. This is a contradiction. Thus, all the edges containing x except h are in the complement of F.

Select one edge say e_x containing x with $e_x \neq h$. Let $F_2 = F_1 \cup \{e_x\}$. Now, $|F_2| = |F| < \gamma_E(G)$. Therefore F_2 cannot be an edge dominating set of G. Therefore, there is an edge g of G $\ni$g does not intersect any member of F_2. In particular $x \notin g$. Now, again F is an edge dominating set of G + h & g is an edge of G. Therefore, g must intersect h. Let $g \cap h = \{y\}$ then $y \in h$. again if there is an edge e_y containing y $\ni e_y \in F$ then $e_y \in F_2$ & $g \cap e_y \neq \phi$. This is again a contradiction. Therefore, there is no edge e_y containing y $\ni e_y \neq h$ & $e_y \in F$.

i.e. All the edges containing y except h are in the complement of F.

■

Theorem 4.8: Let G be a hypergraph & h be as above. Suppose there is a minimum edge dominating set F of G, there are two distinct vertices x & y in h

& there are two distinct edges e_x & e_y containing x & y respectively ∍ every edge adjacent to e_x or e_y is in F or is adjacent to some member of F then $\gamma_E(G + h) < \gamma_E(G)$.

Proof: Let $F_1 = (F - \{e_x, e_y\}) \cup \{h\}$ then $|F_1| < |F|$. Let f be any edge of G + h ∍ f $\notin F_1$. If f is adjacent to e_x in F (in G) then by our assumption f is adjacent to some member of F different from e_x. This means that f is adjacent to some member of F_1. Similarly, if f is adjacent to e_y then also it adjacent to some member of F_1.

If f is adjacent to some edge g of F with $g \neq e_x$, $g \neq e_y$ then $g \in F_1$ & in this case also f is adjacent to some member of F_1. Thus, F_1 is an edge dominating set of G + h.

$\therefore \gamma_E(G + h) \leq |F_1| < |F| = \gamma_E(G)$.

$\therefore \gamma_E(G + h) < \gamma_E(G)$.

■

Theorem 4.9: Let G be a hypergraph & h be as above. Suppose $\gamma_E(G + h) < \gamma_E(G)$ then $\gamma_E(G) - |h| + 1 \leq \gamma_E(G + h) < \gamma_E(G)$.

Proof: Let F_0 be a minimum edge dominating set of G + h. Let h = $\{x_1, x_2, \ldots, x_k\}$. Consider any edge e_{xi} of G containing x_i ∍ (1) $x_i \in e_{xi}$ & (2) $e_{xi} \neq h$ (i = 1, 2, ……., k)

Let $F = F_0 \cup \{e_{x1}, e_{x2}, \ldots, e_{xk}\} - \{h\}$ then F is an edge dominating set of G.

$\therefore |F| = |F_0| + |h| - 1 \geq \gamma_E(G)$.

$\therefore |F_0| \geq \gamma_E(G) - |h| + 1$

Thus, $\gamma_E(G) - |h| + 1 \leq \gamma_E(G + h) < \gamma_E(G)$.

■

Theorem 4.10: Let G be a hypergraph & h be as above then $\gamma_E(G + h) = \gamma_E(G) - |h| + 1$ iff there is a minimum edge dominating set F of G & distinct edges $e_1, e_2, \ldots, e_k$(Where k = |h|) in F ∍$e_i \cap h \neq \phi$, $\forall i$ & every edge f which is adjacent to e_i is either in F or is adjacent to some member of F.

Proof: Suppose the condition holds. Let F be a minimum edge dominating set of G ∍ the above condition is satisfied for F.

Let $F_0 = F - \{e_1, e_2, \ldots e_k\} \cup \{h\}$ then $|F_0| = \gamma_E(G) - |h| + 1$.

Now, we prove that F_0 is a minimum edge dominating set of G + h. Let g be any edge of G + h ∍g $\notin F_0$ then g $\notin$ F also.

Also g is an edge of G. Therefore, g is adjacent to some member of F. If g is adjacent to e_i for some i then by our assumption there is some edge f in F ∍ g is adjacent to f. Note that f is edge of G + h also. Therefore, g is adjacent to some member of F_0. We may note that if $g \cap e_i = h \cap e_i$ then g is adjacent to h, which is a member of F_0.

In other cases it is obvious that g is adjacent to some member of F_0.

Thus, F_0 is an edge dominating set of G + h.

By the above theorem F_0 is a minimum edge dominating set of G + h.

$\therefore |F_0| = \gamma_E(G) - |h| + 1 = \gamma_E(G + h)$

Conversely suppose $\gamma_E(G + h) = \gamma_E(G) - |h| + 1$. Let F_0 be a minimum edge dominating set of G + h then $h \in F_0$. Let h = $\{x_1, x_2, \ldots, x_k\}$ then by proposition 4.7 there are two distinct vertices say x_1 & x_2 such that all the edges incident at x_1 except h & all the edges incident at x_2 except h are in the complement of F_0.

Let e_{x1} & e_{x2} be two distinct edges containing x_1 & x_2 respectively.

Let $F_1 = F_0 \cup \{e_{x1}, e_{x2}\} - \{h\}$, suppose |h| = 2. Let f be any edge of G which is not in F_1. If f is adjacent to h at x_1 then f is adjacent to e_{x1} & similarly f is adjacent to h at x_2 then f is adjacent to e_{x2}. For other edges which are not in F_1 it can be verified that they are adjacent to some member of F_1.

Let g be any edge of G ∍g is adjacent to e_{x1} & suppose g $\notin$ F. Since $e_{x1} \notin F_0$ there is an edge incident at e_{x1} which is in F_0. Therefore, the condition is satisfied. Similarly, if g is adjacent to e_{x2} then g is adjacent to some member of F_0. Thus, the condition is satisfied.

Suppose $|h| \geq k$. Then again there are two vertices x_1 & x_2 in F_0∍ all the edges containing x_1 or x_2 are in the complement of F_0. Select two edges e_{x1} & e_{x2} containing x_1 & x_2 respectively ∍$e_{xi} \neq h$ for i = 1, 2 Let $F_1 = F_0 \cup \{x_1, x_2\} - \{h\}$ then $|F| < \gamma_E(G)$ (because $\gamma_E(G) = \gamma_E(G + h) + |h| - 1$, $|h| \geq 3$). Therefore, F_1 cannot be an edge dominating set of G. Therefore, there is an edge e′ of G ∍ e′ is not adjacent with any member of F_1 but e′ must be adjacent with some member of F_0. Therefore, e′ is adjacent with only one member of F_0 namely h. Then $e' \cap h = \{x_3\}$ (Say).

Now, all the edges containing x_3 except h are in the complement of F_1. Select an edge e_{x3} containing x_3∍ $e_{x3} \neq h$. Now, let $F_2 = F_1 \cup \{e_{x3}\}$. If |h| = 3 then $|F_2| = \gamma_E(G)$. Here also we can prove that F_2 is an edge dominating set of G & required conditions are satisfied.

In general if k > 3 then by selecting edges $e_{x1}, e_{x2}, \ldots, e_{xk}$ containing $x_1, x_2, \ldots, x_k$ respectively which are not in F_0 & by considering the set $F = F_0 \cup \{e_{x1}, e_{x2}, \ldots, e_{xk}\} - \{h\}$ we can prove that F is an edge dominating set of G also $|F| = |F_0| + |h| - 1 = \gamma_E(G + h) + |h| - 1 = \gamma_E(G)$

∴ F is a minimum edge dominating set of G. Also the conditions are satisfied by F.

■

5. Conclusions

In this paper we have established the conditions under which the edge domination number increases or decreases when an edge is removed or added to the hypergraph. Further one can consider the following problems:

(1) What is the minimum number of edges which must be removed or added to increases or decreases the edge domination number of a hypergraph.

(2) One can also study the minimal edge dominating set with maximum cardinality in hypergraph.

Acknowledgment

The authors are thankful to the referees for their valuable suggestions to improve the quality of this paper.

REFERENCES

[1] Acharya B., *Domination in Hypergraphs*, AKCE J. Graphs. Combin., 4, NO. 2(2007) 111 – 126

[2] Behr A., Camarinopoulos L., *On the domination of hypergraphs by their edges*, Discrete Mathematics,187(1998), 31 - 38

[3] Berge C.,Graphs and Hypergraphs, North-Holland, Amsterdam (1973)

[4] Berge C., *Hypergraphs*, North – Holland Mathematical Library, New York, Volume – 45 (1989)

[5] Haynes T., Hedetniemi S. and Slater P., *Domination in Graphs Advanced Topics*, Marcel Dekker, Inc., New York, (1998).

[6] Haynes T., Hedetniemi S. and Slater P., *Fundamental of Domination in Graphs*, Marcel Dekker, Inc., New York, (1998)

[7] Thakkar D.and Dave V., *Edge Domination in Hypergraph*, Communicated for publication.

[8] Thakkar D.and Dave V., *More aboutEdge Domination in Hypergraph*, Communicated for publication.

[9] Thakkar D.and Kothiya A., *Uniqueness of* χ^f *– Partition of a Graph*,IJMTT, Vol. – 33(1)(2016) 16 – 18.

[10] Thakkar D.and Badiyani S., *Edge deletion & Restrained Sets in Graphs*, IJMTT, Vol. – 37(4)(2016) 260 – 262.