

Eulerian integral associated with product of two multivariable

Aleph-functions and a class of polynomials

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ABSTRACT

The present paper is evaluated a new Eulerian integral associated with the product of two multivariable Aleph-functions , a multivariable A-function, a generalized Lauricella function and a class of multivariable polynomials with general arguments . We will study the cases concerning the multivariable I-function defined by Sharma et al [3] and Srivastava-Daoust polynomial [4].

Keywords: Eulerian integral, multivariable A-function, generalized Lauricella function of several variables, multivariable Aleph-function, generalized hypergeometric function, class of polynomials, Srivastava-Daoust polynomial

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1. Introduction

In this paper, we consider a general class of Eulerian integral concerning the product of two Multivariable Aleph-functions, a multivariable A-function and a general class of multivariable polynomials defined by Srivastava et al [7].

The Aleph-function of several variables is an extension of the multivariable I-function defined by Sharma and Ahmad [3], itself is a generalisation of G and H-functions of several variables defined by Srivastava et al [7].

The serie representation of the multivariable A-function is given by Gautam [1] as

$$A[u_1, \dots, u_v] = A_{A,C:(M',N'); \dots; (M^{(v)}, N^{(v)})}^{0,\lambda:(\alpha',\beta'); \dots; (\alpha^{(v)}, \beta^{(v)})} \left(\begin{matrix} u_1 \\ \cdot \\ \cdot \\ u_v \end{matrix} \left| \begin{matrix} [(\mathbf{g}_j); \gamma', \dots, \gamma^{(v)}]_{1,A} : \\ \cdot \\ \cdot \\ [(\mathbf{f}_j); \xi', \dots, \xi^{(v)}]_{1,C} : \end{matrix} \right. \right)$$

$$\left(\begin{matrix} (q^{(1)}, \eta^{(1)})_{1,M^{(1)}}; \dots; (q^{(v)}, \eta^{(v)})_{1,M^{(v)}} \\ \cdot \\ \cdot \\ (p^{(1)}, \epsilon^{(1)})_{1,N^{(1)}}; \dots; (p^{(v)}, \epsilon^{(v)})_{1,N^{(v)}} \end{matrix} \right) = \sum_{G_i=1}^{\alpha^{(i)}} \sum_{g_i=1}^{\infty} \phi_1 \frac{\prod_{i=1}^v \phi_i u_i^{\eta_{G_i, g_i}} (-)^{\sum_{i=1}^v g_i}}{\prod_{i=1}^v \epsilon_{G_i}^{(i)} g_i!} \tag{1.1}$$

where

$$\phi_1 = \frac{\prod_{j=1}^{\lambda} \Gamma(1 - g_j + \sum_{i=1}^v \gamma_j^{(i)} \eta_{G_i, g_i})}{\prod_{j=\lambda'+1}^A \Gamma(g_j - \sum_{i=1}^v \gamma_j^{(i)} U_i) \prod_{j=1}^C \Gamma(1 - f_j + \sum_{i=1}^v \xi_j^{(i)} \eta_{G_i, g_i})} \tag{1.2}$$

$$\phi_i = \frac{\prod_{j=1, j \neq m_i}^{\alpha^{(i)}} \Gamma(p_j^{(i)} - \epsilon_j^{(i)} \eta_{G_i, g_i}) \prod_{j=1}^{\beta^{(i)}} \Gamma(1 - q_j^{(i)} + \eta_j^{(i)} \eta_{G_i, g_i})}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}} \Gamma(1 - p_j^{(i)} + \epsilon_j^{(i)} \eta_{G_i, g_i}) \prod_{j=\beta^{(i)}+1}^{M^{(i)}} \Gamma(q_j^{(i)} - \eta_j^{(i)} \eta_{G_i, g_i})}, i = 1, \dots, v \tag{1.3}$$

$$\text{and } \eta_{G_i, g_i} = \frac{p_{G_i}^{(i)} + g_i}{\epsilon_{G_i}^{(i)}}, i = 1, \dots, v \tag{1.4}$$

$$\text{which is valid under the following conditions : } \epsilon_{m_i}^{(i)} [p_j^{(i)} + p'_i] \neq \epsilon_j^{(i)} [p_{m_i} + g_i] \tag{1.5}$$

and

$$u_i \neq 0, \sum_{j=1}^A \gamma_j^{(i)} - \sum_{j=1}^C \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \epsilon_j^{(i)} < 0, i = 1, \dots, v \tag{1.6}$$

Here $\lambda, A, C, \alpha_i, \beta_i, M_i, N_i \in \mathbb{N}^*$; $i = 1, \dots, v$; $f_j, g_j, p_j^{(i)}, q_j^{(i)}, \gamma_j^{(i)}, \xi_j^{(i)}, \eta_j^{(i)}, \epsilon_j^{(i)} \in \mathbb{C}$

We have : $\aleph(z_1, \dots, z_r) = \aleph^{0, n; m_1, n_1, \dots, m_r, n_r}_{p_i, q_i, \tau_i; R: p_i(1), q_i(1), \tau_i(1); R^{(1)}; \dots; p_i(r), q_i(r), \tau_i(r); R^{(r)}}$ $\left(\begin{array}{c} z_1 \\ \vdots \\ z_r \end{array} \right)$

$$[(a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)})_{1, n}] , [\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \dots, \alpha_{ji}^{(r)})_{n+1, p_i}] : \dots \dots \dots , [\tau_i(b_{ji}; \beta_{ji}^{(1)}, \dots, \beta_{ji}^{(r)})_{m+1, q_i}] :$$

$$\left([(c_j^{(1)}, \gamma_j^{(1)})_{1, n_1}], [\tau_{i(1)}(c_{ji(1)}, \gamma_{ji(1)})_{n_1+1, p_i(1)}]; \dots ; [(c_j^{(r)}, \gamma_j^{(r)})_{1, n_r}], [\tau_{i(r)}(c_{ji(r)}, \gamma_{ji(r)})_{n_r+1, p_i(r)}] \right)$$

$$\left([(d_j^{(1)}, \delta_j^{(1)})_{1, m_1}], [\tau_{i(1)}(d_{ji(1)}, \delta_{ji(1)})_{m_1+1, q_i(1)}]; \dots ; [(d_j^{(r)}, \delta_j^{(r)})_{1, m_r}], [\tau_{i(r)}(d_{ji(r)}, \delta_{ji(r)})_{m_r+1, q_i(r)}] \right)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L'_1} \dots \int_{L'_r} \psi(s_1, \dots, s_r) \prod_{k=1}^r \theta_k(s_k) z_k^{s_k} ds_1 \dots ds_r \tag{1.8}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1, \dots, s_r) = \frac{\prod_{j=1}^n \Gamma(1 - a_j + \sum_{k=1}^r \alpha_j^{(k)} s_k)}{\sum_{i=1}^R [\tau_i \prod_{j=n+1}^{p_i} \Gamma(a_{ji} - \sum_{k=1}^r \alpha_{ji}^{(k)} s_k) \prod_{j=1}^{q_i} \Gamma(1 - b_{ji} + \sum_{k=1}^r \beta_{ji}^{(k)} s_k)]} \tag{1.9}$$

and $\theta_k(s_k) = \frac{\prod_{j=1}^{m_k} \Gamma(d_j^{(k)} - \delta_j^{(k)} s_k) \prod_{j=1}^{n_k} \Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}} [\tau_{i^{(k)}} \prod_{j=m_k+1}^{q_{i^{(k)}}} \Gamma(1 - d_{ji^{(k)}} + \delta_{ji^{(k)}}^{(k)} s_k) \prod_{j=n_k+1}^{p_{i^{(k)}}} \Gamma(c_{ji^{(k)}}^{(k)} - \gamma_{ji^{(k)}}^{(k)} s_k)]} \tag{1.10}$

Suppose , as usual , that the parameters

- $a_j, j = 1, \dots, p; b_j, j = 1, \dots, q;$
- $c_j^{(k)}, j = 1, \dots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \dots, p_{i^{(k)}};$
- $d_j^{(k)}, j = 1, \dots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \dots, q_{i^{(k)}};$

with $k = 1 \dots, r, i = 1, \dots, R, i^{(k)} = 1, \dots, R^{(k)}$

are complex numbers , and the $\alpha' s, \beta' s, \gamma' s$ and $\delta' s$ are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^n \alpha_j^{(k)} + \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)}$$

$$-\tau_{i^{(k)}} \sum_{j=m_k+1}^{q_i^{(k)}} \delta_{ji^{(k)}}^{(k)} \leq 0 \tag{1.11}$$

The reals numbers τ_i are positives for $i = 1$ to R , $\tau_{i^{(k)}}$ are positives for $i^{(k)} = 1$ to $R^{(k)}$

The contour L_k is in the s_k -p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop, if necessary, ensure that the poles of $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to m_k are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^r \alpha_j^{(i)} s_k)$ with $j = 1$ to n and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to n_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi, \text{ where}$$

$$A_i^{(k)} = \sum_{j=1}^n \alpha_j^{(k)} - \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_i^{(k)}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1 \dots, r, i = 1, \dots, R, i^{(k)} = 1, \dots, R^{(k)} \tag{1.12}$$

The complex numbers z_i are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \dots, z_r) = O(|z_1|^{\alpha_1}, \dots, |z_r|^{\alpha_r}), \max(|z_1|, \dots, |z_r|) \rightarrow 0$$

$$\aleph(z_1, \dots, z_r) = O(|z_1|^{\beta_1}, \dots, |z_r|^{\beta_r}), \min(|z_1|, \dots, |z_r|) \rightarrow \infty$$

where $k = 1, \dots, r : \alpha_k = \min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \dots, m_k$ and

$$\beta_k = \max[Re((c_j^{(k)} - 1) / \gamma_j^{(k)})], j = 1, \dots, n_k$$

We will use these following notations in this paper

$$U = p_i, q_i, \tau_i; R; V = m_1, n_1; \dots; m_r, n_r \tag{1.13}$$

$$W = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \dots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.14}$$

$$A = \{(a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \dots, \alpha_{ji}^{(r)})_{n+1,p_i}\} \tag{1.15}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \dots, \beta_{ji}^{(r)})_{m+1,q_i}\} \tag{1.16}$$

$$C = \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}, \tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}\}, \dots, \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}}\} \tag{1.17}$$

$$D = \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}\}, \dots, \{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}}\} \tag{1.18}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \dots, z_r) = \aleph_{U:W}^{0, n:V} \left(\begin{array}{c|c} z_1 & A : C \\ \cdot & \cdot \cdot \cdot \\ \cdot & \cdot \cdot \cdot \\ \cdot & \cdot \cdot \cdot \\ z_r & B : D \end{array} \right) \tag{1.19}$$

Consider the Aleph-function of s variables

$$\aleph(z_1, \dots, z_s) = \aleph_{p'_i, q'_i, l_i; r': p'_{i(1)}, q'_{i(1)}, l_{i(1)}; r^{(1)}; \dots; p'_{i(s)}, q'_{i(s)}, l_{i(s)}; r^{(s)}}^{0, n': m'_1, n'_1, \dots, m'_s, n'_s} \left(\begin{array}{c|c} z_1 & \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s & \end{array} \right)$$

$$[(u_j; \mu_j^{(1)}, \dots, \mu_j^{(r')})_{1, n'}] \quad , [l_i(u_{ji}; \mu_{ji}^{(1)}, \dots, \mu_{ji}^{(r')})_{n'+1, p'_i}] :$$

$$\dots \quad , [l_i(v_{ji}; \nu_{ji}^{(1)}, \dots, \nu_{ji}^{(r')})_{m'+1, q'_i}] :$$

$$[(a_j^{(1)}; \alpha_j^{(1)})_{1, n'_1}], [l_{i(1)}(a_{ji(1)}; \alpha_{ji(1)}^{(1)})_{n'_1+1, p'_{i(1)}}]; \dots ; [(a_j^{(s)}; \alpha_j^{(s)})_{1, n'_s}], [l_{i(s)}(a_{ji(s)}; \alpha_{ji(s)}^{(s)})_{n'_s+1, p'_{i(s)}}]$$

$$[(b_j^{(1)}; \beta_j^{(1)})_{1, m'_1}], [l_{i(1)}(b_{ji(1)}; \beta_{ji(1)}^{(1)})_{m'_1+1, q'_{i(1)}}]; \dots ; [(b_j^{(s)}; \beta_j^{(s)})_{1, m'_s}], [l_{i(s)}(b_{ji(s)}; \beta_{ji(s)}^{(s)})_{m'_s+1, q'_{i(s)}}]$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L''_1} \dots \int_{L''_s} \zeta(t_1, \dots, t_s) \prod_{k=1}^s \phi_k(t_k) z_k^{t_k} dt_1 \dots dt_s \tag{1.20}$$

with $\omega = \sqrt{-1}$

$$\zeta(t_1, \dots, t_s) = \frac{\prod_{j=1}^{n'} \Gamma(1 - u_j + \sum_{k=1}^s \mu_j^{(k)} t_k)}{\sum_{i=1}^{r'} [l_i \prod_{j=n'+1}^{P_i} \Gamma(u_{ji} - \sum_{k=1}^s \mu_{ji}^{(k)} t_k) \prod_{j=1}^{q'_i} \Gamma(1 - v_{ji} + \sum_{k=1}^s \nu_{ji}^{(k)} t_k)]} \tag{1.21}$$

$$\text{and } \phi_k(t_k) = \frac{\prod_{j=1}^{m'_k} \Gamma(b_j^{(k)} - \beta_j^{(k)} t_k) \prod_{j=1}^{n'_k} \Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{Q_i^{(k)}} [l_{i^{(k)}} \prod_{j=m'_k+1}^{Q_i^{(k)}} \Gamma(1 - b_{ji^{(k)}} + \beta_{ji^{(k)}}^{(k)} t_k) \prod_{j=n'_k+1}^{P_i^{(k)}} \Gamma(a_{ji^{(k)}}^{(k)} - \alpha_{ji^{(k)}}^{(k)} s_k)]} \tag{1.22}$$

Suppose , as usual , that the parameters

$$u_j, j = 1, \dots, p'; v_j, j = 1, \dots, q';$$

$$a_j^{(k)}, j = 1, \dots, n'_k; a_{j_i^{(k)}}^{(k)}, j = n_k + 1, \dots, p'_{i^{(k)}};$$

$$b_{j_i^{(k)}}^{(k)}, j = m'_k + 1, \dots, q'_{i^{(k)}}; b_j^{(k)}, j = 1, \dots, m'_k;$$

$$\text{with } k = 1 \dots, s, i = 1, \dots, r', i^{(k)} = 1, \dots, r^{(k)}$$

are complex numbers , and the α' 's, β' 's, γ' 's and δ' 's are assumed to be positive real numbers for standardization purpose such that

$$\begin{aligned}
 U_i^{(k)} &= \sum_{j=1}^{n'} \mu_j^{(k)} + \iota_i \sum_{j=n'+1}^{p'_i} \mu_{ji}^{(k)} + \sum_{j=1}^{n'_k} \alpha_j^{(k)} + \iota_{i^{(k)}} \sum_{j=n'_k+1}^{p'_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)} - \iota_i \sum_{j=1}^{q'_i} v_{ji}^{(k)} - \sum_{j=1}^{m'_k} \beta_j^{(k)} \\
 &- \iota_{i^{(k)}} \sum_{j=m'_k+1}^{q'_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} \leq 0
 \end{aligned} \tag{1.23}$$

The reals numbers τ_i are positives for $i = 1, \dots, s$, $\iota_{i^{(k)}}$ are positives for $i^{(k)} = 1 \dots r^{(k)}$

The contour L_k is in the t_k -p lane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ is a real number with loop, if necessary, ensure that the poles of $\Gamma(b_j^{(k)} - \beta_j^{(k)} t_k)$ with $j = 1$ to m'_k are separated from those of $\Gamma(1 - u_j + \sum_{i=1}^s \mu_j^{(k)} t_k)$ with $j = 1$ to N and $\Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} t_k)$ with $j = 1$ to n'_k to the left of the contour L_k . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$\begin{aligned}
 |arg z_k| &< \frac{1}{2} B_i^{(k)} \pi, \text{ where} \\
 B_i^{(k)} &= \sum_{j=1}^{n'} \mu_j^{(k)} - \iota_i \sum_{j=n'+1}^{p'_i} \mu_{ji}^{(k)} - \iota_i \sum_{j=1}^{q'_i} v_{ji}^{(k)} + \sum_{j=1}^{n'_k} \alpha_j^{(k)} - \iota_{i^{(k)}} \sum_{j=n'_k+1}^{p'_{i^{(k)}}} \alpha_{ji^{(k)}}^{(k)} \\
 &+ \sum_{j=1}^{m'_k} \beta_j^{(k)} - \iota_{i^{(k)}} \sum_{j=m'_k+1}^{q'_{i^{(k)}}} \beta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1, \dots, s, i = 1, \dots, r, i^{(k)} = 1, \dots, r^{(k)}
 \end{aligned} \tag{1.24}$$

The complex numbers z_i are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \dots, z_s) = O(|z_1|^{\alpha'_1}, \dots, |z_s|^{\alpha'_s}), \max(|z_1|, \dots, |z_s|) \rightarrow 0$$

$$\aleph(z_1, \dots, z_s) = O(|z_1|^{\beta'_1}, \dots, |z_s|^{\beta'_s}), \min(|z_1|, \dots, |z_s|) \rightarrow \infty$$

where $k = 1, \dots, s : \alpha'_k = \min[Re(b_j^{(k)} / \beta_j^{(k)})], j = 1, \dots, m'_k$ and

$$\beta'_k = \max[Re((a_j^{(k)} - 1) / \alpha_j^{(k)})], j = 1, \dots, n'_k$$

We will use these following notations in this paper

$$U' = p'_i, q'_i, \iota_i; r'; V' = m'_1, n'_1; \dots; m'_s, n'_s \tag{1.25}$$

$$W' = p'_{i(1)}, q'_{i(1)}, \iota_{i(1)}; r^{(1)}, \dots, p'_{i(r)}, q'_{i(r)}, \iota_{i(s)}; r^{(s)} \tag{1.26}$$

$$A' = \{(u_j; \mu_j^{(1)}, \dots, \mu_j^{(s)})_{1, n'}\}, \{\iota_i(u_{ji}; \mu_{ji}^{(1)}, \dots, \mu_{ji}^{(s)})_{n'+1, p'_i}\} \tag{1.27}$$

$$B' = \{\iota_i(v_{ji}; v_{ji}^{(1)}, \dots, v_{ji}^{(s)})_{m'+1, q'_i}\} \tag{1.28}$$

$$C' = (a_j^{(1)}; \alpha_j^{(1)})_{1, n'_1}, \iota_{i(1)}(a_{ji(1)}^{(1)}; \alpha_{ji(1)}^{(1)})_{n'_1+1, p'_{i(1)}}, \dots, (a_j^{(s)}; \alpha_j^{(s)})_{1, n'_s}, \iota_{i(s)}(a_{ji(s)}^{(s)}; \alpha_{ji(s)}^{(s)})_{n'_s+1, p'_{i(s)}} \tag{1.29}$$

$$D' = (b_j^{(1)}; \beta_j^{(1)})_{1, m'_1}, \iota_{i(1)}(b_{ji(1)}^{(1)}; \beta_{ji(1)}^{(1)})_{m'_1+1, q'_{i(1)}}, \dots, (b_j^{(s)}; \beta_j^{(s)})_{1, m'_s}, \iota_{i(s)}(\beta_{ji(s)}^{(s)}; \beta_{ji(s)}^{(s)})_{m'_s+1, q'_{i(s)}} \quad (1.30)$$

The multivariable Aleph-function write :

$$\aleph(z_1, \dots, z_s) = \aleph_{U'; W'}^{0, n'; V'} \left(\begin{array}{c} z_1 \\ \cdot \\ \cdot \\ z_s \end{array} \middle| \begin{array}{l} A' : C' \\ \cdot \\ \cdot \\ B' : D' \end{array} \right) \quad (1.31)$$

Srivastava and Garg [5] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1, \dots, h_u} [z_1, \dots, z_u] = \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} (-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u) \frac{z_1^{R_1} \dots z_u^{R_u}}{R_1! \dots R_u!} \quad (1.32)$$

The coefficients are $B[E; R_1, \dots, R_u]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^P \Gamma(A_j)}{\prod_{j=1}^Q \Gamma(B_j)} {}_P F_Q [(A_P); (B_Q); -(x_1 + \dots + x_r)]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \frac{\prod_{j=1}^P \Gamma(A_j + s_1 + \dots + s_r)}{\prod_{j=1}^Q \Gamma(B_j + s_1 + \dots + s_r)} \Gamma(-s_1) \dots \Gamma(-s_r) x_1^{s_1} \dots x_r^{s_r} ds_1 \dots ds_r \quad (2.1)$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j + s_1 + \dots + s_r)$ are separated from those of $\Gamma(-s_j), j = 1, \dots, r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j), j = 1, \dots, r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^l [1 - \tau_j (t-a)^{h_j}]^{-\lambda_j} \prod_{j=1}^k (f_j t + g_j)^{\sigma_j} dt = (b-a)^{\alpha+\beta-1} B(\alpha, \beta) \prod_{j=1}^k (af_j + g_j)^{\sigma_j}$$

$$F_{1:0, \dots, 0; 0, \dots, 0}^{1:1, \dots, 1; 1, \dots, 1} \left(\begin{array}{l} (\alpha : h_1, \dots, h_l, 1, \dots, 1) : (\lambda_1 : 1), \dots, (\lambda_l : 1); (-\sigma_1 : 1), \dots, (-\sigma_k : 1) \\ \cdot \\ \cdot \\ (\alpha + \beta : h_1, \dots, h_l, 1, \dots, 1) : -, \dots, -; -, \dots, - \end{array} \right)$$

$$; \tau_1 (b-a)^{h_1}, \dots, \tau_l (b-a)^{h_l}, -\frac{(b-a)f_1}{af_1 + g_1}, \dots, -\frac{(b-a)f_k}{af_k + g_k} \quad (2.2)$$

where $a, b \in \mathbb{R}(a < b), \alpha, \beta, f_i, g_i, \sigma_i, \tau_j, h_j \in \mathbb{C}, \lambda_j \in \mathbb{R}^+(i = 1, \dots, k; j = 1, \dots, l)$

$$\min(\operatorname{Re}(\alpha), \operatorname{Re}(\beta)) > 0, \max_{1 \leq j \leq l} \{|\tau_j(b-a)^{h_j}|\} < 1, \max_{1 \leq j \leq k} \left\{ \left| \frac{(b-a)f_i}{af_i + g_i} \right| \right\} < 1,$$

and $F_{1:0, \dots, 0; 0, \dots, 0}^{1:1, \dots, 1; 1, \dots, 1}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust [4, page 454] and [6] given by :

$$F_{1:0, \dots, 0; 0, \dots, 0}^{1:1, \dots, 1; 1, \dots, 1} \left(\begin{matrix} (\alpha : h_1, \dots, h_l, 1, \dots, 1) : (\lambda_1 : 1), \dots, (\lambda_l : 1); (-\sigma_1 : 1), \dots, (-\sigma_k : 1) \\ \dots \\ (\alpha + \beta : h_1, \dots, h_l, 1, \dots, 1) : -, \dots, -; -, \dots, - \end{matrix} \right) ; \tau_1(b-a)^{h_1}, \dots, \tau_l(b-a)^{h_l}, -\frac{(b-a)f_1}{af_1 + g_1}, \dots, -\frac{(b-a)f_k}{af_k + g_k} = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \prod_{j=1}^l \Gamma(\lambda_j) \prod_{j=1}^k \Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}} \int_{L_1} \dots \int_{L_{l+k}} \frac{\Gamma(\alpha + \sum_{j=1}^l h_j s_j + \sum_{j=1}^k s_{l+j})}{\Gamma(\alpha + \beta + \sum_{j=1}^l h_j s_j + \sum_{j=1}^k s_{l+j})} \prod_{j=1}^l \Gamma(\lambda_j + s_j) \prod_{j=1}^k \Gamma(-\sigma_j + s_{l+j})$$

$$\prod_{j=1}^{l+k} \Gamma(-s_j) z_1^{s_1} \dots z_l^{s_l} z_{l+1}^{s_{l+1}} \dots z_{l+k}^{s_{l+k}} ds_1 \dots ds_{l+k} \tag{2.3}$$

Here the contour $L_j s$ are defined by $L_j = L_{w\zeta_j\infty}(\operatorname{Re}(\zeta_j) = v_j'')$ starting at the point $v_j'' - \omega\infty$ and terminating at the point $v_j'' + \omega\infty$ with $v_j'' \in \mathbb{R}(j = 1, \dots, l)$ and each of the remaining contour L_{l+1}, \dots, L_{l+k} run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha} = \sum_{r=0}^{\infty} \frac{(\alpha)_r}{r!} z^r (|z| < 1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [2, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [4, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i = \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j^{(i)}}, \zeta_j^{(i)} > 0 (i = 1, \dots, r); \theta'_i = \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j'^{(i)}}, \zeta_j'^{(i)} > 0 (i = 1, \dots, s)$$

$$\theta''_i = \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j''^{(i)}}, \zeta_j''^{(i)} > 0 (i = 1, \dots, u)$$

$$\theta'''_i = \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j'''^{(i)}}, \zeta_j'''^{(i)} > 0 (i = 1, \dots, v) \tag{3.1}$$

$$K_2 = (1 - \beta - \sum_{i=1}^u R_i b_i - \sum_{i=1}^v \eta_{G_i, g_i} b'_i; \rho_1, \dots, \rho_r, \rho'_1, \dots, \rho'_s, 0, \dots, 0, 0 \dots, 0) \quad (3.3)$$

$$K_j = [1 - \lambda_j - \sum_{i=1}^u R_i \zeta_j^{(i)} - \sum_{i=1}^v \eta_{G_i, g_i} \zeta_j^{(i)}; \zeta_j^{(1)}, \dots, \zeta_j^{(r)}, \zeta_j^{(1)} \dots, \zeta_j^{(s)}, 0, \dots, 1, \dots, 0, 0 \dots, 0]_{1, l} \quad (3.4)$$

$$K'_j = [1 + \sigma_j - \sum_{i=1}^u R_i \lambda_j^{(i)} - \sum_{i=1}^v \eta_{G_i, g_i} \lambda_j^{(i)}; \lambda_j^{(1)}, \dots, \lambda_j^{(r)}, \lambda_j^{(1)} \dots, \lambda_j^{(s)}, 0, \dots, 0, 0 \dots, 1, \dots, 0]_{1, k} \quad (3.5)$$

$$L_1 = (1 - \alpha - \beta - \sum_{i=1}^u R_i (a_i + b_i) - \sum_{i=1}^v (a'_i + b'_i) \eta_{G_i, g_i}; \mu_1 + \rho_1, \dots, \mu_r + \rho_r, \mu'_1 + \rho'_1, \dots, \mu'_r + \rho'_r, h_1, \dots, h_l, 1, \dots, 1) \quad (3.6)$$

$$L_j = [1 - \lambda_j - \sum_{i=1}^u R_i \zeta_j^{(i)} - \sum_{i=1}^s \zeta_j^{(i)} \eta_{G_i, g_i}; \zeta_j^{(1)}, \dots, \zeta_j^{(r)}, \zeta_j^{(1)} \dots, \zeta_j^{(s)}, 0, \dots, 0, 0 \dots, 0]_{1, l} \quad (3.7)$$

$$L'_j = [1 + \sigma_j - \sum_{i=1}^u R_i \lambda_j^{(i)} - \sum_{i=1}^v \lambda_j^{(i)} \eta_{G_i, g_i}; \lambda_j^{(1)}, \dots, \lambda_j^{(r)}, \lambda_j^{(1)} \dots, \lambda_j^{(s)}, 0, \dots, 0, 0 \dots, 0]_{1, k} \quad (3.8)$$

$$P_1 = (b - a)^{\alpha + \beta - 1} \left\{ \prod_{j=1}^h (af_j + g_j)^{\sigma_j} \right\} \quad (3.9)$$

$$B_{u, v} = (b - a)^{\sum_{i=1}^v (a'_i + b'_i) \eta_{G_i, g_i} + \sum_{i=1}^u (a_i + b_i) R_i} \left\{ \prod_{j=1}^h (af_j + g_j)^{-\sum_{i=1}^v \lambda_i^{(i)} \eta_{G_i, g_i} - \sum_{i=1}^u \lambda_i^{(i)} R_i} \right\} G_v \quad (3.10)$$

where $G_v = \phi_1 \frac{\prod_{i=1}^v \phi_i u_i^{\eta_{G_i, g_i} (-)^{\sum_{i=1}^v g_i}}}{\prod_{i=1}^v \epsilon_{G_i}^{(i)} g_i!}$

ϕ_1, ϕ_i for $i = 1, \dots, v$ are defined respectively by (1.2) and (1.3)

$$B_u = \frac{(-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u)}{R_1! \dots R_u!} \quad (3.11)$$

$$V_1 = V; V'; 1, 0; \dots; 1, 0; 1, 0; \dots; 1, 0; W_1 = W; W'; 0, 1; \dots; 0, 1; 0, 1; \dots; 0, 1 \quad (3.12)$$

$$C_1 = C; C'; (1, 0), \dots, (1, 0); (1, 0), \dots, (1, 0); D_1 = D; D'; (0, 1), \dots, (0, 1); (0, 1), \dots, (0, 1) \quad (3.13)$$

We have the general Eulerian integral

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^l [1 - \tau_j (t-a)^{h_j}]^{-\lambda_j} \prod_{j=1}^k (f_j t + g_j)^{\sigma_j}$$

$$S_L^{h_1, \dots, h_u} \left(\begin{matrix} z_1'' \theta_1'' (t-a)^{a_1} (b-t)^{b_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \vdots \\ z_u'' \theta_u'' (t-a)^{a_u} (b-t)^{b_u} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(u)}} \end{matrix} \right)$$

$$A \left(\begin{matrix} z_1''' \theta_1''' (t-a)^{a'_1} (b-t)^{b'_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)'}} \\ \vdots \\ z_v''' \theta_v''' (t-a)^{a'_v} (b-t)^{b'_v} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(v)'}} \end{matrix} \right)$$

$$\mathcal{N} \left(\begin{matrix} z_1 \theta_1 (t-a)^{\mu_1} (b-t)^{\rho_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \vdots \\ z_r \theta_r (t-a)^{\mu_r} (b-t)^{\rho_r} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(r)}} \end{matrix} \right)$$

$$\mathcal{N} \left(\begin{matrix} z'_1 \theta'_1 (t-a)^{\mu'_1} (b-t)^{\rho'_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)'}} \\ \vdots \\ z'_s \theta'_s (t-a)^{\mu'_s} (b-t)^{\rho'_s} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(s)'}} \end{matrix} \right) dt$$

$$= P_1 \sum_{g_1, \dots, g_v=0}^{\infty} \sum_{M_1=0}^{\alpha^{(1)}} \dots \sum_{M_v=0}^{\alpha^{(v)}} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} \prod_{i=1}^v z_i^{\eta_{h_i, k_i}} \prod_{k=1}^u z^{\eta_{R_k}} B_u B_{u,v}$$

$$\begin{array}{c}
 N_{U;U';l+k+2,l+k+1:W_1}^{0,n+n'+l+k+2:V_1} \left(\begin{array}{c}
 \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j^{(1)}}} \\
 \dots \\
 \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j^{(r)}}} \\
 \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^k (af_j+g_j)^{\lambda'_j^{(1)}}} \\
 \dots \\
 \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^k (af_j+g_j)^{\lambda'_j^{(s)}}} \\
 \tau_1(b-a)^{h_1} \\
 \dots \\
 \tau_l(b-a)^{h_l} \\
 \frac{(b-a)f_1}{af_1+g_1} \\
 \dots \\
 \frac{(b-a)f_k}{af_k+g_k}
 \end{array} \right) \begin{array}{c}
 A ; A ; K_1, K_2, K_j, K'_j : C_1 \\
 \dots \\
 B ; B' ; L_1, L_j, L'_j : D_1
 \end{array} \quad (3.14)
 \end{array}$$

We obtain the Aleph-function of $r + s + k + l$ variables. The quantities $A, A', B, B', C, C', C_1, D_1, V_1$ and W_1 are defined above.

- (A) $a, b \in \mathbb{R}(a < b); \mu_i, \mu'_u, \rho_i, \rho'_u, \lambda_j^{(i)}, \lambda_j^{(u)}, h_v \in \mathbb{R}^+, f_i, g_j, \tau_v, \sigma_j, \lambda_v \in \mathbb{C} (i = 1, \dots, r; j = 1, \dots, k; u = 1, \dots, s; v = 1, \dots, l), a_i, b_i, \lambda_j^{(i)}, \zeta_j^{(i)} \in \mathbb{R}^+, (i = 1, \dots, u; j = 1, \dots, k) a'_i, b'_i, \lambda_j^{(i)}, \zeta_j^{(i)} \in \mathbb{R}^+, (i = 1, \dots, v; j = 1, \dots, k)$

(B) See the section 1

$$(C) \max_{1 \leq j \leq k} \left\{ \left| \frac{(b-a)f_j}{af_j+g_j} \right| \right\} < 1, \max_{1 \leq j \leq l} \{ |\tau_j(b-a)^{h_j}| \} < 1$$

$$(D) Re \left[\alpha + \sum_{j=1}^v a'_j \min_{1 \leq k \leq \alpha^{(i)}} \frac{p_k^{(j)}}{\epsilon_k^{(j)}} + \sum_{j=1}^r \mu_j \min_{1 \leq k \leq m_i} \frac{d_k^{(j)}}{\delta_k^{(j)}} + \sum_{j=1}^s \mu'_i \min_{1 \leq k \leq m'_i} \frac{b_k^{(j)}}{\beta_k^{(j)}} \right] > 0$$

$$Re \left[\beta + \sum_{j=1}^v b'_j \min_{1 \leq k \leq \alpha^{(i)}} \frac{p_k^{(j)}}{\epsilon_k^{(j)}} + \sum_{j=1}^r \rho_j \min_{1 \leq k \leq m_i} \frac{d_k^{(j)}}{\delta_k^{(j)}} + \sum_{j=1}^s \rho'_j \min_{1 \leq k \leq m'_i} \frac{b_k^{(j)}}{\beta_k^{(j)}} \right] > 0$$

$$(E) \operatorname{Re} \left(\alpha + \sum_{i=1}^v \eta_{G_i, g_i} a'_i + \sum_{i=1}^u R_i a_i + \sum_{i=1}^r \mu_i s_i + \sum_{i=1}^s t_i \mu'_i \right) > 0$$

$$\operatorname{Re} \left(\beta + \sum_{i=1}^v \eta_{G_i, g_i} b'_i + \sum_{i=1}^u R_i b_i + \sum_{i=1}^r v_i s_i + \sum_{i=1}^s t_i \rho'_i \right) > 0$$

$$\operatorname{Re} \left(\lambda_j + \sum_{i=1}^v \eta_{G_i, g_i} \lambda_j^{(i)} + \sum_{i=1}^u R_i \lambda_j^{(i)} + \sum_{i=1}^r s_i \zeta_j^{(i)} + \sum_{i=1}^s t_i \zeta_j^{(i)} \right) > 0 (j = 1, \dots, l);$$

$$\operatorname{Re} \left(-\sigma_j + \sum_{i=1}^v \eta_{G_i, g_i} \lambda_j^{(i)} + \sum_{i=1}^u R_i \lambda_j^{(i)} + \sum_{i=1}^r s_i \lambda_j^{(i)} + \sum_{i=1}^s t_i \lambda_j^{(i)} \right) > 0 (j = 1, \dots, k);$$

$$(F) U_i^{(k)} = \sum_{j=1}^n \alpha_j^{(k)} + \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i(k)} \sum_{j=n_k+1}^{p_i(k)} \gamma_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i(k)} \sum_{j=m_k+1}^{q_i(k)} \delta_{ji}^{(k)} \leq 0$$

$$U_i^{(k)} = \sum_{j=1}^{n'} \mu_j^{(k)} + \iota_i \sum_{j=n'+1}^{p'_i} \mu_{ji}^{(k)} + \sum_{j=1}^{n'_k} \alpha_j^{(k)} + \iota_{i(k)} \sum_{j=n'_k+1}^{p'_i(k)} \alpha_{ji}^{(k)} - \iota_i \sum_{j=1}^{q'_i} \nu_{ji}^{(k)} - \sum_{j=1}^{m'_k} \beta_j^{(k)} - \iota_{i(k)} \sum_{j=m'_k+1}^{q'_i(k)} \beta_{ji}^{(k)} \leq 0$$

$$(G) A_i^{(k)} = \sum_{j=1}^n \alpha_j^{(k)} - \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i(k)} \sum_{j=n_k+1}^{p_i(k)} \gamma_{ji}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i(k)} \sum_{j=m_k+1}^{q_i(k)} \delta_{ji}^{(k)} - \sum_{l=1}^k \lambda_j^{(i)} - \sum_{l=1}^l \zeta_j^{(i)} - \mu_k - \rho_k > 0, \text{ with } k = 1 \dots, r,$$

$$i = 1, \dots, R, i^{(k)} = 1, \dots, R^{(k)}$$

$$B_i^{(k)} = \sum_{j=1}^{n'} \mu_j^{(k)} - \iota_i \sum_{j=n'+1}^{p'_i} \mu_{ji}^{(k)} - \iota_i \sum_{j=1}^{q'_i} \nu_{ji}^{(k)} + \sum_{j=1}^{n'_k} \alpha_j^{(k)} - \iota_{i(k)} \sum_{j=n'_k+1}^{p'_i(k)} \alpha_{ji}^{(k)}$$

$$+ \sum_{j=1}^{m'_k} \beta_j^{(k)} - \iota_{i(k)} \sum_{j=m'_k+1}^{q'_i(k)} \beta_{ji}^{(k)} - \sum_{l=1}^k \lambda_j^{(i)} - \sum_{l=1}^l \zeta_j^{(i)} - \mu'_k - \rho'_k > 0, \text{ with } k = 1, \dots, s,$$

$$i = 1, \dots, r, i^{(k)} = 1, \dots, r^{(k)}$$

$$(H) \left| \arg \left(z_i \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\zeta_j^{(i)}} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(i)}} \right) \right| < \frac{1}{2} A_i^{(k)} \pi \quad (a \leq t \leq b; i = 1, \dots, r)$$

$$\left| \arg \left(z'_i \prod_{j=1}^l [1 - \tau'_j(t-a)^{h'_i}]^{-\zeta'_j{}^{(i)}} \prod_{j=1}^k (f_j t + g_j)^{-\lambda'_j{}^{(i)}} \right) \right| < \frac{1}{2} B_i^{(k)} \pi \quad (a \leq t \leq b; i = 1, \dots, s)$$

(I) The multiple series occurring on the right-hand side of (3.14) is absolutely and uniformly convergent.

Proof

To prove (3.14), first, we express in series the multivariable A-function with the help of (1.1), a class of multivariable polynomials defined by Srivastava et al [5] $S_L^{h_1, \dots, h_u}[\cdot]$ in series with the help of (1.32) and we interchange the order of summations and t-integral (which is permissible under the conditions stated). Expressing the Aleph-functions of r-variables and s-variables in terms of Mellin-Barnes type contour integral with the help of (1.8) and (1.20) respectively and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collect the power of $[1 - \tau_j(t-a)^{h_i}]$ with $(i = 1, \dots, r; j = 1, \dots, l)$ and collect the power of $(f_j t + g_j)$ with $j = 1, \dots, k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r + s + k + l)$ dimensional Mellin-Barnes integral in multivariable Aleph-function, we obtain the equation (3.14).

Remarks

If a) $\rho_1 = \dots, \rho_r = \rho'_1 = \dots, \rho'_s = 0$; b) $\mu_1 = \dots, \mu_r = \mu'_1 = \dots, \mu'_s = 0$, we obtain the similar formulas that (3.14) with the corresponding simplifications.

4. Particular cases

a) If $\tau_i, \tau_{i(1)}, \dots, \tau_{i(r)}, l_i, l_{i(1)}, \dots, l_{i(s)} \rightarrow 1$, the multivariable Aleph-functions of r and s-variables reduces to multivariable I-functions of r and s-variables defined by Sharma and al [3] respectively and we have

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^l [1 - \tau_j(t-a)^{h_i}]^{-\lambda_j} \prod_{j=1}^k (f_j t + g_j)^{\sigma_j}$$

$$S_L^{h_1, \dots, h_u} \left(\begin{matrix} z_1'' \theta_1'' (t-a)^{a_1} (b-t)^{b_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j''^{(1)}} \\ \vdots \\ z_u'' \theta_u'' (t-a)^{a_u} (b-t)^{b_u} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j''^{(u)}} \end{matrix} \right)$$

$$A \left(\begin{matrix} z_1''' \theta_1''' (t-a)^{a'_1} (b-t)^{b'_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'''^{(1)}} \\ \vdots \\ z_v''' \theta_v''' (t-a)^{a'_v} (b-t)^{b'_v} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'''^{(v)}} \end{matrix} \right)$$

$$\begin{aligned}
 & I \begin{pmatrix} z_1 \theta_1 (t-a)^{\mu_1} (b-t)^{\rho_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \vdots \\ z_r \theta_r (t-a)^{\mu_r} (b-t)^{\rho_r} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(r)}} \end{pmatrix} \\
 & I \begin{pmatrix} z'_1 \theta'_1 (t-a)^{\mu'_1} (b-t)^{\rho'_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'^{(1)}} \\ \vdots \\ z'_s \theta'_s (t-a)^{\mu'_s} (b-t)^{\rho'_s} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'^{(s)}} \end{pmatrix} dt \\
 & = P_1 \sum_{g_1, \dots, g_v=0}^{\infty} \sum_{M_1=0}^{\alpha^{(1)}} \dots \sum_{M_v=0}^{\alpha^{(v)}} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} \prod_{i=1}^v z_i^{\eta_{h_i, k_i}} \prod_{k=1}^u z''^{R_k} B_u B_{u,v} \\
 & I_{U; U'; l+k+2, l+k+1: W_1}^{0, n+n'+l+k+2: V_1} \left(\begin{array}{c|c} \frac{z_1 (b-a)^{\mu_1 + \rho_1}}{\prod_{j=1}^k (a f_j + g_j)^{\lambda_j^{(1)}}} & \vdots \\ \vdots & \vdots \\ \frac{z_r (b-a)^{\mu_r + \rho_r}}{\prod_{j=1}^k (a f_j + g_j)^{\lambda_j^{(r)}}} & \vdots \\ \frac{z'_1 (b-a)^{\mu'_1 + \rho'_1}}{\prod_{j=1}^k (a f_j + g_j)^{\lambda_j'^{(1)}}} & \vdots \\ \vdots & \vdots \\ \frac{z'_s (b-a)^{\mu'_s + \rho'_s}}{\prod_{j=1}^k (a f_j + g_j)^{\lambda_j'^{(s)}}} & \vdots \\ \tau_1 (b-a)^{h_1} & \vdots \\ \vdots & \vdots \\ \tau_l (b-a)^{h_l} & \vdots \\ \frac{(b-a) f_1}{a f_1 + g_1} & \vdots \\ \vdots & \vdots \\ \frac{(b-a) f_k}{a f_k + g_k} & \vdots \end{array} \right) \begin{array}{l} A ; A ; K_1, K_2, K_j, K'_j : C_1 \\ \\ \\ \\ \\ \\ \\ B ; B' ; L_1, L_j, L'_j : D_1 \end{array} \quad (4.1)
 \end{aligned}$$

under the same conditions and notations that (3.14) with $\tau_i, \tau_{i(1)}, \dots, \tau_{i(r)}, l_i, l_{i(1)}, \dots, l_{i(s)} \rightarrow 1$

$$b) \text{ If } B(L; R_1, \dots, R_u) = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1 \theta'_j + \dots + R_u \theta_j^{(u)}} \prod_{j=1}^{B'} (b'_j)_{R_1 \phi'_j} \dots \prod_{j=1}^{B^{(u)}} (b_j^{(u)})_{R_u \phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{R_1 \psi'_j + \dots + R_u \psi_j^{(u)}} \prod_{j=1}^{D'} (d'_j)_{R_1 \delta'_j} \dots \prod_{j=1}^{D^{(u)}} (d_j^{(u)})_{R_u \delta_j^{(u)}}} \quad (4.2)$$

then the general class of multivariable polynomial $S_L^{h_1, \dots, h_u} [z_1, \dots, z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [4]. We have

$$\int_a^b (t-a)^{\alpha-1} (b-t)^{\beta-1} \prod_{j=1}^l [1 - \tau_j (t-a)^{h_i}]^{-\lambda_j} \prod_{j=1}^k (f_j t + g_j)^{\sigma_j}$$

$$F_{\bar{C}; D'; \dots; D^{(u)}}^{1+\bar{A}; B'; \dots; B^{(u)}} \left(\begin{matrix} z_1'' \theta_1'' (t-a)^{a_1} (b-t)^{b_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j''(1)} \\ \vdots \\ z_u'' \theta_u'' (t-a)^{a_u} (b-t)^{b_u} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j''(u)} \end{matrix} \right)$$

$$A \left(\begin{matrix} z_1''' \theta_1''' (t-a)^{a'_1} (b-t)^{b'_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'''(1)} \\ \vdots \\ z_v''' \theta_v''' (t-a)^{a'_v} (b-t)^{b'_v} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'''(v)} \end{matrix} \right)$$

$$\mathfrak{N} \left(\begin{matrix} z_1 \theta_1 (t-a)^{\mu_1} (b-t)^{\rho_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \vdots \\ z_r \theta_r (t-a)^{\mu_r} (b-t)^{\rho_r} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j^{(r)}} \end{matrix} \right)$$

$$\mathfrak{N} \left(\begin{matrix} z'_1 \theta'_1 (t-a)^{\mu'_1} (b-t)^{\rho'_1} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'^{(1)}} \\ \vdots \\ z'_s \theta'_s (t-a)^{\mu'_s} (b-t)^{\rho'_s} \prod_{j=1}^k (f_j t + g_j)^{-\lambda_j'^{(s)}} \end{matrix} \right) dt$$

$$= P_1 \sum_{g_1, \dots, g_u=0}^{\infty} \sum_{M_1=0}^{\alpha^{(1)}} \dots \sum_{M_v=0}^{\alpha^{(v)}} \sum_{R_1, \dots, R_u=0}^{h_1 R_1 + \dots + h_u R_u \leq L} \prod_{i=1}^v z_i^{\prime\prime\prime \eta_{h_i, k_i}} \prod_{k=1}^u z^{\prime\prime R_k} B'_u B_{u,v}$$

$$\begin{matrix} N^{0, n+n'+l+k+2; V_1} \\ U; U'; l+k+2, l+k+1; W_1 \end{matrix} \left(\begin{array}{c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j^{(1)}}} \\ \dots \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^k (af_j+g_j)^{\lambda_j^{(r)}}} \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^k (af_j+g_j)^{\lambda'_j^{(1)}}} \\ \dots \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^k (af_j+g_j)^{\lambda'_j^{(s)}}} \\ \tau_1(b-a)^{h_1} \\ \dots \\ \tau_l(b-a)^{h_l} \\ \frac{(b-a)f_1}{af_1+g_1} \\ \dots \\ \frac{(b-a)f_k}{af_k+g_k} \end{array} \middle| \begin{array}{l} A; A; K_1, K_2, K_j, K'_j : C_1 \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ B; B'; L_1, L_j, L'_j : D_1 \end{array} \right) \quad (4.3)$$

under the same notations and conditions that (3.14)

where $B'_u = \frac{(-L)_{h_1 R_1 + \dots + h_u R_u} B(E; R_1, \dots, R_u)}{R_1! \dots R_u!}$, $B[E; R_1, \dots, R_v]$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extended to product of any finite number of multivariable Aleph-functions and a class of multivariable polynomials defined by Srivastava et al [4].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of two multivariable Aleph-function, an expansion of multivariable A-function and a class of multivariable polynomials defined by Srivastava et al [5] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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