Eulerian integral associated with product of three multivariable A-functions, generalized Lauricella function and a class of polynomial

$F.Y.\ AYANT^1$

1 Teacher in High School , France

ABSTRACT
The present paper is evaluated a new Eulerian integral associated with the product of three multivariable A-functions defined by Gautam et al [1] a generalized Lauricella function and a class of multivariable polynomials with general arguments . We will study the case concerning the multivariable H-function defined by Srivastava et al [7] and Srivastava-Doust polynomial [4].

Keywords: Eulerian integral, multivariable I-function, generalized Lauricella function of several variables, multivariable A-function, generalized hypergeometric function, class of polynomials,multivariable A-function.

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1. Introduction

In this paper, we evaluate a new Eulerian integral of most general characters associated with the products of three multivariable A-functions defined by Gautam et [1] and a class of polynomials with general arguments.
First time, we define the multivariable I-function by :

The serie representation of the multivariable A-function is given by Gautam [1] as

$$A[u_1,\cdots,u_v] = A^{0,\lambda:(\alpha',\beta');\cdots;(\alpha^{(v)},\beta^{(v)})}_{A,C:(M',N');\cdots;(M^{(v)},N^{(v)})}\left(\begin{array}{c} \mathrm{u}_1 \\ \cdot \\ \cdot \\ \mathrm{u}_v \end{array}\middle| \begin{array}{c} [(\mathrm{g}_j);\gamma',\cdots,\gamma^{(v)}]_{1,A}: \\ \cdots \\ [(\mathrm{f}_j);\xi',\cdots,\xi^{(v)}]_{1,C}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{q}^{(1)},\eta^{(1)})_{1,M^{(1)}};\cdots;(q^{(v)},\eta^{(v)})_{1,M^{(v)}} \\ \cdots \\ \cdots \\ (\mathrm{p}^{(1)},\epsilon^{(1)})_{1,N^{(1)}};\cdots;(p^{(v)},\epsilon^{(v)})_{1,N^{(v)}} \end{array}\right) = \sum_{G_i=1}^{\alpha^{(i)}}\sum_{g_i=1}^{\infty}\phi_1\frac{\prod_{i=1}^{v}\phi_i u_i^{\eta_{G_i,g_i}}(-)^{\sum_{i=1}^{v}g_i}}{\prod_{i=1}^{v}\epsilon^{(i)}_{G_i}g_i!} \tag{1.1}$$

where

$$\phi_1 = \frac{\prod_{j=1}^{\lambda}\Gamma\left(1-g_j+\sum_{i=1}^{v}\gamma_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\lambda'+1}^{A}\Gamma\left(g_j-\sum_{i=1}^{v}\gamma_j^{(i)}U_i\right)\prod_{j=1}^{C}\Gamma\left(1-f_j+\sum_{i=1}^{v}\xi_j^{(i)}\eta_{G_i,g_i}\right)} \tag{1.2}$$

$$\phi_i = \frac{\prod_{j=1,j\neq m_i}^{\alpha^{(i)}}\Gamma\left(p_j^{(i)}-\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=1}^{\beta^{(i)}}\Gamma\left(1-q_j^{(i)}+\eta_j^{(i)}\eta_{G_i,g_i}\right)}{\prod_{j=\alpha^{(i)}+1}^{N^{(i)}}\Gamma\left(1-p_j^{(i)}+\epsilon_j^{(i)}\eta_{G_i,g_i}\right)\prod_{j=\beta^{(i)}+1}^{M^{(i)}}\Gamma\left(q_j^{(i)}-\eta_j^{(i)}\eta_{G_i,g_i}\right)}, i=1,\cdots,v \tag{1.3}$$

and $\eta_{G_i,g_i} = \dfrac{p^{(i)}_{G_i}+g_i}{\epsilon^{(i)}_{G_i}}, i=1,\cdots,v$ (1.4)

which is valid under the following conditions : $\epsilon^{(i)}_{m_i}[p_j^{(i)}+p_i'] \neq \epsilon_j^{(i)}[p_{m_i}+g_i]$ (1.5) and

$$u_i \neq 0, \sum_{j=1}^{A} \gamma_j^{(i)} - \sum_{j=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \epsilon_j^{(i)} < 0, i = 1, \cdots, v \tag{1.6}$$

Here $\lambda, A, C, \alpha_i, \beta_i, M_i, N_i \in \mathbb{N}^*; i = 1, \cdots, v ; f_j, g_j, p_j^{(i)}, q_j^{(i)}, \gamma_j^{(i)}, \xi_j^{(i)}, \eta_j^{(i)}, \epsilon_j^{(i)} \in \mathbb{C}$

The A-function is defined and represented in the following manner.

$$A(z_1, \cdots, z_r) = A_{p,q:p_1,q_1;\cdots;p_r,q_r}^{m,n:m_1,n_1;\cdots;m_r,n_r} \left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j; A_j^{(1)}, \cdots, A_j^{(r)})_{1,p} : \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j; B_j^{(1)}, \cdots, B_j^{(r)})_{1,q} : \end{array} \right.$$

$$\left. \begin{array}{c} (\mathrm{c}_j^{(1)}, C_j^{(1)})_{1,p_1}; \cdots ; (c_j^{(r)}, C_j^{(r)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)}, D_j^{(1)})_{1,q_1}; \cdots ; (d_j^{(r)}, D_j^{(r)})_{1,q_r} \end{array} \right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots, s_r) \prod_{i=1}^{r} \theta_i(s_i) z_i^{s_i} \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.9}$$

where $\phi(s_1, \cdots, s_r), \theta_i(s_i), i = 1, \cdots, r$ are given by :

$$\phi(s_1, \cdots, s_r) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \sum_{i=1}^{r} B_j^{(i)} s_i) \prod_{j=1}^{n} \Gamma(1 - a_j + \sum_{i=1}^{r} A_j^{(i)} s_j)}{\prod_{j=n+1}^{p} \Gamma(a_j - \sum_{i=1}^{r} A_j^{(i)} s_j) \prod_{j=m+1}^{q} \Gamma(1 - b_j + \sum_{i=1}^{r} B_j^{(i)} s_j)} \tag{1.10}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma(1 - c_j^{(i)} + C_j^{(i)} s_i) \prod_{j=1}^{m_i} \Gamma(d_j^{(i)} - D_j^{(i)} s_i)}{\prod_{j=n_i+1}^{p_i} \Gamma(c_j^{(i)} - C_j^{(i)} s_i) \prod_{j=m_i+1}^{q_i} \Gamma(1 - d_j^{(i)} + D_j^{(i)} s_i)} \tag{1.11}$$

Here $m, n, p, m_i, n_i, p_i, c_i \in \mathbb{N}^*; i = 1, \cdots, r ; a_j, b_j, c_j^{(i)}, d_j^{(i)}, A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i) z_k| < \frac{1}{2} \eta_k \pi, \xi^* = 0, \eta_i > 0 \tag{1.12}$$

$$\Omega_i = \prod_{j=1}^{p} \{A_j^{(i)}\}^{A_j^{(i)}} \prod_{j=1}^{q} \{B_j^{(i)}\}^{-B_j^{(i)}} \prod_{j=1}^{q_i} \{D_j^{(i)}\}^{D_j^{(i)}} \prod_{j=1}^{p_i} \{C_j^{(i)}\}^{-C_j^{(i)}}; i = 1, \cdots, r \tag{1.13}$$

$$\xi_i^* = Im(\sum_{j=1}^{p} A_j^{(i)} - \sum_{j=1}^{q} B_j^{(i)} + \sum_{j=1}^{q_i} D_j^{(i)} - \sum_{j=1}^{p_i} C_j^{(i)}); i = 1, \cdots, r \tag{1.14}$$

$$\eta_i = Re \left(\sum_{j=1}^{n} A_j^{(i)} - \sum_{j=n+1}^{p} A_j^{(i)} + \sum_{j=1}^{m} B_j^{(i)} - \sum_{j=m+1}^{q} B_j^{(i)} + \sum_{j=1}^{m_i} D_j^{(i)} - \sum_{j=m_i+1}^{q_i} D_j^{(i)} + \sum_{j=1}^{n_i} C_j^{(i)} - \sum_{j=n_i+1}^{p_i} C_j^{(i)} \right)$$

$$i = 1, \cdots, r \tag{1.15}$$

Consider the second multivariable A-function.

$$A(z'_1,\cdots,z'_s) = A^{m',n':m'_1,n'_1;\cdots;m'_r,n'_r}_{p',q':p'_1,q'_1;\cdots;p'_r,q'_r}\left(\begin{array}{c|c} z_1 & (a'_j;A'^{(1)}_j,\cdots,A'^{(s)}_j)_{1,p'}: \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s & (b'_j;B'^{(1)}_j,\cdots,B'^{(s)}_j)_{1,q'}: \end{array}\right.$$

$$\left.\begin{array}{c} (c'^{(1)}_j,C'^{(1)}_j)_{1,p'_1};\cdots;(c'^{(s)}_j,C'^{(s)}_j)_{1,p'_s} \\ \\ (d'^{(1)}_j,D'^{(1)}_j)_{1,q'_1};\cdots;(d'^{(s)}_j,D'^{(s)}_j)_{1,q'_s} \end{array}\right) \quad (1.16)$$

$$= \frac{1}{(2\pi\omega)^s}\int_{L'_1}\cdots\int_{L'_s}\phi'(t_1,\cdots,t_s)\prod_{i=1}^{s}\theta'_i(t_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (1.17)$$

where $\phi'(t_1,\cdots,t_s),\theta'_i(t_i), i=1,\cdots,s$ are given by :

$$\phi'(t_1,\cdots,t_s) = \frac{\prod_{j=1}^{m'}\Gamma(b'_j-\sum_{i=1}^{s}B'^{(i)}_j t_i)\prod_{j=1}^{n'}\Gamma(1-a'_j+\sum_{i=1}^{s}A'^{(i)}_j t_j)}{\prod_{j=n'+1}^{p'}\Gamma(a'_j-\sum_{i=1}^{s}A'^{(i)}_j t_j)\prod_{j=m'+1}^{q'}\Gamma\left(1-b'_j+\sum_{i=1}^{s}B'^{(i)}_j t_j\right)} \quad (1.18)$$

$$\theta'_i(t_i) = \frac{\prod_{j=1}^{n'_i}\Gamma(1-c'^{(i)}_j+C'^{(i)}_j t_i)\prod_{j=1}^{m'_i}\Gamma(d'^{(i)}_j-D'^{(i)}_j t_i)}{\prod_{j=n'_i+1}^{p'_i}\Gamma(c'^{(i)}_j-C'^{(i)}_j t_i)\prod_{j=m'_i+1}^{q'_i}\Gamma(1-d'^{(i)}_j+D'^{(i)}_j t_i)} \quad (1.19)$$

Here $m',n',p',m'_i,n'_i,p'_i,c'_i\in\mathbb{N}^*; i=1,\cdots,r$; $a'_j,b'_j,c'^{(i)}_j,d'^{(i)}_j,A'^{(i)}_j,B'^{(i)}_j,C'^{(i)}_j,D'^{(i)}_j\in\mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega'_i)z'_k| < \frac{1}{2}\eta'_k\pi, \xi'^* = 0, \eta'_i > 0 \quad (1.20)$$

$$\Omega'_i = \prod_{j=1}^{p'}\{A'^{(i)}_j\}^{A'^{(i)}_j}\prod_{j=1}^{q'}\{B'^{(i)}_j\}^{-B'^{(i)}_j}\prod_{j=1}^{q'_i}\{D'^{(i)}_j\}^{D'^{(i)}_j}\prod_{j=1}^{p'_i}\{C'^{(i)}_j\}^{-C'^{(i)}_j}; i=1,\cdots,s \quad (1.21)$$

$$\xi'^*_i = Im(\sum_{j=1}^{p'}A'^{(i)}_j - \sum_{j=1}^{q'}B'^{(i)}_j + \sum_{j=1}^{q'_i}D'^{(i)}_j - \sum_{j=1}^{p'_i}C'^{(i)}_j); i=1,\cdots,s \quad (1.22)$$

$$\eta'_i = Re\left(\sum_{j=1}^{n'}A'^{(i)}_j - \sum_{j=n'+1}^{p'}A'^{(i)}_j + \sum_{j=1}^{m'}B'^{(i)}_j - \sum_{j=m'+1}^{q'}B'^{(i)}_j + \sum_{j=1}^{m'_i}D'^{(i)}_j - \sum_{j=m'_i+1}^{q'_i}D'^{(i)}_j + \sum_{j=1}^{n'_i}C'^{(i)}_j - \sum_{j=n'_i+1}^{p'_i}C'^{(i)}_j\right)$$

$$i=1,\cdots,s \quad (1.23)$$

Srivastava and Garg [5] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]= \sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L} (-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)\frac{z_1^{R_1}\cdots z_u^{R_u}}{R_1!\cdots R_u!} \quad (1.24)$$

The coefficients are $B[E;R_1,\ldots,R_v]$ arbitrary constants, real or complex.

2. Integral representation of generalized Lauricella function of several variables

The following generalized hypergeometric function in terms of multiple contour integrals is also required [6 ,page 39 eq .30]

$$\frac{\prod_{j=1}^{P}\Gamma(A_j)}{\prod_{j=1}^{Q}\Gamma(B_j)}\,{}_PF_Q\left[(A_P);(B_Q);-(x_1+\cdots+x_r)\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\frac{\prod_{j=1}^{P}\Gamma(A_j+s_1+\cdots+s_r)}{\prod_{j=1}^{Q}\Gamma(B_j+s_1+\cdots+s_r)}\Gamma(-s_1)\cdots\Gamma(-s_r)x_1^{s_1}\cdots x_r^{s_r}\mathrm{d}s_1\cdots\mathrm{d}s_r \quad (2.1)$$

where the contours are of Barnes type with indentations, if necessary, to ensure that the poles of $\Gamma(A_j+s_1+\cdots+s_r)$ are separated from those of $\Gamma(-s_j),j=1,\cdots,r$. The above result (1.23) can be easily established by an appeal to the calculus of residues by calculating the residues at the poles of $\Gamma(-s_j),j=1,\cdots,r$

In order to evaluate a number of integrals of multivariable I-function, we first establish the formula

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}\mathrm{d}t = (b-a)^{\alpha+\beta-1}B(\alpha,\beta)\prod_{j=1}^{k}(af_j+g_j)^{\sigma_j}$$

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$\left.;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\right) \quad (2.2)$$

where $a,b\in\mathbb{R}(a<b),\alpha,\beta,f_i,g_i,\sigma_i,\tau_j,h_j\in\mathbb{C},\lambda_j\in\mathbb{R}^+(i=1,\cdots,k;j=1,\cdots,l)$

$$min(Re(\alpha),Re(\beta))>0,\ \max_{1\leqslant j\leqslant l}\left\{\left|\tau_j(b-a)^{h_j}\right|\right\}<1,\ \max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1,$$

and $F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}$ is a particular case of the generalized Lauricella function introduced by Srivastava-Daoust[3,page 454] given by :

$$F_{1:0,\cdots,0;0,\cdots,0}^{1:1,\cdots,1;1\cdots,1}\left(\begin{array}{c}(\alpha:h_1,\cdots,h_l,1,\cdots,1):(\lambda_1:1),\cdots,(\lambda_l:1);(-\sigma_1:1),\cdots,(-\sigma_k:1)\\ \cdots\\ (\alpha+\beta:h_1,\cdots,h_l,1,\cdots,1):-,\cdots,-;-,\cdots,-\end{array}\right.$$

$$;\tau_1(b-a)^{h_1},\cdots,\tau_l(b-a)^{h_l},-\frac{(b-a)f_1}{af_1+g_1},\cdots,-\frac{(b-a)f_k}{af_k+g_k}\Bigg)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\prod_{j=1}^{l}\Gamma(\lambda_j)\prod_{j=1}^{k}\Gamma(-\sigma_j)}$$

$$\frac{1}{(2\pi\omega)^{l+k}}\int_{L_1}\cdots\int_{L_{l+k}}\frac{\Gamma\left(\alpha+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}{\Gamma\left(\alpha+\beta+\sum_{j=1}^{l}h_js_j+\sum_{j=1}^{k}s_{l+j}\right)}\prod_{j=1}^{l}\Gamma(\lambda_j+s_j)\prod_{j=1}^{k}\Gamma(-\sigma_j+s_{l+j})$$

$$\prod_{j=1}^{l+k}\Gamma(-s_j)z_1^{s_1}\cdots z_l^{s_l}z_{l+1}^{s_{l+1}}\cdots,z_{l+k}^{s_{l+k}}\,\mathrm{d}s_1\cdots\mathrm{d}s_{l+k} \tag{2.3}$$

Here the contour $L_j's$ are defined by $L_j=L_{w\zeta_j\infty}(Re(\zeta_j)=v_j'')$ starting at the point $v_j''-\omega\infty$ and terminating at the point $v_j''+\omega\infty$ with $v_j''\in\mathbb{R}(j=1,\cdots,l)$ and each of the remaining contour $L_{l+1},\cdots,L_{l+k}$ run from $-\omega\infty$ to $\omega\infty$

(2.2) can be easily established by expanding $\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}$ by means of the formula :

$$(1-z)^{-\alpha}=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{r!}z^r(|z|<1) \tag{2.4}$$

integrating term by term with the help of the integral given by Saigo and Saxena [3, page 93, eq.(3.2)] and applying the definition of the generalized Lauricella function [5, page 454].

3. Eulerian integral

In this section , we note :

$$\theta_i=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}},\zeta_j^{(i)}>0(i=1,\cdots,r)\,;\,\theta_i'=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'^{(i)}},\zeta_j'^{(i)}>0(i=1,\cdots,s)$$

$$\theta_i''=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j''^{(i)}},\zeta_j''^{(i)}>0(i=1,\cdots,u)$$

$$\theta_i'''=\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j'''^{(i)}},\zeta_j'''^{(i)}>0(i=1,\cdots,v) \tag{3.1}$$

$$X=m_1,n_1;\cdots;m_r,n_r;m_1',n_1';\cdots;m_s',n_s';1,0;\cdots;1,0;1,0;\cdots;1,0 \tag{3.2}$$

$$Y=p_1,q_1;\cdots;p_r,q_r;p_1',q_1';\cdots;p_s',q_s';0,1;\cdots;0,1;0,1;\cdots;0,1 \tag{3.3}$$

$$A=(a_j;A_j^{(1)},\cdots,A_j^{(r)},0,\cdots,0,0,\cdots,0,0,\cdots,0)_{1,p} \tag{3.4}$$

$$B=(b_j;B_j^{(1)},\cdots,B_j^{(r)},0\cdots,0,0\cdots,0,0\cdots,0)_{1,q} \tag{3.5}$$

$$A'=(a_j';0,\cdots,0,A_j'^{(1)},\cdots,A_j'^{(s)},0,\cdots,0,0,\cdots,0)_{1,p'} \tag{3.6}$$

$$B'=(b_j';0,\cdots,0,B_j'^{(1)},\cdots,B_j'^{(s)},0,\cdots,0,0,\cdots,0)_{1,q'} \tag{3.7}$$

$$\mathrm{C} = (\mathrm{c}_j^{(1)}, C_j^{(1)})_{1,p_1}; \cdots ; (c_j^{(r)}, C_j^{(r)})_{1,p_r}; (c_j'^{(1)}, C_j'^{(1)})_{1,p_1'}; \cdots ; (c_j'^{(r)}, C_j'^{(s)})_{1,p_s'}$$

$$(1,0); \cdots ; (1,0); (1,0); \cdots ; (1,0) \tag{3.8}$$

$$D = (\mathrm{d}_j^{(1)}, D_j^{(1)})_{1,q_1}; \cdots ; (d_j^{(r)}, D_j^{(r)})_{1,q_r}; (\mathrm{d'}_j^{(1)}, D_j'^{(1)})_{1,q_1'}; \cdots ; (d_j'^{(s)}, D_j'^{(s)})_{1,q_s'};$$

$$(0,1); \cdots ; (0,1); (0,1); \cdots ; (0,1) \tag{3.9}$$

$$K_1 = (1 - \alpha - \sum_{i=1}^{u} R_i a_i - \sum_{i=1}^{v} \eta_{G_i,g_i} a_i'; \mu_1, \cdots, \mu_r, \mu_1', \cdots, \mu_s', h_1, \cdots, h_l, 1, \cdots, 1) \tag{3.10}$$

$$K_2 = (1 - \beta - \sum_{i=1}^{u} R_i b_i - \sum_{i=1}^{v} \eta_{G_i,g_i} b_i'; \rho_1, \cdots, \rho_r, \rho_1', \cdots, \rho_s', 0, \cdots, 0, 0 \cdots, 0) \tag{3.11}$$

$$K_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \zeta_j'''^{(i)}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)},$$

$$0, \cdots, \underset{\mathrm{j}}{1}, \cdots, 0, 0 \cdots, 0]_{1,l} \tag{3.12}$$

$$K_j' = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \eta_{G_i,g_i} \lambda_j'''^{(i)}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)},$$

$$0, \cdots, 0, 0 \cdots, \underset{\mathrm{j}}{1}, \cdots, 0]_{1,k} \tag{3.13}$$

$$L_1 = (1 - \alpha - \beta - \sum_{i=1}^{u} R_i(a_i + b_i) - \sum_{i=1}^{v} (a_i' + b_i') \eta_{G_i,g_i}; \mu_1 + \rho_1, \cdots, \mu_r + \rho_r, \mu_1' + \rho_1', \cdots, \mu_r' + \rho_r',$$

$$h_1, \cdots, h_l, 1, \cdots, 1) \tag{3.14}$$

$$L_j = [1 - \lambda_j - \sum_{i=1}^{u} R_i \zeta_j''^{(i)} - \sum_{i=1}^{s} \zeta_j'''^{(i)} \eta_{G_i,g_i}; \zeta_j^{(1)}, \cdots, \zeta_j^{(r)}, \zeta_j'^{(1)} \cdots, \zeta_j'^{(s)}, 0, \cdots, 0, 0 \cdots, 0]_{1,l} \tag{3.15}$$

$$L_j' = [1 + \sigma_j - \sum_{i=1}^{u} R_i \lambda_j''^{(i)} - \sum_{i=1}^{v} \lambda_j'''^{(i)} \eta_{G_i,g_i}; \lambda_j^{(1)}, \cdots, \lambda_j^{(r)}, \lambda_j'^{(1)} \cdots, \lambda_j'^{(s)}, 0, \cdots, 0, 0, \cdots, 0]_{1,k} \tag{3.16}$$

$$\mathfrak{A} = A, A'; \mathfrak{B} = B, B' \tag{3.17}$$

$$P_1 = (b-a)^{\alpha+\beta-1} \left\{ \prod_{j=1}^{h} (af_j + g_j)^{\sigma_j} \right\} \tag{3.18}$$

$$B_{u,v} = (b-a)^{\sum_{i=1}^{v} (\mu_i + \mu_i' + \rho_i + \rho_i') \eta_{G_i,g_i}} \left\{ \prod_{j=1}^{h} (af_j + g_j)^{-\sum_{i=1}^{v} (\lambda_i + \lambda_i') \eta_{G_i,g_i}} \right\} G_v \tag{3.19}$$

where $G_v = \phi_1 \frac{\prod_{i=1}^{v} \phi_i u_i^{\eta_{G_i,g_i}} (-)^{\sum_{i=1}^{v} g_i}}{\prod_{i=1}^{v} \epsilon_{G_i}^{(i)} g_i!}$

ϕ_1, ϕ_i for $i = 1, \cdots, v$ are defined respectively by (1.2) and (1.3)

$$B_u = \frac{(-L)_{h_1 R_1 + \cdots + h_u R_u} B(E; R_1, \cdots, R_u)}{R_1! \cdots R_u!} \tag{3.20}$$

We have the general Eulerian integral.

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1} \prod_{j=1}^{l} \left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j} \prod_{j=1}^{k} (f_j t + g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u} \left(\begin{array}{c} z_1'' \theta_1'' (t-a)^{a_1} (b-t)^{b_1} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_u'' \theta_u'' (t-a)^{a_u} (b-t)^{b_u} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j''^{(u)}} \end{array} \right)$$

$$A \left(\begin{array}{c} z_1''' \theta_1''' (t-a)^{a_1'} (b-t)^{b_1'} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_v''' \theta_v''' (t-a)^{a_v'} (b-t)^{b_v'} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'''^{(v)}} \end{array} \right)$$

$$A \left(\begin{array}{c} z_1 \theta_1 (t-a)^{\mu_1} (b-t)^{\rho_1} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_r \theta_r (t-a)^{\mu_r} (b-t)^{\rho_r} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j^{(r)}} \end{array} \right)$$

$$A \left(\begin{array}{c} z_1' \theta_1' (t-a)^{\mu_1'} (b-t)^{\rho_1'} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ z_s' \theta_s' (t-a)^{\mu_s'} (b-t)^{\rho_s'} \prod_{j=1}^{k} (f_j t + g_j)^{-\lambda_j'^{(s)}} \end{array} \right) \mathrm{d}t$$

$$= P_1 \frac{\prod_{j=1}^{Q} \Gamma(B_j)}{\prod_{j=1}^{P} \Gamma(A_j)} \sum_{g_1,\cdots,g_v=0}^{\infty} \sum_{M_1=0}^{\alpha^{(1)}} \cdots \sum_{M_v=0}^{\alpha^{(v)}} \sum_{R_1,\cdots,R_u=0}^{h_1 R_1 + \cdots h_u R_u \leqslant L} \prod_{i=1}^{v} z_i'''^{\eta_{h_i,k_i}} \prod_{k=1}^{u} z''^{R_k} B_u B_{u,v}$$

$$A^{m+m',n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A}\ ,\mathrm{K}_1,K_2,K_j,K'_j:C \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(1)}_j}} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(s)}_j}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \tau_l(b-a)^{h_l} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdots & \cdot \\ \cdots & \\ \frac{(b-a)f_k}{af_k+g_k} & \mathfrak{B}\ ,\mathrm{L}_1,L_j,L'_j:D \end{array}\right) \quad (3.21)$$

where $\mathfrak{A},\mathfrak{B},C,D,X,K_1,K_2,K_j,K'_j,L_1,L_j,L'_j,P_1,P_u,B_u$ and $\mathfrak{B}_1$ are defined above.

Provided that

(A) $a,b\in\mathbb{R}(a<b);\mu_i,\mu'_u,\rho_i,\rho'_u,\lambda_j^{(i)},\lambda_j'^{(u)},h_v\in\mathbb{R}^+,f_i,g_j,\tau_v,\sigma_j,\lambda_v\in\mathbb{C}\ \ (i=1,\cdots,r;j=1,\cdots;k;$

$u=1,\cdots,s;v=1,\cdots,l),a_i,b_i,\lambda_j''^{(i)},\zeta_j''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,u;j=1,\cdots,k)$

$a'_i,b'_i,\lambda_j'''^{(i)},\zeta_j'''^{(i)}\in\mathbb{R}^+,(i=1,\cdots,v;j=1,\cdots,k)$

(B) $m,n,p,m_i,n_i,p_i,c_i\in\mathbb{N}^*;i=1,\cdots,r$ **;** $a_j,b_j,c_j^{(i)},d_j^{(i)},A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{C}$

$m',n',p',m'_i,n'_i,p'_i,c'_i\in\mathbb{N}^*;i=1,\cdots,r\ ;a'_j,b'_j,c_j'^{(i)},d_j'^{(i)},A_j'^{(i)},B_j'^{(i)},C_j'^{(i)},D_j'^{(i)}\in\mathbb{C}$

(C) $\max_{1\leqslant j\leqslant k}\left\{\left|\frac{(b-a)f_i}{af_i+g_i}\right|\right\}<1$

(D) $Re\big[\alpha+\sum_{j=1}^{v}a'_j\min_{1\leqslant k\leqslant\alpha^{(i)}}\frac{p_k^{(j)}}{\epsilon_k^{(j)}}+\sum_{j=1}^{r}\mu_j\min_{1\leqslant k\leqslant m_i}\frac{d_k^{(j)}}{D_k^{(j)}}+\sum_{j=1}^{s}\mu'_i\min_{1\leqslant k\leqslant m'_i}\frac{d_k'^{(j)}}{D_k'^{(j)}}\big]>0$

$$Re\big[\beta+\sum_{j=1}^{v}b'_j\min_{1\leqslant k\leqslant\alpha^{(i)}}\frac{p_k^{(j)}}{\epsilon_k^{(j)}}+\sum_{j=1}^{r}\rho_j\min_{1\leqslant k\leqslant m_i}\frac{D_k^{(j)}}{D_k^{(j)}}+\sum_{j=1}^{s}\rho'_j\min_{1\leqslant k\leqslant m'_i}\frac{d_k'^{(j)}}{D_k'^{(j)}}\big]>0$$

(E) $Re\left(\alpha+\sum_{i=1}^{v}\eta_{G_i,g_i}a'_i+\sum_{i=1}^{u}R_ia_i+\sum_{i=1}^{r}\mu_is_i+\sum_{i=1}^{s}t_i\mu'_i\right)>0$

$$Re\left(\beta+\sum_{i=1}^{v}\eta_{G_i,g_i}b'_i+\sum_{i=1}^{u}R_ib_i+\sum_{i=1}^{r}v_is_i+\sum_{i=1}^{s}t_i\rho'_i\right)>0$$

$$Re\left(\lambda_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda_j'''^{(i)}+\sum_{i=1}^{u}R_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\zeta_j^{(i)}+\sum_{i=1}^{s}t_i\zeta_j'^{(i)}\right)>0(j=1,\cdots,l);$$

$$Re\left(-\sigma_j+\sum_{i=1}^{v}\eta_{G_i,g_i}\lambda'''^{(i)}+\sum_{i=1}^{u}R_i\lambda_j''^{(i)}+\sum_{i=1}^{r}s_i\lambda_j^{(i)}+\sum_{i=1}^{s}t_i\lambda_j'^{(i)}\right)>0(j=1,\cdots,k);$$

(F) $|arg(\Omega_i)z_k|<\frac{1}{2}\eta_i\pi,\xi^*=0,\eta_i>0$

$$\Omega_i=\prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}};i=1,\cdots,r$$

$$\xi_i^*=Im(\sum_{j=1}^{p}A_j^{(i)}-\sum_{j=1}^{q}B_j^{(i)}+\sum_{j=1}^{q_i}D_j^{(i)}-\sum_{j=1}^{p_i}C_j^{(i)});i=1,\cdots,r$$

$$\eta_i=Re\left(\sum_{j=1}^{n}A_j^{(i)}-\sum_{j=n+1}^{p}A_j^{(i)}+\sum_{j=1}^{m}B_j^{(i)}-\sum_{j=m+1}^{q}B_j^{(i)}+\sum_{j=1}^{m_i}D_j^{(i)}-\sum_{j=m_i+1}^{q_i}D_j^{(i)}+\sum_{j=1}^{n_i}C_j^{(i)}-\sum_{j=n_i+1}^{p_i}C_j^{(i)}\right)$$

$$-\mu_i-\rho_i-\sum_{l=1}^{k}\lambda_j^{(i)}-\sum_{l=1}^{l}\zeta_j^{(i)}>0\,;\ i=1,\cdots,r$$

$$|arg(\Omega'_i)z'_k|<\frac{1}{2}\eta'_i\pi,\xi'^*=0,\eta'_i>0$$

$$\Omega'_i=\prod_{j=1}^{p'}\{A_j'^{(i)}\}^{A_j'^{(i)}}\prod_{j=1}^{q'}\{B_j'^{(i)}\}^{-B_j'^{(i)}}\prod_{j=1}^{q'_i}\{D_j'^{(i)}\}^{D_j'^{(i)}}\prod_{j=1}^{p'_i}\{C_j'^{(i)}\}^{-C_j'^{(i)}};i=1,\cdots,s$$

$$\xi_i'^*=Im(\sum_{j=1}^{p'}A_j'^{(i)}-\sum_{j=1}^{q'}B_j'^{(i)}+\sum_{j=1}^{q'_i}D_j'^{(i)}-\sum_{j=1}^{p'_i}C_j'^{(i)});i=1,\cdots,s$$

$$\eta'_i=Re\left(\sum_{j=1}^{n'}A_j'^{(i)}-\sum_{j=n'+1}^{p'}A_j'^{(i)}+\sum_{j=1}^{m'}B_j'^{(i)}-\sum_{j=m'+1}^{q'}B_j'^{(i)}+\sum_{j=1}^{m'_i}D_j'^{(i)}-\sum_{j=m'_i+1}^{q'_i}D_j'^{(i)}+\sum_{j=1}^{n'_i}C_j'^{(i)}-\sum_{j=n'_i+1}^{p'_i}C_j'^{(i)}\right)$$

$$-\mu'_i-\rho'_i-\sum_{l=1}^{k}\lambda_j'^{(i)}-\sum_{l=1}^{l}\zeta_j'^{(i)}>0\,;i=1,\cdots,s$$

(H) $\left|arg\left(z_i\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\zeta_j^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j^{(i)}}\right)\right|<\frac{1}{2}\eta_i\pi\ \ (a\leqslant t\leqslant b;i=1,\cdots,r)$

$$\left|arg\left(z'_i\prod_{j=1}^{l}\left[1-\tau'_j(t-a)^{h'_i}\right]^{-\zeta_j'^{(i)}}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda_j'^{(i)}}\right)\right|<\frac{1}{2}\eta'_i\pi\ (a\leqslant t\leqslant b;i=1,\cdots,s)$$

(I) The multiple series occuring on the right-hand side of (3.21) is absolutely and uniformly convergent.

Proof

To prove (3.21), first, we express in serie the multivariable A-function with the help of (1.1), a class of multivariable polynomials defined by Srivastava $S_{N_1,\cdots,N_u}^{M_1,\cdots,M_u}[.]$ in serie with the help of (1.24) and we interchange the order of summations and x-integral (which is permissible under the conditions stated). Expressing the I-functions of r-variables and s-variables defined by Gautam et al [1] in terms of Mellin-Barnes type contour integral with the help of (1.9) and (1.17) respectively and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process . Now collect the power of $\left[1-\tau_j(t-a)^{h_i}\right]$ with $(i=1,\cdots,r;j=1,\cdots,l)$ and collect the power of (f_jt+g_j) with $j=1,\cdots,k$. Use the equations (2.2) and (2.3) and express the result in Mellin-Barnes contour integral. Interpreting the $(r+s+k+l)$ dimensional Mellin-Barnes integral in multivariable A-function defined by Gautam et al [1], we obtain the equation (3.20).

Remarks

If a) $\rho_1=\cdots,\rho_r=\rho'_1=\cdots,\rho'_s=0$; b) $\mu_1=\cdots,\mu_r=\mu'_1=\cdots,\mu'_s=0$, we obtain the similar formulas that (3.21) with the corresponding simplifications.

4. Particular cases

a) If $A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{R}$,$m=0$ and $A'^{(i)}_j,B'^{(i)}_j,C'^{(i)}_j,D'^{(i)}_j\in\mathbb{R}$ and $m'=0$, the multivariable A-functions reduces to multivariable H-functions defined by Srivastava et al [7], we obtain the following result.

$$\int_a^b(t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$S_L^{h_1,\cdots,h_u}\begin{pmatrix} z''_1\theta''_1(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda''^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ z''_u\theta''_u(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda''^{(u)}_j}\end{pmatrix}$$

$$A\begin{pmatrix} z'''_1\theta'''_1(t-a)^{a'_1}(b-t)^{b'_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda'''^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ z'''_v\theta'''_v(t-a)^{a'_v}(b-t)^{b'_v}\prod_{j=1}^k(f_jt+g_j)^{-\lambda'''^{(v)}_j}\end{pmatrix}$$

$$H\begin{pmatrix} z_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda^{(1)}_j}\\ \cdot\\ \cdot\\ \cdot\\ z_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda^{(r)}_j}\end{pmatrix}$$

$$H\left(\begin{array}{c} z'_1\theta'_1(t-a)^{\mu'_1}(b-t)^{\rho'_1}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(1)}_j} \\ . \\ . \\ . \\ z'_s\theta'_s(t-a)^{\mu'_s}(b-t)^{\rho'_s}\prod_{j=1}^{k}(f_jt+g_j)^{-\lambda'^{(s)}_j} \end{array}\right)\mathrm{d}t$$

$$= P_1\frac{\prod_{j=1}^{Q}\Gamma(B_j)}{\prod_{j=1}^{P}\Gamma(A_j)}\sum_{g_1,\cdots,g_v=0}^{\infty}\sum_{M_1=0}^{\alpha^{(1)}}\cdots\sum_{M_v=0}^{\alpha^{(v)}}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{i=1}^{v}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}B_uB_{u,v}$$

$$H^{0,n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A}\ ,\ \mathrm{K}_1, K_2, K_j, K'_j : C \\ \cdots & . \\ \cdots & . \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & . \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(1)}_j}} & . \\ \cdots & . \\ \cdots & . \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda'^{(s)}_j}} & . \\ \tau_1(b-a)^{h_1} & . \\ \cdots & . \\ \cdots & . \\ \tau_l(b-a)^{h_l} & . \\ \frac{(b-a)f_1}{af_1+g_1} & . \\ \cdots & . \\ \cdots & . \\ \frac{(b-a)f_k}{af_k+g_k} & \mathfrak{B}\ ,\ \mathrm{L}_1, L_j, L'_j : D \end{array}\right) \quad (4.1)$$

under the same notations and validity conditions that (3.21) with $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$, $m = 0$ and $A'^{(i)}_j, B'^{(i)}_j, C'^{(i)}_j, D'^{(i)}_j \in \mathbb{R}$ and $m' = 0$

b) If $B(L;R_1,\cdots,R_u) = \dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{R_1\psi'_j+\cdots+R_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}}$ (4.2) then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_u}[z_1,\cdots,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [4]. We have

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}\prod_{j=1}^{l}\left[1-\tau_j(t-a)^{h_i}\right]^{-\lambda_j}\prod_{j=1}^{k}(f_jt+g_j)^{\sigma_j}$$

$$F_{\bar{C}:D';\cdots;D^{(u)}}^{1+\bar{A}:B';\cdots;B^{(u)}}\left(\begin{array}{c} \mathrm{z}_1''\theta_1''(t-a)^{a_1}(b-t)^{b_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_u''\theta_u''(t-a)^{a_u}(b-t)^{b_u}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j''^{(u)}} \end{array}\right)$$

$$\bar{I}\left(\begin{array}{c} \mathrm{z}_1'''\theta_1'''(t-a)^{a_1'}(b-t)^{b_1'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'''^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_v'''\theta_v'''(t-a)^{a_v'}(b-t)^{b_v'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'''^{(v)}} \end{array}\right)$$

$$A\left(\begin{array}{c} \mathrm{z}_1\theta_1(t-a)^{\mu_1}(b-t)^{\rho_1}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r\theta_r(t-a)^{\mu_r}(b-t)^{\rho_r}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j^{(r)}} \end{array}\right)$$

$$A\left(\begin{array}{c} \mathrm{z}_1'\theta_1'(t-a)^{\mu_1'}(b-t)^{\rho_1'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(1)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_s'\theta_s'(t-a)^{\mu_s'}(b-t)^{\rho_s'}\prod_{j=1}^k(f_jt+g_j)^{-\lambda_j'^{(s)}} \end{array}\right)\mathrm{d}t$$

$$=P_1\frac{\prod_{j=1}^Q\Gamma(B_j)}{\prod_{j=1}^P\Gamma(A_j)}\sum_{g_1,\cdots,g_v=0}^{\infty}\sum_{M_1=0}^{\alpha^{(1)}}\cdots\sum_{M_v=0}^{\alpha^{(v)}}\sum_{R_1,\cdots,R_u=0}^{h_1R_1+\cdots h_uR_u\leqslant L}\prod_{i=1}^{v}z_i'''^{\eta_{h_i,k_i}}\prod_{k=1}^{u}z''^{R_k}B_uB_{u,v}$$

$$A^{m+m',n+n'+l+k+2;X}_{p+p'+l+k+2,q+q'+l+k+1;Y}\left(\begin{array}{c|c} \frac{z_1(b-a)^{\mu_1+\rho_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(1)}}} & \mathfrak{A}\ ,\mathrm{K}_1,K_2,K_j,K'_j:C \\ \cdot\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ \frac{z_r(b-a)^{\mu_r+\rho_r}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j^{(r)}}} & \cdot \\ \frac{z'_1(b-a)^{\mu'_1+\rho'_1}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(1)}}} & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ \frac{z'_s(b-a)^{\mu'_s+\rho'_s}}{\prod_{j=1}^{k}(af_j+g_j)^{\lambda_j'^{(s)}}} & \cdot \\ \tau_1(b-a)^{h_1} & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ \tau_l(b-a)^{h_l} & \cdot \\ \frac{(b-a)f_1}{af_1+g_1} & \cdot \\ \cdot\ \cdot\ \cdot & \mathfrak{B}\ ,\mathrm{L}_1,L_j,L'_j:D \\ \cdot\ \cdot\ \cdot & \\ \frac{(b-a)f_k}{af_k+g_k} & \end{array}\right) \quad (4.3)$$

under the same conditions and notations that (3.21)

where $B'_u = \dfrac{(-L)_{h_1R_1+\cdots+h_uR_u}B(E;R_1,\cdots,R_u)}{R_1!\cdots R_u!}$, $B[E;R_1,\ldots,R_v]$ is defined by (4.2)

Remark:

By the following similar procedure, the results of this document can be extented to product of any finite number of multivariable A-functions and a class of multivariable polynomials defined by Srivastava [5].

5. Conclusion

In this paper we have evaluated a generalized Eulerian integral involving the product of three multivariable A-functions defined by Gautam et al [1] and a class of multivariable polynomials defined by Srivastava et al [5] with general arguments. The formulae established in this paper is very general nature. Thus, the results established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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Personal adress : 411 Avenue Joseph Raynaud
Le parc Fleuri , Bat B
83140 , Six-Fours les plages
Tel : 06-83-12-49-68
Department : VAR
Country : FRANCE