Fractional derivative associated with the multivariable I-function, the generalized M-series and multivariable polynomials

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ABSTRACT
The aim of present paper is to derive a fractional derivative of the multivariable I-function of Prasad [4], associated with a general class of multivariable polynomials defined by Srivastava [9], the Aleph-function of one variable, the generalized M-serie and the generalized Lauricella functions defined by Srivastava and Daoust [10]. We will see the case concerning the multivariable H-function. The results derived here are of a very general nature and hence encompass several cases of interest hittherto scattered in the literature.

Keywords : multivariable I-function, Aleph-function, class of multivariable polynomials, fractional derivative, Lauricella function, binomial expansion, H-function of several variables, M-serie.

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1.Introduction and preliminaries.

Shekhawat et al [7,8] , Pandey et al [3] have studied the fractional derivative and fractional integral of product of special functions, respectively. In this present paper, the fractional derivative of the product of the Lauricella function, the general class of polynomials, the generalized M-serie, the multivariable I-function defined by Prasad [4] and the Aleph-function of one variable is derived. In recent years, several authors have found that derivatives and integrals of fractional order are suitable for description of properties of various real materials. The main advantages of fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of varies materials and process.

By Oldham and Spanner [2] the fractional derivative of a function $f(t)$ of complex order γ

$$ {}_aD_t^{\gamma}\left\{f(t)\right\} = \begin{cases} \frac{1}{\Gamma(-\gamma)}\int_0^t (t-x)^{-\gamma-1}f(x)\mathrm{d}x, Re(\gamma)<0 \\ \frac{d^m}{dt^m}\ {}_aD_t^{\gamma-m}\left\{f(t)\right\}, 0\leqslant Re(\gamma)\leqslant m \end{cases} \tag{1.1}$$

Where m is a positive integer.

The Aleph- function , introduced by Südland [13] et al , however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\aleph(z)=\aleph_{P_i,Q_i,c_i;r'}^{M,N}\left(\mathrm{z}\ \middle|\ \begin{matrix}(\mathrm{a}_j,A_j)_{1,\mathfrak{n}},[c_i(a_{ji},A_{ji})]_{\mathfrak{n}+1,p_i;r'}\\ (\mathrm{b}_j,B_j)_{1,m},[c_i(b_{ji},B_{ji})]_{m+1,q_i;r'}\end{matrix}\right)=\frac{1}{2\pi\omega}\int_L \Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)z^{-s}\mathrm{d}s \tag{1.2}$$

for all z different to 0 and

$$\Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)=\frac{\prod_{j=1}^M\Gamma(b_j+B_js)\prod_{j=1}^N\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r'}c_i\prod_{j=N+1}^{P_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}-B_{ji}s)} \tag{1.3}$$

With :

$|argz|<\frac{1}{2}\pi\Omega$ Where $\Omega=\sum_{j=1}^M\beta_j+\sum_{j=1}^N\alpha_j-c_i(\sum_{j=M+1}^{Q_i}\beta_{ji}+\sum_{j=N+1}^{P_i}\alpha_{ji})>0$ with $i=1,\cdots,r'$

For convergence conditions and other details of Aleph-function , see Südland et al [13].
Serie representation of Aleph-function is given by Chaurasia et al [1].

$$\aleph_{P_i,Q_i,c_i;r'}^{M,N}(z) = \sum_{G=1}^{M}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M,N}(s)}{B_G g!}z^{-s} \tag{1.4}$$

With $s = \eta_{G,g} = \dfrac{b_G+g}{B_G}, P_i < Q_i, |z| < 1$ and $\Omega_{P_i,Q_i,c_i;r'}^{M,N}(s)$ is given in (1.2) (1.5)

The generalized polynomials of multivariables defined by Srivastava [9], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.6}$$

where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients are $A[N_1,K_1;\cdots;N_s,K_s]$ arbitrary constants, real or complex.

The generalized M-serie is the extension of the both Mittag-Leffler function and generalized hypergeometric function, see [5,6]

We have :

$${}_p^{\lambda}M_q^{\mu}(c_1,\cdots,c_p;d_1,\cdots,d_q;z) = {}_p^{\lambda}M_q^{\mu}(z) = \sum_{k=0}^{\infty}\frac{(c_1)_k\cdots(c_p)_k}{(d_1)_k\cdots(d_q)_k}\frac{z^k}{\Gamma(\lambda k+\mu)}, z,\lambda,\mu\in\mathbb{C}, Re(\lambda)>0 \tag{1.7}$$

The multivariable I-function defined by Prasad [4] is an extension of the multivariable H-function defined by Srivastava and Panda [11]. She is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots;(\mathrm{a}_{rj};\alpha'_{rj},\cdots,\alpha_{rj}^{(r)})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}} \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r & (\mathrm{b}_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots;(\mathrm{b}_{rj};\beta'_{rj},\cdots,\beta_{rj}^{(r)})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}} \end{array}\right) \tag{1.8}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(s_1,\cdots,s_r)\prod_{i=1}^{s}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.9}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.
The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) +$$

$$+ \left(\sum_{k=1}^{n_r} \alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r} \alpha_{rk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_r} \beta_{rk}^{(i)} \right) \tag{1.10}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1'}, \cdots, |z_r|^{\alpha_r'}), max(|z_1|, \cdots, |z_r|) \rightarrow 0$$

$$I(z_1, \cdots, z_r) \; = 0(|z_1|^{\beta_r'}, \cdots, |z_r|^{\beta_s'}), min(|z_1| , \cdots, |z_r|) \rightarrow \infty$$

where $k = 1, \cdots, z : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k' = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

The Srivastava-Daoust function is defined by (see 10):

$$F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}} \left(\begin{array}{c} z_1 \\ \cdot\cdot\cdot \\ \cdot\cdot\cdot \\ z_r \end{array} \middle| \begin{array}{l} [(\mathrm{a});\theta', \cdots, \theta^{(r)}] : [(b'); \phi']; \cdots ; [(b^{(r)}); \phi^{(r)}] \\ [(\mathrm{c});\psi', \cdots, \psi^{(r)}] : [(d'); \delta']; \cdots ; [(d^{(r)}); \delta^{(r)}] \end{array} \right)$$

$$= \sum_{m_1, \cdots, m_r = 0}^{\infty} B(m_1, \cdots, m_r) z_1^{m_1} \cdots z_r^{m_r} \tag{1.11}$$

where

$$B(m_1, \cdots, m_r) = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{m_1\theta_j' + \cdots + m_r\theta_j^{(r)}} \prod_{j=1}^{B'} (b_j')_{m_1\phi_j'} \cdots \prod_{j=1}^{B^{(r)}} (b_j^{(r)})_{m_r\phi_j^{(r)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{m_1\psi_j' + \cdots + m_r\psi_j^{(r)}} \prod_{j=1}^{D'} (d_j')_{m_1\delta_j'} \cdots \prod_{j=1}^{D^{(r)}} (d_j^{(r)})_{m_r\delta_j^{(r)}}} \tag{1.12}$$

2. Required results

Lemme 1

$${}_aD_t^{\gamma}(t^{\mu}) = \frac{\Gamma(\mu + 1)}{\Gamma(\mu - \gamma + 1)} t^{\mu - \gamma}, Re(\mu) > -1 \tag{2.1}$$

Lemme 2

If $\lambda \geqslant 0, 0 < x < 1, Re(1 + p) > 0, Re(q) > -1, \lambda_i > 0$ and $\Delta_i > 0$, then (see 12)

$$x^{\lambda}F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\left(\begin{array}{c} \mathrm{z}_1x^{\lambda_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_rx^{\lambda_r} \end{array}\right)=\sum_{M=0}^{\infty}\frac{(1+p+q+2M)(-\lambda)_M(1+p)_\lambda}{M!(1+p+q+M)_{\lambda+1}}$$

$$F_M(z_1,\cdots,z_r)\,{}_2F_1\left[\begin{array}{c}\text{-M,1+p+q+M} \\ \text{1+p}\end{array};x\right] \tag{2.2}$$

where $M\geqslant 0$ and

$$F_M(z_1,\cdots,z_r)=F_{\bar{C}+2:D';\cdots;D^{(r)}}^{\bar{A}+2:B';\cdots;B^{(r)}}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_r \end{array}\left|\begin{array}{c} (\text{1+p+}\lambda;\lambda_1,\cdots,\lambda_r),(1+\lambda;\lambda_1,\cdots,\lambda_r), \\ \cdot\ \cdot\ \cdot \\ (\text{2+p+q+M+}\lambda;\lambda_1,\cdots,\lambda_r),(1-M+\lambda;\lambda_1,\cdots,\lambda_r), \end{array}\right.\right.$$

$$\left.\begin{array}{c}[\text{(a)};\theta',\cdots,\theta^{(r)}]:[(b');\phi'];\cdots;[(b^{(r)});\phi^{(r)}] \\ [\text{(c)};\psi',\cdots,\psi^{(r)}]:[(d');\delta'];\cdots;[(d^{(r)});\delta^{(r)}]\end{array}\right) \tag{2.3}$$

Lemme 3

Binomial expansion is given by :

$$(x+\zeta)^{\lambda}=\zeta^{\lambda}\sum_{m=0}\binom{\lambda}{m}\left(\frac{x}{\zeta}\right)^m,\left|\frac{x}{\zeta}\right|<1 \tag{2.4}$$

Lemme 4

$$D_x^{\mu}\left\{f(x)g(x)\right\}=\sum_{m=0}^{\infty}\binom{\mu}{m}D_x^{\mu-m}\left\{f(x)g(x)\right\} \tag{2.5}$$

3. Main result

In this section, the fractional derivative of the product of the Lauricella function, the general class of polynomials, the generalized M-serie, the multivariable I-function defined by Prasad [4] and the Aleph-function of one variable are derived.

Let

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};V=0,n_2;0,n_3;\cdots;0,n_{s-1} \tag{3.1}$$

$$W=(p',q');\cdots;(p^{(r)},q^{(r)});X=(m',n');\cdots;(m^{(r)},n^{(r)}) \tag{3.2}$$

$$A=(a_{2k},\alpha'_{2k},\alpha''_{2k})_{1,p_2};\cdots;(a_{(r-1)k},\alpha'_{(r-1)k},\alpha''_{(r-1)k},\cdots,\alpha^{(r-1)}_{(r-1)k})_{1,p_{r-1}} \tag{3.3}$$

$$B=(b_{2k},\beta'_{2k},\beta''_{2k})_{1,q_2};\cdots;(b_{(r-1)k},\beta'_{(r-1)k},\beta''_{(r-1)k},\cdots,\beta^{(r-1)}_{(r-1)k})_{1,q_{r-1}} \tag{3.4}$$

$$\mathbb{A}=(a_{rk};\alpha'_{rk},\alpha''_{rk},\cdots,\alpha^{(r)}_{rk})_{1,p_r},(-\lambda-\sum_{i=1}^{s}a_iK_i-a_1m'-a_2\eta_{G,g};\lambda_1,\cdots,\lambda_r),$$

$$(-\tau-k-\sum_{i=1}^{s}b_iK_i-b_1m'-b_2\eta_{G,g};\tau_1,\cdots,\tau_r),(-m-l;\lambda_1+\tau_1,\cdots,\lambda_r+\tau_r) \quad (3.5)$$

$$\mathbb{B}=(b_{rk};\beta'_{rk},\beta''_{rk},\cdots,\beta^{(r)}_{rk})_{1,q_r},(m-\lambda-\sum_{i=1}^{s}a_iK_i-a_1m'-a_2\eta_{G,g};\lambda_1,\cdots,\lambda_r),$$

$$(-\eta-\lambda-k-\sum_{i=1}^{s}b_iK_i-b_1m'-b_2\eta_{G,g};\tau_1,\cdots,\tau_r),(-m-l-\mu;\lambda_1+\tau_1,\cdots,\lambda_r+\tau_r) \quad (3.6)$$

$$A'=(a'_k,\alpha'_k)_{1,p'};\cdots;(a^{(r)}_k,\alpha^{(r)}_k)_{1,p^{(r)}};B'=(b'_k,\beta'_k)_{1,p'};\cdots;(b^{(r)}_k,\beta^{(r)}_k)_{1,q^{(r)}} \quad (3.7)$$

$$A_s=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s] \quad (3.8)$$

$$C_{m'}=\frac{(c_1)_{m'}\cdots(c_p)_{m'}}{(d_1)_{m'}\cdots(d_q)_{m'}}\frac{1}{\Gamma(\lambda'm'+\mu')} \quad (3.9)$$

$$\Delta=\frac{(-)^l(-\zeta)^{-1}(1+p+q+2M)(-\lambda)_M}{k!M!(1+p+q+M)_{\lambda+1}(1+p)_k}\times\frac{(1+p+q+M)_k(1+p)_\lambda}{m!l!}x^k(-\zeta)^{\lambda-m+\sum_{i=1}^{s}K_ia_i+a_1m'+a_2\eta_{G,g}}$$

$$(\eta)^{\lambda-m+\sum_{i=1}^{s}K_ib_i+b_1m'+b_2\eta_{G,g}}F_M(\gamma_1,\cdots,\gamma_r) \quad \textbf{(3.10)}$$

then we have the following result :

Theorem

$$D^{\mu}_t\left\{(t-\zeta)^{\lambda}x^{\lambda}(\eta-t)^{\lambda+\tau}\,\aleph^{M',N}_{P_i,Q_i,c_i;r'}((t-\zeta)^{a_2}(\eta-t)^{b_2})\;{}^{\lambda'}_{p}M^{\mu'}_{q}((t-\zeta)^{a_1}(\eta-t)^{b_1})\right.$$

$$\left.F^{\bar{A}:B';\cdots;B^{(r)}}_{\bar{C}:D';\cdots;D^{(r)}}\begin{pmatrix}\gamma_1\{x(\eta-t)\}^{\lambda_1}\\ \cdots\\ \cdots\\ \gamma_r\{x(\eta-t)\}^{\lambda_r}\end{pmatrix}S^{M_1,\cdots,M_s}_{N_1,\cdots,N_s}\begin{pmatrix}(t-\zeta)^{a'_1}(\eta-t)^{b'_1}\\ \cdots\\ \cdots\\ (t-\zeta)^{a'_s}(\eta-t)^{b'_s}\end{pmatrix}I\begin{pmatrix}z_1\{t(t-\zeta)\}^{\lambda_1}\{t(\eta-t)\}^{\tau_1}\\ \cdots\\ \cdots\\ z_r\{t(t-\zeta)\}^{\lambda_r}\{t(\eta-t)\}^{\tau_r}\end{pmatrix}\right\}$$

$$=\sum_{m,l,k,M=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{m'=0}^{\infty}\frac{(-)^g\Omega^{M',N}_{P_i,Q_i,c_i,r'}(\eta_{G,g})}{B_Gg!}A_sC_{m'}\Delta$$

$$I^{V;0,n_r+3:X}_{U;p_r+3,q_r+3:W}\left(\begin{matrix}z_1(-\zeta)^{\lambda_1}\eta^{\tau_1}t^{\lambda_1+\tau_1}\\ \cdots\\ \cdots\\ z_r(-\zeta)^{\lambda_r}\eta^{\tau_r}t^{\lambda_r+\tau_r}\end{matrix}\;\middle|\;\begin{matrix}\mathrm{A}\;;\;\mathbb{A}:\;\mathrm{A'}\\ \cdots\\ \cdots\\ \mathrm{B}\;;\;\mathbb{B}:\;\mathrm{B'}\end{matrix}\right) \quad \textbf{(3.11)}$$

Provided that

$$Re\left[\lambda+m'a_1+a_2\min_{1\leqslant j\leqslant M'}\frac{b_j}{B_j}+\sum_{i=1}^{r}\lambda_i\min_{1\leqslant j\leqslant m^{(i)}}\frac{b^{(i)}_j}{\beta^{(i)}_j}\right]>-1$$

$$Re\left[\tau+m'b_1+b_2\min_{1\leqslant j\leqslant M'}\frac{b_j}{B_j}+\sum_{i=1}^{r}\tau_i\min_{1\leqslant j\leqslant m^{(i)}}\frac{b^{(i)}_j}{\beta^{(i)}_j}\right]>-1$$

$a_1, a_2, b_1, b_2, \lambda_i, \tau_i, a'_j, b'_j > 0, i = 1, \cdots, r, j = 1, \cdots, s, \lambda', \mu' \in \mathbb{C}, Re(\lambda') > 0$

Proof

To prove the theorem, we first express the Srivastava-Daoust function with the help of Lemme 2, express $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[.]$, $\aleph_{P_i,Q_i,c_i;r'}^{M',N}(.)$ and ${}_p^{\lambda'}M_q^{\mu'}(.)$ in series with the help of (1.6), (1.4) and (1.7) respectively and the I-function of r variables defined by Prasad [4] in Mellin-Barnes contour integral with the help of (1.9), changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). Now collect the powers of $(t-\zeta)$ and $(\eta-t)$ and apply the binomial expansion with the help of Lemme 3 and use the Lemme 1. Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

4. Multivariable H-function

If $U = V = A = B = 0$, the multivariable I-function defined by Prasad [4] reduces to multivariable H-function defined by Srivastava et al [11]. We have the following result.

Corollary 1

$$D_t^{\mu}\left\{(t-\zeta)^{\lambda}x^{\lambda}(\eta-t)^{\lambda+\tau}\,\aleph_{P_i,Q_i,c_i;r'}^{M',N}((t-\zeta)^{a_2}(\eta-t)^{b_2})\;{}_p^{\lambda'}M_q^{\mu'}((t-\zeta)^{a_1}(\eta-t)^{b_1})\right.$$

$$\left.F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\begin{pmatrix}\gamma_1\{x(\eta-t)\}^{\lambda_1}\\ \cdots\\ \cdots\\ \gamma_r\{x(\eta-t)\}^{\lambda_r}\end{pmatrix} S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}(t-\zeta)^{a'_1}(\eta-t)^{b'_1}\\ \cdots\\ \cdots\\ (t-\zeta)^{a'_s}(\eta-t)^{b'_s}\end{pmatrix} H\begin{pmatrix}z_1\{t(t-\zeta)\}^{\lambda_1}\{t(\eta-t)\}^{\tau_1}\\ \cdots\\ \cdots\\ z_r\{t(t-\zeta)\}^{\lambda_r}\{t(\eta-t)\}^{\tau_r}\end{pmatrix}\right\}$$

$$= \sum_{m,l,k,M=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G=1}^{M}\sum_{g=0}^{\infty}\sum_{m'=0}^{\infty}\frac{(-)^g\Omega_{P_i,Q_i,c_i,r'}^{M',N}(\eta_{G,g})}{B_G g!}A_s C_{m'}\Delta$$

$$H_{p_r+3,q_r+3:W}^{0,n_r+3:X}\left(\begin{matrix}z_1(-\zeta)^{\lambda_1}\eta^{\tau_1}t^{\lambda_1+\tau_1}\\ \cdots\\ \cdots\\ z_r(-\zeta)^{\lambda_r}\eta^{\tau_r}t^{\lambda_r+\tau_r}\end{matrix}\ \middle|\ \begin{matrix}\mathbb{A} : A'\\ \cdots\\ \cdots\\ \mathbb{B} : B'\end{matrix}\right) \tag{4.1}$$

Under the same notations and conditions that (3.11) with $U = V = A = B = 0$

Corollary 2

If $n_r = p_r = q_r = 0$, **the multivariable H-function breaks into product of Fox's H-function of one variable and we have.**

$$D_t^{\mu}\left\{(t-\zeta)^{\lambda}x^{\lambda}(\eta-t)^{\lambda+\tau}\,\aleph_{P_i,Q_i,c_i;r'}^{M',N}((t-\zeta)^{a_2}(\eta-t)^{b_2})\;{}_p^{\lambda'}M_q^{\mu'}((t-\zeta)^{a_1}(\eta-t)^{b_1})\right.$$

$$F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\begin{pmatrix}\gamma_1\{x(\eta-t)\}^{\lambda_1}\\ \cdots\\ \cdots\\ \gamma_r\{x(\eta-t)\}^{\lambda_r}\end{pmatrix} S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}(t-\zeta)^{a'_1}(\eta-t)^{b'_1}\\ \cdots\\ \cdots\\ (t-\zeta)^{a'_s}(\eta-t)^{b'_s}\end{pmatrix}$$

$$\left.\prod_{i=1}^{r}H_{p^{(i)},q^{(i)}}^{m^{(i)},n^{(i)}}\left(z_i\{t(t-\zeta)\}^{\lambda_i}\{t(\eta-t)\}^{\tau_i}\ \middle|\ \begin{matrix}(a_j^{(i)},\alpha_j^{(i)})_{1,p^{(i)}}\\ \cdots\\ (b_j^{(i)},\beta_j^{(i)})_{1,q^{(i)}}\end{matrix}\right)\right]$$

$$= \sum_{m,l,k,M=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{m'=0}^{\infty} \frac{(-)^g \Omega_{P_i,Q_i,c_i,r'}^{M',N}(\eta_{G,g})}{B_G g!} A_s C_{m'} \Delta$$

$$H_{3,3:W}^{0,3:X}\left(\begin{array}{c|c} z_1(-\zeta)^{\lambda_1}\eta^{\tau_1}t^{\lambda_1+\tau_1} & \mathbb{A}' : A' \\ \cdots & \cdots \\ \cdots & \cdots \\ z_r(-\zeta)^{\lambda_r}\eta^{\tau_r}t^{\lambda_r+\tau_r} & \mathbb{B}' : B' \end{array}\right) \quad \textbf{(4.2)}$$

Under the same notations and conditions that (3.11) with $U = V = A = B = 0 = n_r = p_r = q_r$

where

$$\mathbb{A}' = (-\lambda - \sum_{i=1}^{s} a_i K_i - a_1 m' - a_2\eta_{G,g}; \lambda_1, \cdots, \lambda_r), \quad (-\tau - k - \sum_{i=1}^{s} b_i K_i - b_1 m' - b_2\eta_{G,g}; \tau_1, \cdots, \tau_r),$$

$$(-m - l; \lambda_1 + \tau_1, \cdots, \lambda_r + \tau_r) \quad (4.3)$$

$$\mathbb{B}' = (m - \lambda - \sum_{i=1}^{s} a_i K_i - a_1 m' - a_2\eta_{G,g}; \lambda_1, \cdots, \lambda_r), (-\eta - \lambda - k - \sum_{i=1}^{s} b_i K_i - b_1 m' - b_2\eta_{G,g}; \tau_1, \cdots, \tau_r),$$

$$(-m - l - \mu; \lambda_1 + \tau_1, \cdots, \lambda_r + \tau_r) \quad \textbf{(4.4)}$$

If $\alpha^{(i)} = \beta^{(i)} = 1$**, the Fox's H-function reduces to Meijer's G-function and we obtain**

$$D_t^{\mu}\left\{(t-\zeta)^{\lambda} x^{\lambda} (\eta - t)^{\lambda+\tau}\, \aleph_{P_i,Q_i,c_i;r'}^{M',N}((t-\zeta)^{a_2}(\eta-t)^{b_2})\; {}_{p}^{\lambda'}M_q^{\mu'}((t-\zeta)^{a_1}(\eta-t)^{b_1})\right.$$

$$F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\left(\begin{array}{c} \gamma_1\{x(\eta-t)\}^{\lambda_1} \\ \cdots \\ \cdots \\ \gamma_r\{x(\eta-t)\}^{\lambda_r} \end{array}\right) S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\left(\begin{array}{c} (t-\zeta)^{a'_1}(\eta-t)^{b'_1} \\ \cdots \\ \cdots \\ (t-\zeta)^{a'_s}(\eta-t)^{b'_s} \end{array}\right)$$

$$\left.\prod_{i=1}^{r} G_{p^{(i)},q^{(i)}}^{m^{(i)},n^{(i)}}\left(z_i\{t(t-\zeta)\}^{\lambda_i}\{t(\eta-t)\}^{\tau_i}\,\middle|\, \begin{array}{c} (a_j^{(i)})_{1,p^{(i)}} \\ \cdots \\ (b_j^{(i)})_{1,q^{(i)}} \end{array}\right)\right]$$

$$= \sum_{m,l,k,M=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{m'=0}^{\infty} \frac{(-)^g \Omega_{P_i,Q_i,c_i,r'}^{M',N}(\eta_{G,g})}{B_G g!} A_s C_{m'} \Delta$$

$$H_{3,3:W}^{0,3:X}\left(\begin{array}{c|c} z_1(-\zeta)^{\lambda_1}\eta^{\tau_1}t^{\lambda_1+\tau_1} & \mathbb{A}', (a_j^{(i)})_{1,p^{(i)}} \\ \cdots & \cdots \\ \cdots & \\ z_r(-\zeta)^{\lambda_r}\eta^{\tau_r}t^{\lambda_r+\tau_r} & \mathbb{B}', (b_j^{(i)})_{1,q^{(i)}} \end{array}\right) \quad \textbf{(4.5)}$$

Under the same notations and conditions that (3.11) with $U = V = A = B = 0 = n_r = p_r = q_r$ **and** $\alpha_j^{(i)} = \beta_j^{(i)} = 1$

5. Conclusion

The multivariable I-function defined by Prasad [4] in terms of the Mellin-Barnes contour integrals is most general character which involves a number of special functions of one and several variables. The fractional derivative operator involving various special functions have signifiant importance and applications in Physics, Mechanics ,Biology, etc.

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