Fractional derivative associated with the multivariable I-function, the generalized Wright function and multivariable polynomials

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ABSTRACT
The aim of present paper is to derive a fractional derivative of the multivariable I-function of Prathima [4], associated with a general class of multivariable polynomials defined by Srivastava [8], the I-function of one variable defined by Rathie, the generalized Wright function and the generalized Lauricella functions defined by Srivastava and Daoust [9]. We will see the case concerning the multivariable H-function. The results derived here are of a very general nature and hence encompass several cases of interest hittherto scattered in the literature.

Keywords : multivariable I-function, I-function, class of multivariable polynomials, fractional derivative, Lauricella function, binomial expansion, H-function of several variables, Wright function.

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1. Introduction

Shekhawat et al [6,7] , Pandey et al [3] have studied the fractional derivative and fractional integral of product of special functions, respectively. In this present paper, the fractional derivative of the product of the Lauricella function, the general class of polynomials, the generalized Wright function, the multivariable I-function defined by Prathima et al [4] and the I-function of one variable is derived. In recent years, several authors have found that derivatives and integrals of fractional order are suitable for description of properties of various real materials. The main advantages of fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of varies materials and process.

By Oldham and Spanner [2] the fractional derivative of a function $f(t)$ of complex order γ

$$ {}_aD_t^{\gamma}\left\{f(t)\right\} = \left\{ \begin{array}{l} \frac{1}{\Gamma(-\gamma)}\int_0^t (t-x)^{-\gamma-1}f(x)\mathrm{d}x, Re(\gamma)<0 \\ \\ \frac{d^m}{dt^m}\ {}_aD_t^{\gamma-m}\left\{f(t)\right\}, 0 \leqslant Re(\gamma) \leqslant m \end{array} \right. \tag{1.1}$$

Where m is a positive integer.

The multivariable I-function defined by Prathima [4] is an extension of the multivariable H-function defined by Srivastava and Panda [11]. She is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I_{p,q:p_1,q_1;\cdots;p_r,q_r}^{0,n:m_1,n_1;\cdots;m_r,n_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)},\bar{\delta}_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\bar{\delta_j}^{(r)};D_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

where $\phi(s_1,\cdots,s_r),\theta_i(s_i), i=1,\cdots,r$ are given by :

$$\phi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\bar{\delta}_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\bar{\delta}_j^{(i)}s_i\right)} \tag{1.4}$$

where $i=1,\cdots,r$. Also $z_i\neq 0$ for $i=1,\cdots,r$

The parameters $m_j,n_j,p_j,q_j(j=1,\cdots,r),n,p,q$ are non negative integers (for more details, see Prathima et al [4])

$\alpha_j^{(i)}(j=1,\cdots,p;i=1,\cdots,r),\beta_j^{(i)}(j=1,\cdots,q;i=1,\cdots,r),\gamma_j^{(i)}(j=1,\cdots,p_i;i=1,\cdots,r)$ and $\delta_j^{(i)}$

$(j=1,\cdots,q_i;i=1,\cdots,r)$ are assumed to be positive quantities for standardisation purpose.

$a_j(j=1,\cdots,p),b_j(j=1,\cdots,q),c_j^{(i)}(j=1,\cdots,p_i,i=1,\cdots,r),d_j^{(i)}(j=1,\cdots,q_i,i=1,\cdots,r)$ are complex numbers.

The exposants $A_j(j=1,\cdots,p),B_j(j=1,\cdots,q),C_j^{(i)}(j=1,\cdots,p_i;i=1,\cdots,r),D_j^{(i)}(j=1,\cdots,q_i;i=1,\cdots,r)$

of various gamma function involved in (2.2) and (2.3) may take non integer values.

The contour L_i in the complex s_i-plane is of Mellin Barnes type which runs from $c-i\infty$ to $c+i\infty$ (c real) with indentation, if necessary, in such a manner that all singularities of $\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}s_i\right),j=1,\cdots,m_i$lie to the right and $\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}-\gamma_j^{(i)}s_i\right),j=1,\cdots,n_i$ are to the left of L_i.

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i=\sum_{j=1}^{p}A_j\alpha_j^{(i)}-\sum_{j=1}^{q}B_j\beta_j^{(i)}+\sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=1}^{q_i}D_j^{(i)}\bar{\delta}_j^{(i)}, i=1,\cdots,r \tag{1.5}$$

The integral (2.1) converges absolutely if

$|arg(z_k)|<\frac{1}{2}\Delta_k\pi, k=1,\cdots,r$ where

$$\Delta_k=-\sum_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}\bar{\delta}_j^{(k)}+\sum_{j=1}^{n_k}C_j^{(k)}\gamma_j^{(k)}-\sum_{j=n_k+1}^{p_k}C_j^{(k)}\gamma_j^{(k)}>0 \tag{1.6}$$

The I- function , introduced by Rathie [5], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral :

$$\bar{I}(z)=I_{p,q}^{m,n}\left(\mathrm{z}\ \middle|\ \begin{matrix}(\mathrm{a}_j,\alpha_i;A_j)_{n,n+1},(a_j,\alpha_i;A_j)_p\\ (\mathrm{b}_j,\beta_j;1)_{m,m+1},(b_j,\beta_j;B_j)_q\end{matrix}\right)=\frac{1}{2\pi\omega}\int_L \Omega_{p,q}^{m,n}(s)z^{-s}\mathrm{d}s \tag{1.7}$$

for all z different to 0 and

$$\Omega_{p,q}^{m,n}(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\prod_{j=1}^{n}\Gamma^{A_j}(1-a_j-\alpha_j s)}{\prod_{j=n+1}^{p}\Gamma^{A_i}(a_j+\alpha_j s)\prod_{j=m+1}^{q}\Gamma^{B_j}(1-b_j-\beta_j s)} \tag{1.8}$$

When the poles of $\Gamma(b_j-\beta_j s), j=1,\cdots,m$, are simples the integral (1.8) can be evaluate with the help of the Residue Theorem. We obtain

$$\bar{I}(z) = \sum_{G=1}^{m}\sum_{g=0}^{\infty}\frac{(-)^g\Omega_{p,q}^{m,n}(s)}{B_G g!}z^s \tag{1.9}$$

with $s=\eta_{G,g}=\dfrac{b_G+g}{B_G}, p<q, |z|<1$ and $\Omega_{p,q}^{m,n}(s)$ is given in (1.8)

For more detail, see Rathie [5].

The generalized polynomials of multivariables defined by Srivastava [8], is given in the following manner :

$$S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[y_1,\cdots,y_s] = \sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}$$

$$A[N_1,K_1;\cdots;N_s,K_s]y_1^{K_1}\cdots y_s^{K_s} \tag{1.10}$$

where $M_1,\cdots,M_s$ are arbitrary positive integers and the coefficients are $A[N_1,K_1;\cdots;N_s,K_s]$ arbitrary constants, real or complex.

The generalized Wright function ${}_p\psi_q$ defined for $z\in\mathbb{C}$, $a_i,b_j\in\mathbb{C}$ and $\alpha_i,\beta_j\in\mathbb{R}$ ($\alpha_i,\beta_j\neq 0; i=1,\cdots,p$; $j=1,\cdots,q$) is given by the serie :

$${}_p\psi_q(z) = {}_p\psi_q\left(\begin{array}{c}(\mathrm{a}_i,\alpha_i)_{1,p}\\(\mathrm{b}_j,\beta_j)_{1,q}\end{array}\middle|\ \mathrm{z}\right) = \sum_{l=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+\alpha_i l)z^l}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j l)l!} \tag{1.11}$$

For more details, see Wright [12] and [13].

We will note $b_l = \dfrac{\prod_{i=1}^{p}\Gamma(a_i+\alpha_i l)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j l)}$ (1.12)

The Srivastava-Daoust function is defined by (see 9):

$$F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\left(\begin{array}{c}\mathrm{z}_1\\ \cdots\\ \cdots\\ \mathrm{z}_r\end{array}\middle| \begin{array}{l}[(\mathrm{a});\theta',\cdots,\theta^{(r)}]:[(b');\phi'];\cdots;[(b^{(r)});\phi^{(r)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(r)}]:[(d');\delta'];\cdots;[(d^{(r)});\delta^{(r)}]\end{array}\right)$$

$$= \sum_{m_1,\cdots,m_r=0}^{\infty} B(m_1,\cdots,m_r)z_1^{m_1}\cdots z_r^{m_r} \tag{1.13}$$

where

$$B(m_1,\cdots,m_r) = \frac{\prod_{j=1}^{\bar{A}}(a_j)_{m_1\theta_j'+\cdots+m_r\theta_j^{(r)}}\prod_{j=1}^{B'}(b_j')_{m_1\phi_j'}\cdots\prod_{j=1}^{B^{(r)}}(b_j^{(r)})_{m_r\phi_j^{(r)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi_j'+\cdots+m_r\psi_j^{(r)}}\prod_{j=1}^{D'}(d_j')_{m_1\delta_j'}\cdots\prod_{j=1}^{D^{(r)}}(d_j^{(r)})_{m_r\delta_j^{(r)}}}$$

2. Required results

Lemme 1

$$ {}_aD_t^{\gamma}(t^{\mu}) = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\gamma+1)} t^{\mu-\gamma}, Re(\mu) > -1 \quad (2.1)$$

Lemme 2

If $\lambda \geqslant 0, 0 < x < 1, Re(1+p) > 0, Re(q) > -1, \lambda_i > 0$ and $\Delta_i > 0$, then we have (see 10)

$$ x^{\lambda} F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}} \left(\begin{array}{c} \mathrm{z}_1 x^{\lambda_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_r x^{\lambda_r} \end{array} \right) = \sum_{M=0}^{\infty} \frac{(1+p+q+2M)(-\lambda)_M (1+p)_{\lambda}}{M!(1+p+q+M)_{\lambda+1}} $$

$$ F_M(z_1, \cdots, z_r) {}_2F_1 \left[\begin{array}{c} \text{-M,1+p+q+M} \\ \text{1+p} \end{array} ; x \right] \quad (2.2)$$

with $M \geqslant 0$ and

$$ F_M(z_1, \cdots, z_r) = F_{\bar{C}+2:D';\cdots;D^{(r)}}^{\bar{A}+2:B';\cdots;B^{(r)}} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{z}_r \end{array} \middle| \begin{array}{c} (1\text{+p+}\lambda; \lambda_1, \cdots, \lambda_r), (1+\lambda; \lambda_1, \cdots, \lambda_r), \\ \cdot\ \cdot\ \cdot \\ (2\text{+p+q+M+}\lambda; \lambda_1, \cdots, \lambda_r), (1-M+\lambda; \lambda_1, \cdots, \lambda_r), \end{array} \right. $$

$$ \left. \begin{array}{c} [(\mathrm{a});\theta', \cdots, \theta^{(r)}] : [(b');\phi']; \cdots; [(b^{(r)});\phi^{(r)}] \\ [(\mathrm{c});\psi', \cdots, \psi^{(r)}] : [(d');\delta']; \cdots; [(d^{(r)});\delta^{(r)}] \end{array} \right) \quad (2.3)$$

Lemme 3

Binomial expansion is given by :

$$ (x+\zeta)^{\lambda} = \zeta^{\lambda} \sum_{m=0} \binom{\lambda}{m} \left(\frac{x}{\zeta}\right)^m, \left|\frac{x}{\zeta}\right| < 1 \quad (2.4)$$

Lemme 4

$$ D_x^{\mu} \{f(x)g(x)\} = \sum_{m=0}^{\infty} \binom{\mu}{m} D_x^{\mu-m} \{f(x)g(x)\} \quad (2.5)$$

3. Main result

In this section, the fractional derivative of the product of the Lauricella function, the general class of polynomials, the generalized Wright function, the multivariable I-function defined by Prathima et al [4] and the I-function of one variable are derived.

Let

$$ A = (a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}; A_j)_{1,p}, \quad (-\lambda - \sum_{i=1}^{s} a_i K_i - a_1 m'' - a_2 \eta_{G,g}; \lambda_1, \cdots, \lambda_r; 1), $$

$$(-\tau-k-\sum_{i=1}^{s}b_iK_i-b_1m''-b_2\eta_{G,g};\tau_1,\cdots,\tau_r;1),\ (-m'-l;\lambda_1+\tau_1,\cdots,\lambda_r+\tau_r;1) \tag{3.1}$$

$$B=(b_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q}\ (m'-\lambda-\sum_{i=1}^{s}a_iK_i-a_1m''-a_2\eta_{G,g};\lambda_1,\cdots,\lambda_r;1),$$

$$(-\eta-\lambda-k-\sum_{i=1}^{s}b_iK_i-b_1m''-b_2\eta_{G,g};\tau_1,\cdots,\tau_r;1),\quad (-m'-l-\mu;\lambda_1+\tau_1,\cdots,\lambda_r+\tau_r;1) \tag{3.2}$$

$$C=(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \tag{3.3}$$

$$D=(d_j^{(1)},\bar{\delta}_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\bar{\delta}_j^{(r)};D_j^{(r)})_{1,q_r} \tag{3.4}$$

$$X=m_1,n_1;\cdots;m_r,n_r\ ;Y=p_1,q_1;\cdots;p_r,q_r \tag{3.5}$$

$$A_s=\frac{(-N_1)_{M_1K_1}}{K_1!}\cdots\frac{(-N_s)_{M_sK_s}}{K_s!}A[N_1,K_1;\cdots;N_s,K_s] \tag{3.6}$$

$$\Delta=\frac{(-)^l(-\zeta)^{-1}(1+p+q+2M)(-\lambda)_M}{k!M!(1+p+q+M)_{\lambda+1}(1+p)_k}\times\frac{(1+p+q+M)_k(1+p)_\lambda}{m!l!}\,x^k(-\zeta)^{\lambda-m'+\sum_{i=1}^{s}K_ia_i+a_1m''+a_2\eta_{G,g}}$$

$$(\eta)^{\lambda-m'+\sum_{i=1}^{s}K_ib_i+b_1m''+b_2\eta_{G,g}}\quad F_M(\gamma_1,\cdots,\gamma_r) \tag{3.7}$$

$b_{m''}$ is defined by (1.12)

Then, we have the following result

Theorem

$$D_t^\mu\left\{(t-\zeta)^\lambda x^\lambda(\eta-t)^{\lambda+\tau}\,\bar{I}_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}((t-\zeta)^{a_2}(\eta-t)^{b_2})\ {}_{p'}\psi_{q'}\left((t-\zeta)^{a_1}(\eta-t)^{b_1}\right)\right.$$

$$\left.F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\begin{pmatrix}\gamma_1\{x(\eta-t)\}^{\lambda_1}\\ \cdots\\ \cdots\\ \gamma_r\{x(\eta-t)\}^{\lambda_r}\end{pmatrix}S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}\text{(t-}\zeta)^{a'_1}(\eta-t)^{b'_1}\\ \cdots\\ \cdots\\ \text{(t-}\zeta)^{a'_s}(\eta-t)^{b'_s}\end{pmatrix}I\begin{pmatrix}\mathrm{z}_1\{t(t-\zeta)\}^{\lambda_1}\{t(\eta-t)\}^{\tau_1}\\ \cdots\\ \cdots\\ \mathrm{z}_r\{t(t-\zeta)\}^{\lambda_r}\{t(\eta-t)\}^{\tau_r}\end{pmatrix}\right\}$$

$$=\sum_{m',l,k,M=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G=1}^{\ddot{m}}\sum_{g=0}^{\infty}\sum_{m''=0}^{\infty}\frac{(-)^g\Omega_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}(\eta_{G,g})}{B_Gg!}\quad A_sb_{m''}\Delta$$

$$I_{p+3,q+3:Y}^{0,n+3:X}\left(\begin{array}{c|c}\mathrm{z}_1(-\zeta)^{\lambda_1}\eta^{\tau_1}t^{\lambda_1+\tau_1} & \text{A: C}\\ \cdots & \cdots\\ \cdots & \cdots\\ \mathrm{z}_r(-\zeta)^{\lambda_r}\eta^{\tau_r}t^{\lambda_r+\tau_r} & \text{B : D}\end{array}\right) \tag{3.8}$$

Provided that

$$Re\left[\lambda_i + a_1 m'' + a_2 \min_{1\leqslant j\leqslant \ddot{m}} \frac{b_j}{\beta_j} + \sum_{i=1}^{r} \lambda_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}\right] > -1$$

$$Re\left[\mu_i + a_1 m'' + a_2 \min_{1\leqslant j\leqslant \ddot{m}} \frac{b_j}{\beta_j} + \sum_{i=1}^{r} \mu_i \min_{1\leqslant j\leqslant m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}}\right] > -1$$

$a_i, b_j \in \mathbb{C}, \alpha_i, \beta_j \in \mathbb{R}\,(\alpha_i, \beta_j \neq 0; i = 1, \cdots, p'; j = 1, \cdots, q')$

Proof

To prove the theorem, we first express the Srivastava-Daoust function with the help of Lemme 2, express $S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}[.]$, $\bar{I}_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}(.)$and ${}_{p'}\psi_{q'}(.)$ in series with the help of (1.10), (1.9) and (1.11) respectively and the I-function of r variables defined by Prathima et al [4] in Mellin-Barnes contour integral with the help of (1.2), changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). Now collect the powers of $(t-\zeta)$ and $(\eta - t)$ and apply the binomial expansion with the help of Lemme 3 and use the Lemme 1. Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

4. Multivariable H-function

If $A_j = B_j = C_j^{(i)} = D_j^{(i)} = 1$, the multivariable I-function defined by Prathima et al [4] reduces to multivariable H-function defined by Srivastava et al [11]. We have the following result.

Corollary 1

$$D_t^{\mu}\left\{(t-\zeta)^{\lambda} x^{\lambda} (\eta - t)^{\lambda+\tau}\, \bar{I}_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}((t-\zeta)^{a_2}(\eta-t)^{b_2})\; {}_{p'}\psi_{q'}\left((t-\zeta)^{a_1}(\eta-t)^{b_1}\right)\right.$$

$$\left. F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\begin{pmatrix}\gamma_1\{x(\eta-t)\}^{\lambda_1}\\ \cdots\\ \cdots\\ \gamma_r\{x(\eta-t)\}^{\lambda_r}\end{pmatrix} S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}(t-\zeta)^{a_1'}(\eta-t)^{b_1'}\\ \cdots\\ \cdots\\ (t-\zeta)^{a_s'}(\eta-t)^{b_s'}\end{pmatrix} H\begin{pmatrix}z_1\{t(t-\zeta)\}^{\lambda_1}\{t(\eta-t)\}^{\tau_1}\\ \cdots\\ \cdots\\ z_r\{t(t-\zeta)\}^{\lambda_r}\{t(\eta-t)\}^{\tau_r}\end{pmatrix}\right\}$$

$$= \sum_{m',l,k,M=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{G=1}^{\ddot{m}} \sum_{g=0}^{\infty} \sum_{m''=0}^{\infty} \frac{(-)^g \Omega_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}(\eta_{G,g})}{B_G g!} \quad A_s b_{m''} \Delta$$

$$H_{p+3,q+3:Y}^{0,n+3:X}\left(\begin{array}{c|c} z_1(-\zeta)^{\lambda_1}\eta^{\tau_1}t^{\lambda_1+\tau_1} & \mathrm{A: C}\\ \cdots & \cdots\\ \cdots & \cdots\\ z_r(-\zeta)^{\lambda_r}\eta^{\tau_r}t^{\lambda_r+\tau_r} & \mathrm{B : D}\end{array}\right) \tag{4.1}$$

Under the same notations and conditions that (3.8) with $A_j = B_j = C_j^{(i)} = D_j^{(i)} = 1$

Corollary 2

If $n = p = q = 0$, **the multivariable H-function breaks into product of Fox's H-function of one variable and we have.**

$$D_t^{\mu}\left\{(t-\zeta)^{\lambda} x^{\lambda} (\eta - t)^{\lambda+\tau}\, \bar{I}_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}((t-\zeta)^{a_2}(\eta-t)^{b_2})\; {}_{p'}\psi_{q'}\left((t-\zeta)^{a_1}(\eta-t)^{b_1}\right)\right.$$

$$F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\begin{pmatrix}\gamma_1\{x(\eta-t)\}^{\lambda_1}\\ \cdots\\ \cdots\\ \gamma_r\{x(\eta-t)\}^{\lambda_r}\end{pmatrix} S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}(\text{t-}\zeta)^{a'_1}(\eta-t)^{b'_1}\\ \cdots\\ \cdots\\ (\text{t-}\zeta)^{a'_s}(\eta-t)^{b'_s}\end{pmatrix}$$

$$\prod_{i=1}^{r} H_{p^{(i)},q^{(i)}}^{m^{(i)},n^{(i)}}\left(\mathrm{z}_i\{t(t-\zeta)\}^{\lambda_i}\{t(\eta-t)\}^{\tau_i}\middle|\begin{matrix}(\mathrm{c}_j^{(i)},\gamma_j^{(i)})_{1,p^{(i)}}\\ \cdots\\ (\mathrm{d}_j^{(i)},\bar{\delta}_j^{(i)})_{1,q^{(i)}}\end{matrix}\right)\Bigg]$$

$$=\sum_{m',l,k,M=0}^{\infty}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_s=0}^{[N_s/M_s]}\sum_{G=1}^{\ddot{m}}\sum_{g=0}^{\infty}\sum_{m''=0}^{\infty}\frac{(-)^g\Omega_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}(\eta_{G,g})}{B_G g!}\;A_s b_{m''}\Delta$$

$$H_{3,3:Y}^{0,3:X}\left(\begin{matrix}\mathrm{z}_1(-\zeta)^{\lambda_1}\eta^{\tau_1}t^{\lambda_1+\tau_1}\\ \cdots\\ \cdots\\ \mathrm{z}_r(-\zeta)^{\lambda_r}\eta^{\tau_r}t^{\lambda_r+\tau_r}\end{matrix}\middle|\begin{matrix}\text{A}':\text{C}\\ \cdots\\ \cdots\\ \text{B}':\text{D}\end{matrix}\right) \tag{4.2}$$

Under the same notations and conditions that (3.11) with $A_j=B_j=C_j^{(i)}=D_j^{(i)}=1;n=p=q=0$

where

$$A'=(-\lambda-\sum_{i=1}^{s}a_iK_i-a_1m''-a_2\eta_{G,g};\lambda_1,\cdots,\lambda_r),\quad(-\tau-k-\sum_{i=1}^{s}b_iK_i-b_1m''-b_2\eta_{G,g};\tau_1,\cdots,\tau_r),$$

$$(-m-l;\lambda_1+\tau_1,\cdots,\lambda_r+\tau_r) \tag{4.3}$$

$$B'=(m-\lambda-\sum_{i=1}^{s}a_iK_i-a_1m''-a_2\eta_{G,g};\lambda_1,\cdots,\lambda_r),(-\eta-\lambda-k-\sum_{i=1}^{s}b_iK_i-b_1m''-b_2\eta_{G,g};\tau_1,\cdots,\tau_r),$$

$$(-m-l-\mu;\lambda_1+\tau_1,\cdots,\lambda_r+\tau_r) \tag{4.4}$$

If $\gamma^{(i)}=\bar{\delta}^{(i)}=1$**, the Fox's H-function reduces to Meijer's G-function and we obtain**

$$D_t^{\mu}\left\{(t-\zeta)^{\lambda}x^{\lambda}(\eta-t)^{\lambda+\tau}\;\bar{I}_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}((t-\zeta)^{a_2}(\eta-t)^{b_2})\;{}_{p'}\psi_{q'}\left((t-\zeta)^{a_1}(\eta-t)^{b_1}\right)\right.$$

$$F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\begin{pmatrix}\gamma_1\{x(\eta-t)\}^{\lambda_1}\\ \cdots\\ \cdots\\ \gamma_r\{x(\eta-t)\}^{\lambda_r}\end{pmatrix} S_{N_1,\cdots,N_s}^{M_1,\cdots,M_s}\begin{pmatrix}(\text{t-}\zeta)^{a'_1}(\eta-t)^{b'_1}\\ \cdots\\ \cdots\\ (\text{t-}\zeta)^{a'_s}(\eta-t)^{b'_s}\end{pmatrix}$$

$$\prod_{i=1}^{r} G_{p^{(i)},q^{(i)}}^{m^{(i)},n^{(i)}}\left(\mathrm{z}_i\{t(t-\zeta)\}^{\lambda_i}\{t(\eta-t)\}^{\tau_i}\middle|\begin{matrix}(\mathrm{c}_j^{(i)})_{1,p^{(i)}}\\ \cdots\\ (\mathrm{d}_j^{(i)})_{1,q^{(i)}}\end{matrix}\right)\Bigg]$$

$$= \sum_{m,l,k,M=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_s=0}^{[N_s/M_s]} \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{m'=0}^{\infty} \frac{(-)^g \Omega_{\ddot{p},\ddot{q}}^{\ddot{m},\ddot{n}}(\eta_{G,g})}{B_G g!} A_s C_{m'} \Delta$$

$$H_{3,3:W}^{0,3:X} \left(\begin{array}{c|c} \mathrm{z}_1(-\zeta)^{\lambda_1}\eta^{\tau_1}t^{\lambda_1+\tau_1} & \mathbb{A}',(\mathrm{c}_j^{(i)})_{1,p^{(i)}} \\ \cdots & \cdots \\ \cdots & \\ \mathrm{z}_r(-\zeta)^{\lambda_r}\eta^{\tau_r}t^{\lambda_r+\tau_r} & \mathbb{B}',(\mathrm{d}_j^{(i)})_{1,q^{(i)}} \end{array} \right) \quad \textbf{(4.5)}$$

Under the same notations and conditions that (3.8) with $A_j = B_j = C_j^{(i)} = D_j^{(i)} = \gamma_j^{(i)} = \bar{\delta}_j^{(i)} = 1$ **and** $n = p = q = 0$

5. Conclusion

The multivariable I-function defined by Prathima et al [4] in terms of the Mellin-Barnes contour integrals is most general character which involves a number of special functions of one and several variables. The fractional derivative operator involving various special functions have signifiant importance and applications in Physics, Mechanics ,Biology, etc.

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