# Fractional integral formulae involving the Srivastava-Daoust functions and the multivariable Aleph-function

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ABSTRACT
In this paper, we shall establish two fractional integral formulae involving the product of the Srivastava-Daoust functions and the multivariable Aleph-function. Since these functions includes a large number of special functions as its particular cases, therefore, the result establish here will be serve as key formulae. We shall given the particular cases concerning the multivariable H-function and the Aleph-function of two variables.

Keywords :multivariable Aleph-function, Aleph-function of two variables, multivariable H-function, Riemann-Liouville operator, Erdelyi-Kober operator,Srivastava-Daoust function.

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## 1.Introduction and preliminaries.

The multivariable Aleph-function which was introduced by Ayant [1] is an extension of the multivariable I-function recently defined by C.K. Sharma and Ahmad [4] , itself is a generalization of the multivariable H-function defined by Srivastava et al [7]. The multivariable Aleph-function is defined by means of the multiple contour integral :

We have : $\aleph(z_1,\cdots,z_r)=\aleph^{0,\mathfrak{n}:m_1,n_1,\cdots,m_r,n_r}_{p_i,q_i,\tau_i;R:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|} z_1\\ \cdot\\ \cdot\\ \cdot\\ z_r\end{array}\right.$

$[(a_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)})_{1,\mathfrak{n}}]\ ,[\tau_i(a_{ji};\alpha_{ji}^{(1)},\cdots,\alpha_{ji}^{(r)})_{\mathfrak{n}+1,p_i}]:$
$\ldots\ldots\ldots\ldots\ldots\ldots\ ,[\tau_i(b_{ji};\beta_{ji}^{(1)},\cdots,\beta_{ji}^{(r)})_{m+1,q_i}]:$

$$\left.\begin{array}{c}[(c_j^{(1)}),\gamma_j^{(1)})_{1,n_1}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_i^{(1)}}];\cdots;[(c_j^{(r)}),\gamma_j^{(r)})_{1,n_r}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_i^{(r)}}]\\ [(d_j^{(1)}),\delta_j^{(1)})_{1,m_1}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)})_{m_1+1,q_i^{(1)}}];\cdots;[(d_j^{(r)}),\delta_j^{(r)})_{1,m_r}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)})_{m_r+1,q_i^{(r)}}]\end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega=\sqrt{-1}$

$$\psi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{\mathfrak{n}}\Gamma(1-a_j+\sum_{k=1}^r\alpha_j^{(k)}s_k)}{\sum_{i=1}^R[\tau_i\prod_{j=\mathfrak{n}+1}^{p_i}\Gamma(a_{ji}-\sum_{k=1}^r\alpha_{ji}^{(k)}s_k)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\sum_{k=1}^r\beta_{ji}^{(k)}s_k)]} \tag{1.2}$$

and $$\theta_k(s_k)=\frac{\prod_{j=1}^{m_k}\Gamma(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n_k}\Gamma(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m_k+1}^{q_{i^{(k)}}}\Gamma(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n_k+1}^{p_{i^{(k)}}}\Gamma(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

For more details, see Ayant [1]. The condition for absolute convergence of multiple Mellin-Barnes type contour can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_k| < \frac{1}{2}A_i^{(k)}\pi$ , where

$$A_i^{(k)} = \sum_{j=1}^{\mathfrak{n}} \alpha_j^{(k)} - \tau_i \sum_{j=\mathfrak{n}+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_{i^{(k)}}} \gamma_{ji^{(k)}}^{(k)}$$

$$+ \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_{i^{(k)}}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1, \cdots, r, i = 1, \cdots, R \ , i^{(k)} = 1, \cdots, R^{(k)} \tag{1.4}$$

The complex numbers $z_i$ are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \cdots, z_r) = 0(\,|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}\,), max(\,|z_1|, \cdots, |z_r|\,) \rightarrow 0$$

$$\aleph(z_1, \cdots, z_r) = 0(\,|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}\,), min(\,|z_1|, \cdots, |z_r|\,) \rightarrow \infty$$

where $k = 1, \cdots, r : \alpha_k = min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k = max[Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, n_k$$

For convenience, we will use the following notations in this paper.

$$V = m_1, n_1; \cdots; m_r, n_r \tag{1.5}$$

$$\mathrm{W} = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}, \cdots, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.6}$$

$$A = \{(a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)})_{1,n}\}, \{\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{n+1,p_i}\}, \{(c_j^{(1)}; \gamma_j^{(1)})_{1,n_1}\}$$

$$\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}; \gamma_{ji^{(1)}}^{(1)})_{n_1+1,p_{i^{(1)}}}, \cdots, \{(c_j^{(r)}; \gamma_j^{(r)})_{1,n_r}\}, \tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}; \gamma_{ji^{(r)}}^{(r)})_{n_r+1,p_{i^{(r)}}} \tag{1.7}$$

$$B = \{\tau_i(b_{ji}; \beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_i}\}, \{(d_j^{(1)}; \delta_j^{(1)})_{1,m_1}\}, \tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}; \delta_{ji^{(1)}}^{(1)})_{m_1+1,q_{i^{(1)}}}, \cdots,$$

$$\{(d_j^{(r)}; \delta_j^{(r)})_{1,m_r}\}, \tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}; \delta_{ji^{(r)}}^{(r)})_{m_r+1,q_{i^{(r)}}} \tag{1.8}$$

The contracted form concerning the multivariable Aleph-function writes :

$$\aleph(z_1, \cdots, z_r) = \aleph_{p_i,q_i,\tau_i;R:W}^{0,\mathfrak{n}:V}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \mathrm{z}_r & \mathrm{B} \end{array}\right) \tag{1.9}$$

The familiar fractional integral operator is defined and represented in the present paper as :

$$ {}_cI_x^{\mu}\left\{f(t)\right\} = \frac{1}{\Gamma(\mu)}\int_c^x (x-t)^{\mu-1} f(t)\mathrm{d}t\,,\, Re(\mu)>0 \tag{1.10}$$

the special case of the above operator (when $c=0$) is well known in the literature as Riemann-Liouville fractional integral operator and is written as $I_x^{\mu}\left\{f(t)\right\}$.

Also the fractional integral operator investigated by Erdelyi-Kober is defined and represented as Ross ( [2],1975).

$$ I_x^{\eta,\mu}\left\{f(t)\right\} = \frac{x^{-\eta-\mu+1}}{\Gamma(\mu)}\int_0^x (x-t)^{\mu-1} t^{\eta-1} f(t)\mathrm{d}t\,,\, Re(\mu)>0, \eta>0 \tag{1.11}$$

which is obviously a generalization of the Riemann-Liouville fractional integral operator.

The Srivastava-Daoust function is defined by (see [6]):

$$ F_{\bar{C}:D';\cdots;D^{(r)}}^{\bar{A}:B';\cdots;B^{(r)}}\left(\begin{array}{c|c} \mathrm{z}_1 & [(\mathrm{a});\theta',\cdots,\theta^{(r)}]:[(b');\phi'];\cdots;[(b^{(r)});\phi^{(r)}] \\ \cdot & \\ \cdot & \cdot \\ \mathrm{z}_r & [(\mathrm{c});\psi',\cdots,\psi^{(r)}]:[(d');\delta'];\cdots;[(d^{(r)});\delta^{(r)}] \end{array}\right) = \sum_{m_1,\cdots,m_r=0}^{\infty} \mathbb{A}\frac{z_1^{m_1}\cdots z_r^{m_r}}{m_1!\cdots m_r!} \tag{1.12}$$

where

$$ \mathbb{A} = \frac{\prod_{j=1}^{\bar{A}}(a_j)_{m_1\theta_j'+\cdots+m_r\theta_j^{(r)}} \prod_{j=1}^{B'}(b_j')_{m_1\phi_j'}\cdots\prod_{j=1}^{B^{(r)}}(b_j^{(r)})_{m_r\phi_j^{(r)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi_j'+\cdots+m_r\psi_j^{(r)}} \prod_{j=1}^{D'}(d_j')_{m_1\delta_j'}\cdots\prod_{j=1}^{D^{(r)}}(d_j^{(r)})_{m_r\delta_j^{(r)}}} \tag{1.13}$$

The series given by (1.12) converges absolutely if

$$ 1+\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{A}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0; i=1,\cdots,r \tag{1.14}$$

For more details, see Srivastava and Daoust ([6], 1969).

The binomial expansion is given by

$$ (x+\zeta)^{\lambda} = \zeta^{\lambda}\sum_{m=0}\binom{\lambda}{m}\left(\frac{x}{\zeta}\right)^m, \left|\frac{x}{\zeta}\right|<1 \tag{1.15}$$

We have the following formulae

$$ {}_cI_x^{\mu}\left\{x^{\lambda}\right\} = \sum_{s=0}^{\infty}(-)^s\frac{\Gamma(\lambda+1)(x-c)^{s+\mu}x^{\lambda-s}}{\Gamma(\mu)\Gamma(\lambda-s+1)(s+\mu)s!}, Re(\lambda)>-1 \tag{1.16}$$

$$ I_x^{c,\mu}\left\{x^{\lambda}\right\} = \frac{\Gamma(c+\lambda)}{\Gamma(c\lambda+\mu)}x^{\lambda}, Re(\lambda)>-c \tag{1.17}$$

## 2. Results

We also use the following short notations.

$$\alpha' = \sum_{i=1}^{r} m_i v_i + \sum_{i=1}^{r} m_i' v_i' - l; \beta' = \sum_{i=1}^{r} m_i w_i + \sum_{i=1}^{r} m_i' w_i' - t; \gamma = \sum_{i=1}^{r} m_i u_i + \sum_{i=1}^{r} m_i' u_i' + l + t$$

$$L'' = x^{\sigma+\rho}\beta^{\mu}\mathbb{A}\mathbb{A}' a^{\sum_{i=1}^{r} m_i} b^{\sum_{i=1}^{r} m_i'} \prod_{i=1}^{r} \frac{Z_i^{m_i} Z_i'^{m_i'}}{m_i! m_i'! l! t!}$$

$$\rho' = -\rho - \sum_{i=1}^{r} m_i u_i - \sum_{i=1}^{r} m_i' u_i' - l - t; \sigma' = -\sigma - \sum_{i=1}^{r} m_i v_i + \sum_{i=1}^{r} m_i' v_i'; \mu' = -\mu - \sum_{i=1}^{r} m_i w_i + \sum_{i=1}^{r} m_i' w_i'$$

$$Z_i'' = Z_i x^{U_i} \alpha^{V_i} \beta^{W_i}, i = 1, \cdots, r$$

$$\mathbb{A} = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{m_1\theta_j^{(1)}+\cdots+m_r\theta_j^{(r)}} \prod_{j=1}^{B^{(1)}} (b_j^{(1)})_{m_1\phi_j^{(1)}} \cdots \prod_{j=1}^{B^{(r)}} (b_j^{(r)})_{m_r\phi_j^{(r)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{m_1\psi_j^{(1)}+\cdots+m_r\psi_j^{(r)}} \prod_{j=1}^{D^{(1)}} (d_j^{(1)})_{m_1\delta_j^{(1)}} \cdots \prod_{j=1}^{D^{(r)}} (d_j^{(r)})_{m_r\delta_j^{(r)}}}$$

$$\mathbb{A}' = \frac{\prod_{j=1}^{\bar{A}'} (a_j')_{m_1'\theta_j'^{[1]}+\cdots+m_r'\theta_j'^{(r)}} \prod_{j=1}^{B'^{(1)}} (b_j'^{(1)})_{m_1'\phi_j'^{(1)}} \cdots \prod_{j=1}^{B'^{(r)}} (b_j'^{(r)})_{m_r'\phi_j'^{(r)}}}{\prod_{j=1}^{\bar{C}'} (c_j')_{m_1'\psi_j'+\cdots+m_r'\psi_j^{(r)}} \prod_{j=1}^{D'^{(1)}} (d_j'^{(1)})_{m_1'\delta_j'^{(1)}} \cdots \prod_{j=1}^{D'^{(r)}} (d_j'^{(r)})_{m_r'\delta_j'^{(r)}}}$$

$$F = F_{\bar{C}:D^{(1)};\cdots;D^{(r)}}^{\bar{A}:B^{(1)};\cdots;B^{(r)}}; F' = F_{\bar{C}':D'^{(1)};\cdots;D'^{(r)}}^{\bar{A}':B'^{(1)};\cdots;B'^{(r)}}$$

**Theorem 1**

$${}_cI_x^{\upsilon}\Bigg\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu} F \begin{pmatrix} \mathrm{Z}_1 a x^{\mu_1}(x+\alpha)^{v_1}(x+\beta)^{w_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_r a x^{\mu_r}(x+\alpha)^{v_r}(x+\beta)^{w_r} \end{pmatrix} F' \begin{pmatrix} \mathrm{Z'}_1 b x^{\mu_1'}(x+\alpha)^{v_1'}(x+\beta)^{w_1'} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z'}_r b x^{\mu_r'}(x+\alpha)^{v_r'}(x+\beta)^{w_r'} \end{pmatrix}$$

$$\aleph \begin{pmatrix} \mathrm{Z}_1 x^{U_1}(x+\alpha)^{V_1}(x+\beta)^{W_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_r x^{\mu_r}(x+\alpha)^{V_r}(x+\beta)^{W_r} \end{pmatrix} \Bigg\} = \sum_{m_1,\cdots,m_r,m_1',\cdots,m_r',s,l,t=0}^{\infty} (-)^s \frac{\Gamma(\lambda+1)(x-c)^{s+\upsilon}\alpha^{\alpha'}\beta^{\beta'} L'' x^{\gamma-s}}{\Gamma(\upsilon)(s+\upsilon)s!}$$

$$\aleph_{p_i+3,q_i+3,\tau_i;R:W}^{0,\mathfrak{n}+3:V} \left( \begin{matrix} \mathrm{Z}_1 x^{U_1}\alpha^{V_1}\beta^{W_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ \mathrm{Z}_r x^{U_r}\alpha^{V_r}\beta^{W_r} \end{matrix} \left| \begin{matrix} (\rho'; U_1, \cdots, U_r), (\sigma'; V_1, \cdots, V_r), (\mu'; W_1, \cdots, W_r), A \\ \cdot \\ \cdot \\ (\rho'+l+t; U_1, \cdots, U_r), (\sigma'+l; V_1, \cdots, V_r), (\mu'+t; W_1, \cdots, W_r), B \end{matrix} \right. \right) \quad \textbf{(2.1)}$$

Provided that

$Re(\upsilon) > 0; min\{u_i, v_i, w_i, u_i', v_i'; w_i', U_i, V_i, W_i\} > 0, i = 1, \cdots, r$

$$Re\left[\rho+\sum_{i=1}^{r}U_i\min_{1\leqslant j\leqslant \mathfrak{m}_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}\right]>-1;\left|\frac{x}{\alpha}\right|<\pi,\left|\frac{x}{\beta}\right|<\pi;|argZ_k|<\frac{1}{2}A_i^{(k)}\pi\text{ , where } A_i^{(k)}\text{ is defined by (1.4)}$$

$$1+\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{A}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0,\ 1+\sum_{j=1}^{C'}\psi_j'^{(i)}+\sum_{j=1}^{D'^{(i)}}\delta_j'^{(i)}-\sum_{j=1}^{A'}\theta_j'^{(i)}-\sum_{j=1}^{B'^{(i)}}\phi_j'^{(i)}>0,\ i=1,\cdots,r$$

Theorem 2

$$I_x^{c,v}\left\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}F\left(\begin{array}{c}\mathrm{Z}_1ax^{\mu_1}(x+\alpha)^{v_1}(x+\beta)^{w_1}\\ \cdots\\ \cdots\\ \mathrm{Z}_rax^{\mu_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\end{array}\right)F'\left(\begin{array}{c}\mathrm{Z'}_1bx^{\mu_1'}(x+\alpha)^{v_1'}(x+\beta)^{w_1'}\\ \cdots\\ \cdots\\ \mathrm{Z'}_rbx^{\mu_r'}(x+\alpha)^{v_r'}(x+\beta)^{w_r'}\end{array}\right)\right.$$

$$\left.\aleph\left(\begin{array}{c}\mathrm{Z}_1x^{U_1}(x+\alpha)^{V_1}(x+\beta)^{W_1}\\ \cdots\\ \cdots\\ \mathrm{Z}_rx^{\mu_r}(x+\alpha)^{V_r}(x+\beta)^{W_r}\end{array}\right)\right\}=\sum_{m_1,\cdots,m_r,m_1',\cdots,m_r',l,t=0}^{\infty}\alpha^{\alpha'}\beta^{\beta'}L''x^{\gamma}$$

$$\aleph_{p_i+3,q_i+3,\tau_i;R:W}^{0,\mathfrak{n}+3:V}\left(\begin{array}{c}\mathrm{Z}_1x^{U_1}\alpha^{V_1}\beta^{W_1}\\ \cdots\\ \cdots\\ \mathrm{Z}_rx^{U_r}\alpha^{V_r}\beta^{W_r}\end{array}\left|\begin{array}{c}(1\text{-c-}\rho';U_1,\cdots,U_r),(\sigma';V_1,\cdots,V_r),(\mu';W_1,\cdots,W_r),A\\ .\\ .\\ (1\text{-}v-c-\rho';U_1,\cdots,U_r),(\sigma'+l;V_1,\cdots,V_r),(\mu'+t;W_1,\cdots,W_r),B\end{array}\right.\right)\quad\textbf{(2.2)}$$

Provided that

$$Re(v)>0;min\{u_i,v_i,w_i,u_i',v_i';w_i',U_i,V_i,W_i\}>0,i=1,\cdots,r$$

$$Re\left[\rho+\sum_{i=1}^{r}U_i\min_{1\leqslant j\leqslant \mathfrak{m}_i}\frac{b_j^{(i)}}{\beta_j^{(i)}}\right]>-c;\left|\frac{x}{\alpha}\right|<\pi,\left|\frac{x}{\beta}\right|<\pi;|argZ_k|<\frac{1}{2}A_i^{(k)}\pi\text{ , where } A_i^{(k)}\text{ is defined by (1.4)}$$

$$1+\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{A}\theta_j^{(i)}-\sum_{j=1}^{B^{(i)}}\phi_j^{(i)}>0,\ 1+\sum_{j=1}^{C'}\psi_j'^{(i)}+\sum_{j=1}^{D'^{(i)}}\delta_j'^{(i)}-\sum_{j=1}^{A'}\theta_j'^{(i)}-\sum_{j=1}^{B'^{(i)}}\phi_j'^{(i)}>0,\ i=1,\cdots,r$$

Proof

To establish (2.1), we first express the Srivastava-Daoust functions occuring on the left-hand side in series form given by (1.12) and replace the multivariable Aleph-function by its Mellin-Barnes contour integral (1.1), collecting the power of $x,(x+\alpha)$ and $(x+\beta)$ and applying the binomial expansion (1.15). Further, making use of the result (1.16) and interpreting the resulting Mellin-Barnes-contour integral as the multivariable Aleph-function, we obtain the result (2.1).

Following the procedure (2.1) and using the result (1.17) in place (1.16), we obtain the result (2.2) for Erdelyi-Kober operator defined by (1.11).

## 3. Particular cases

a) The multivariable Aleph-function reduces to multivariable H-function defined by Srivastava et al [7]. We have the two following results.

Corollary 1

$$ {}_{c}I_x^{v}\Bigg\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}F\begin{pmatrix} Z_1ax^{\mu_1}(x+\alpha)^{v_1}(x+\beta)^{w_1} \\ \cdots \\ \cdots \\ Z_rax^{\mu_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\end{pmatrix}F'\begin{pmatrix} Z'_1bx^{\mu'_1}(x+\alpha)^{v'_1}(x+\beta)^{w'_1} \\ \cdots \\ \cdots \\ Z'_rbx^{\mu'_r}(x+\alpha)^{v'_r}(x+\beta)^{w'_r}\end{pmatrix}$$

$$H\begin{pmatrix} Z_1x^{U_1}(x+\alpha)^{V_1}(x+\beta)^{W_1} \\ \cdots \\ \cdots \\ Z_rx^{\mu_r}(x+\alpha)^{V_r}(x+\beta)^{W_r}\end{pmatrix}\Bigg\}=\sum_{m_1,\cdots,m_r,m'_1,\cdots,m'_r,s,l,t=0}^{\infty}(-)^s\frac{\Gamma(\lambda+1)(x-c)^{s+v}\alpha^{\alpha'}\beta^{\beta'}L''x^{\gamma-s}}{\Gamma(v)(s+v)s!}$$

$$H^{0,n+3:V}_{p+3,q+3:W}\left(\begin{matrix} Z_1x^{U_1}\alpha^{V_1}\beta^{W_1} \\ \cdots \\ \cdots \\ Z_rx^{U_r}\alpha^{V_r}\beta^{W_r}\end{matrix}\ \middle|\ \begin{matrix}(\rho';U_1,\cdots,U_r),(\sigma';V_1,\cdots,V_r),(\mu';W_1,\cdots,W_r),A \\ . \\ . \\ (\rho'+l+t;U_1,\cdots,U_r),(\sigma'+l;V_1,\cdots,V_r),(\mu'+t;W_1,\cdots,W_r),B\end{matrix}\right) \quad \textbf{(3.1)}$$

Under the same notations and conditions that (2.1).

Corollary 2

$$ I_x^{c,v}\Bigg\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}F\begin{pmatrix} Z_1ax^{\mu_1}(x+\alpha)^{v_1}(x+\beta)^{w_1} \\ \cdots \\ \cdots \\ Z_rax^{\mu_r}(x+\alpha)^{v_r}(x+\beta)^{w_r}\end{pmatrix}F'\begin{pmatrix} Z'_1bx^{\mu'_1}(x+\alpha)^{v'_1}(x+\beta)^{w'_1} \\ \cdots \\ \cdots \\ Z'_rbx^{\mu'_r}(x+\alpha)^{v'_r}(x+\beta)^{w'_r}\end{pmatrix}$$

$$H\begin{pmatrix} Z_1x^{U_1}(x+\alpha)^{V_1}(x+\beta)^{W_1} \\ \cdots \\ \cdots \\ Z_rx^{\mu_r}(x+\alpha)^{V_r}(x+\beta)^{W_r}\end{pmatrix}\Bigg\}=\sum_{m_1,\cdots,m_r,m'_1,\cdots,m'_r,l,t=0}^{\infty}\alpha^{\alpha'}\beta^{\beta'}L''x^{\gamma}$$

$$H^{0,n+3:V}_{p+3,q+3:W}\left(\begin{matrix} Z_1x^{U_1}\alpha^{V_1}\beta^{W_1} \\ \cdots \\ \cdots \\ Z_rx^{U_r}\alpha^{V_r}\beta^{W_r}\end{matrix}\ \middle|\ \begin{matrix}(1\text{-c-}\rho';U_1,\cdots,U_r),(\sigma';V_1,\cdots,V_r),(\mu';W_1,\cdots,W_r),A \\ . \\ . \\ (1\text{-}\upsilon-c-\rho';U_1,\cdots,U_r),(\sigma'+l;V_1,\cdots,V_r),(\mu'+t;W_1,\cdots,W_r),B\end{matrix}\right) \quad \textbf{(3.2)}$$

Under the same notations and conditions that (2.2).

b) If $r=2$, the multivariable Aleph-function reduces to Aleph-function of two variables defined by Sharma [3] and we have the two following relations.

Corollary 3

$$ {}_{c}I_x^{v}\Bigg\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}F\begin{pmatrix} Z_1ax^{\mu_1}(x+\alpha)^{v_1}(x+\beta)^{w_1} \\ \cdots \\ \cdots \\ Z_2ax^{\mu_r}(x+\alpha)^{v_2}(x+\beta)^{w_2}\end{pmatrix}F'\begin{pmatrix} Z'_1bx^{\mu'_1}(x+\alpha)^{v'_1}(x+\beta)^{w'_1} \\ \cdots \\ \cdots \\ Z'_2bx^{\mu'_2}(x+\alpha)^{v'_2}(x+\beta)^{w'_2}\end{pmatrix}$$

$$\aleph\left(\begin{array}{c} Z_1 x^{U_1}(x+\alpha)^{V_1}(x+\beta)^{W_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ Z_2 x^{\mu_2}(x+\alpha)^{V_2}(x+\beta)^{W_2} \end{array}\right)\Bigg\}= \sum_{m_1,m_2,m'_1,m'_2,s,l,t=0}^{\infty} (-)^s \frac{\Gamma(\lambda+1)(x-c)^{s+v}\alpha^{\alpha'}\beta^{\beta'}L''x^{\gamma-s}}{\Gamma(v)(s+v)s!}$$

$$\aleph^{0,\mathfrak{n}+3:V}_{p_i+3,q_i+3,\tau_i;R:W}\left(\begin{array}{c|c} Z_1 x^{U_1}\alpha^{V_1}\beta^{W_1} & (\rho';U_1,U_2),(\sigma';V_1,V_2),(\mu';W_1,W_2),A \\ \cdot\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ Z_2 x^{U_2}\alpha^{V_2}\beta^{W_2} & (\rho'+l+t;U_1,U_2),(\sigma'+l;V_1,V_2),(\mu'+t;W_1,W_2),B \end{array}\right) \tag{3.3}$$

Under the same notations and conditions that (2.1) with $r=2$.

Corollary 4

$$I_x^{c,v}\Bigg\{x^{\rho}(x+\alpha)^{\sigma}(x+\beta)^{\mu}F\left(\begin{array}{c} Z_1 a x^{\mu_1}(x+\alpha)^{v_1}(x+\beta)^{w_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ Z_2 a x^{\mu_r}(x+\alpha)^{v_2}(x+\beta)^{w_2} \end{array}\right) F'\left(\begin{array}{c} Z'_1 b x^{\mu'_1}(x+\alpha)^{v'_1}(x+\beta)^{w'_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ Z'_2 b x^{\mu'_2}(x+\alpha)^{v'_2}(x+\beta)^{w'_2} \end{array}\right)$$

$$\aleph\left(\begin{array}{c} Z_1 x^{U_1}(x+\alpha)^{V_1}(x+\beta)^{W_1} \\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot \\ Z_2 x^{\mu_2}(x+\alpha)^{V_2}(x+\beta)^{W_2} \end{array}\right)\Bigg\}= \sum_{m_1,m_2,m'_1,m'_2,s,l,t=0}^{\infty} \alpha^{\alpha'}\beta^{\beta'}L''x^{\gamma}$$

$$\aleph^{0,\mathfrak{n}+3:V}_{p_i+3,q_i+3,\tau_i;R:W}\left(\begin{array}{c|c} Z_1 x^{U_1}\alpha^{V_1}\beta^{W_1} & (1\text{-c-}\rho';U_1,U_2),(\sigma';V_1,V_2),(\mu';W_1,W_2),A \\ \cdot\ \cdot\ \cdot & \cdot \\ \cdot\ \cdot\ \cdot & \cdot \\ Z_2 x^{U_2}\alpha^{V_2}\beta^{W_2} & (1\text{-}v-c-\rho';U_1,U_2),(\sigma'+l;V_1,V_2),(\mu'+t;W_1,W_2),B \end{array}\right) \tag{3.4}$$

Under the same notations and conditions that (2.2) with $r=2$.

Remarks

By the following similar procedure, the results of this document can be extented to product of any finite number of Srivastava-Daoust functions.

If $\tau_i, \tau_{i'}, \tau_{i''} \rightarrow 1$ and $r=2$ ,we obtain the results of Sharma et al [5].

## 4. Conclusion

The Srivastava-Daoust functions and the multivariable Aleph-function are quite basic in nature. Therefore on specializing the parameters of these functions, we may obtain various other special functions of several variables and one variable such as multivariable H-function, Fox's H-function , Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, MacRobert's E-function, generalized hypergeometric function, Bessel function of first kind, modied Bessel function, Whittaker function, exponential function , binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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