Euler type triple integrals involving, general class of polynomials and multivariable I-function defined by Prasad

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ABSTRACT
The aim of the present document is to evaluate three triple Euler type integrals involving general class of polynomials, special functions and multivariable I-function defined by Prasad [4]. Importance of our findings lies in the fact that they involve the multivariable I-function, which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them. Further we establish some special cases.

KEYWORDS : I-function of several variables, triple Euler type integrals, special function, general class of polynomials, multivariable H-function Srivastava-Doust polynomial

2010 Mathematics Subject Classification. 33C60, 82C31

1.Introduction

In this paper, we evaluate three triple Eulerian integrals involving the multivariable I-function and class of multivariable polynomials with general arguments.
The multivariable I-function defined by Prasad [4] is a extension of the multivariable H-function defined by Srivastava et al [8]. We will use the contracted form.

The I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p^{(1)},q^{(1)};\cdots;p^{(r)},q^{(r)}}\left(\begin{array}{c|c} z_1 & (a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & (b_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots; \end{array}\right.$$

$$\left.\begin{array}{c} (a_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a^{(1)}_j,\alpha^{(1)}_j)_{1,p^{(1)}};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{rj};\beta^{(1)}_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b^{(1)}_j,\beta^{(1)}_j)_{1,q^{(1)}};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(t_i)z_i^{t_i}\mathrm{d}t_1\cdots\mathrm{d}t_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$|argz_i| < \frac{1}{2}\Omega_i\pi$, where

$$\Omega_i = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \tag{1.3}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}) , max(|z_1|, \cdots, |z_r|) \rightarrow 0$$

$$I(z_1, \cdots, z_r) \; = 0(|z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}) , min(|z_1|, \cdots, |z_r|) \rightarrow \infty$$

where $k = 1, \cdots, r : \alpha'_k = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta'_k = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this section :

$$U_r = p_2, q_2; p_3, q_3; \cdots; p_{r-1}, q_{r-1}; V_r = 0, n_2; 0, n_3; \cdots; 0, n_{r-1} \tag{1.4}$$

$$W_r = (p^{(1)}, q^{(1)}); \cdots; (p^{(r)}, q^{(r)}); X_r = (m^{(1)}, n^{(1)}); \cdots; (m^{(r)}, n^{(r)}) \tag{1.5}$$

$$A = (a_{2k}; \alpha_{2k}^{(1)}, \alpha_{2k}^{(2)}); \cdots; (a_{(r-1)k}; \alpha_{(r-1)k}^{(1)}, \alpha_{(r-1)k}^{(2)}, \cdots, \alpha_{(r-1)k}^{(r-1)}) \tag{1.6}$$

$$B = (b_{2k}; \beta_{2k}^{(1)}, \beta_{2k}^{(2)}); \cdots; (b_{(r-1)k}; \beta_{(r-1)k}^{(1)}, \beta_{(r-1)k}^{(2)}, \cdots, \beta_{(r-1)k}^{(r-1)}) \tag{1.7}$$

$$\mathfrak{A} = (a_{rk}; \alpha_{rk}^{(1)}, \alpha_{rk}^{(2)}, \cdots, \alpha_{rk}^{(r)}) : \mathfrak{B} = (b_{rk}; \beta_{rk}^{(1)}, \beta_{rk}^{(2)}, \cdots, \beta_{rk}^{(r)}) \tag{1.8}$$

$$A_1 = (a_k^{(1)}, \alpha_k^{(1)})_{1,p^{(1)}}; \cdots; (a_k^{(r)}, \alpha_k^{(r)})_{1,p^{(r)}} ; B_1 = (b_k^{(1)}, \beta_k^{(1)})_{1,q^{(1)}}; \cdots; (b_k^{(r)}, \beta_k^{(r)})_{1,q^{(r)}} \tag{1.9}$$

;

The multivariable I-function of r-variables write :

$$I(z_1, \cdots, z_r) = I_{U_r:p_r,q_r;W_r}^{V_r;0,n_r;X_r} \left(\begin{array}{c|c} z_1 & A ; \mathfrak{A}; A_1 \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & B; \mathfrak{B}; B_1 \end{array} \right) \tag{1.10}$$

Srivastava and Garg [6] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]=\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{z_1^{R_1}\cdots z_s^{R_s}}{R_1!\cdots R_s!} \quad (1.11)$$

The coefficients $B(E;R_1,\cdots,R_s)$ are arbitrary constants, real or complex.

We will note :$B_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ (1.12)

2 . Results required :

a) $\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x=\dfrac{\pi\Gamma(c)\Gamma(a+b+1/2)\Gamma(c-a-b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma(c-a+1/2)\Gamma(c-b+1/2)}$ (2.1)

Where Re(c) > 0, Re(2c-a-b) > -1 , see Vyas and Rathie [9].

Erdélyi [1] [p.78, eq.(2.4) (1), vol 1]

b) $\int_0^1\int_0^1 t^{b-1}r^{a-1}(1-t)^{c-b-1}(1-r)^{c-a-1}(1-trz)^{-c}\,\mathrm{d}r\mathrm{d}t$

$$=\frac{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}{[\Gamma(c)]^2}\,{}_2F_1(a,b;c;z) \quad (2.2)$$

$Re(a)>0, Re(b)>0, Re(c-a)>0, Re(c-b)>0$

Erdélyi [1] [p.230, eq.(5.8.1) (2), vol 1]

c) $\int_0^1\int_0^1 u^{\beta-1}v^{\beta'-1}(1-u)^{\gamma-\beta-1}(1-v)^{\gamma'-\beta'-1}(1-ux-vy)^{-\alpha}\,\mathrm{d}u\mathrm{d}v$

$$=\frac{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta)\Gamma(\gamma'-\beta')}{\Gamma(\gamma)\Gamma(\gamma')}F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y) \quad (2.3)$$

$Re(\beta)>0, Re(\beta')>0, Re(\gamma-\beta)>0, Re(\gamma'-\beta')>0$

Erdélyi [1] [p.230, eq.(5.8.1) (4), vol 1]

d) $\int_0^1\int_0^1 u^{\alpha-1}v^{\beta-1}(1-u)^{\gamma-\alpha-1}(1-v)^{\gamma'-\beta-1}(1-ux)^{\alpha-\gamma-\gamma'+1}(1-vy)^{\beta-\gamma-\gamma'+1}$

$(1-ux-vy)^{\gamma+\gamma'-\alpha-\beta-1}\mathrm{d}u\mathrm{d}v$

$$= \frac{\Gamma(\beta)\Gamma(\alpha)\Gamma(\gamma-\alpha)\Gamma(\gamma'-\beta)}{\Gamma(\gamma)\Gamma(\gamma')} F_4(\alpha,\beta,\gamma,\gamma';x(1-y),y(1-x)) \tag{2.4}$$

$Re(\beta)>0, Re(\alpha)>0, Re(\gamma-\alpha)>0, Re(\gamma'-\beta)>0$

3. Main results

Theorem 1

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c} \mathrm{y}_1 x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1} \\ \cdots \\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r} \end{array}\right) \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$= \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k=0}^{\infty}\frac{t^k}{k!} B_s y_1^{R_1}\cdots y_s^{R_s} I_{U_r:p_r+6,q_r+4;W_r}^{V_r;0,n_r+6;X_r}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array}\middle| \begin{array}{c} \mathrm{A} \\ \cdots \\ \mathrm{B} \end{array}\right.$$

$$(1-c-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r), \quad (\tfrac{1}{2}-c+a+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots \qquad \cdots$$
$$(\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),(\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1-\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),$$
$$\cdots$$
$$\cdots$$

$$(1-\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r),$$

$$\left.\begin{array}{c}(1-\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),\mathfrak{A}:A' \\ \cdots \\ \mathfrak{B}:\mathrm{B}'\end{array}\right) \tag{3.1}$$

Provided that :

$Re(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$

$Re(2(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$

$Re(\beta+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)>0$

$Re(\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)>0$

$Re(\lambda-\alpha+(\mu_1-\zeta')R_1+\cdots+(\mu_s-\zeta^{(s)})R_s+(\eta_1-\zeta_1)s_1+\cdots+(\eta_r-\zeta_r)s_r)>0$

$Re(\lambda-\beta+(\mu_1-\rho')R_1+\cdots+(\mu_s-\rho^{(s)})R_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$

$|argz_i|<\frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.3)

Theorem 2

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}$$

$$(1-uy-vz)^{-n}\,S_L^{h_1,\cdots,h_s}\begin{pmatrix} \mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'} \\ \cdots \\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}} \end{pmatrix}$$

$$I\begin{pmatrix} \mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1} \\ \cdots \\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r} \end{pmatrix}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}B_sy_1^{R_1}\cdots y_s^{R_s}I_{U_r:p_r+7,q_r+5;W_r}^{V_r;0,n_r+7;X_r}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathrm{A} \\ \cdots & \cdots \\ \mathrm{z}_r & \mathrm{B} \end{array}\right)$$

$$(1\text{-c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ (1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots\qquad\cdots$$
$$(\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ (1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r)$$
$$\cdots$$
$$\cdots$$

$$(1\text{- }\mu-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r)$$
$$\cdots$$
$$(1\text{-n-}\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),(1-\lambda-e'R_1-\cdots-e^{(s)}R_s-k;\eta_1,\cdots,\eta_r)$$

$$(1\text{-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),(1-n-\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),$$
$$\cdots$$
$$\cdots$$

$$\left.\begin{matrix}(1\text{-}\alpha-m-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),\mathfrak{A}:A'\\ \cdots\\ (1\text{-}\mu-m-t'R_1-\cdots-t^{(s)}R_s;t_1,\cdots,t_r),\mathfrak{B}:B'\end{matrix}\right) \tag{3.2}$$

Provided that :

$Re(c+c_1R_1+\cdots+C_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$

$Re(2(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$

$Re(\beta+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)>0$

$Re(\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)>0$

$Re(\lambda-\beta+(e'-\rho')R_1+\cdots+(e^{(s)}-\rho^{(s)})R_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$

$Re(\mu-\alpha+(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+(t_1-\zeta_1)s_1+\cdots+(t_r-\zeta_r)s_r)>0$

$|argz_i|<\frac{1}{2}\Omega_i\pi$, where Ω_i is defined by (1.3)

Theorem 3

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{y}_1x^{\sigma'}y^{\rho'}z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}}\end{pmatrix}$$

$$I\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r}\end{pmatrix}$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$\sum_{k,m=0}^{\infty}\frac{u^k(1-v)^kv^m(1-u)^m}{k!m!}B_sy_1^{R_1}\cdots y_s^{R_s}I^{V_r;0,n_r+6;X_r}_{U_r:p_r+6,q_r+4;W_r}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\ \middle|\ \begin{matrix}\mathrm{A}\\ \cdots\\ \mathrm{B}\end{matrix}\right.$$

$$(1\text{ -c -}\sigma' R_1 - \cdots - \sigma^{(k)} R_s; \sigma_1, \cdots, \sigma_r), \quad (\tfrac{1}{2}\text{-c - }\sigma' R_1 - \cdots - \sigma^{(s)} R_s + a + b; \sigma_1, \cdots, \sigma_r),$$

$$\cdots \qquad\qquad \cdots$$

$$(\tfrac{1}{2}\text{ -c- }\sigma' R_1 - \cdots - \sigma^{(k)} R_s + a; \sigma_1, \cdots, \sigma_r), \quad (\tfrac{1}{2}\text{ -c - }\sigma' R_1 - \cdots - \sigma^{(s)} R_s + b; \sigma_1, \cdots, \sigma_r),$$

$$(1\text{ -}\mu - (t' - \zeta')R_1 + \cdots + (t^{(s)} - \zeta^{(s)})R_s + \beta; t_1 - \zeta_1, \cdots, t_r - \zeta_r),$$

$$\cdots$$

$$(1\text{-}\lambda - (\eta' - \rho')R_1 - \cdots - (\eta^{(s)} - \rho^{(s)})R_s - k; \eta_1, \cdots, \eta_r),$$

$$(1\text{ -}\lambda + \alpha - (\eta' - \rho')R_1 - \cdots - (\eta^{(s)} - \rho^{(s)})R_s; \eta_1 - \rho_1, \cdots, \eta_r - \rho_r),$$

$$\cdots$$

$$(1\text{ -}\mu - m - (t' - \zeta')R_1 - \cdots - (t^{(s)} - \zeta^{(s)})R_s - m; t_1, \cdots, t_r),$$

$$(1\text{ -}\alpha - k - \rho' R_1 - \cdots - \rho^{(s)} R_s - m; \rho_1, \cdots, \rho_r),$$

$$\cdots$$

$$, \cdots\cdots$$

$$\left.(1\text{ -}\beta - k - \zeta' R_1 - \cdots - \zeta^{(s)} R_s; \zeta_1, \cdots, \zeta_r), \mathfrak{A} : A' \atop {\cdots \atop \mathfrak{B}: \text{B'}}\right) \tag{3.3}$$

Provided that :

$$Re(c + \sigma' R_1 + \cdots + \sigma_s^{(s)} R_s + \sigma_1 s_1 + \cdots + \sigma_r s_r) > 0;$$

$$Re(2(c + \sigma_1' R_1 + \cdots + \sigma^{(s)} R_s + \sigma_1 s_1 + \cdots + \sigma_r s_r) - a - b) > -1$$

$$Re(\alpha + \rho' R_1 + \cdots + \rho^{(s)} R_s + \rho_1 s_1 + \cdots + \rho_r s_r) > 0$$

$$Re(\beta + \zeta' R_1 + \cdots + \zeta^{(s)} R_s + \zeta_1 s_1 + \cdots + \zeta_r s_r) > 0$$

$$Re(\lambda - \alpha + (\eta' - \rho')R_1 + \cdots + (\eta^{(s)} - \rho^{(s)})R_s + (\eta_1 - \rho_1)s_1 + \cdots + (\eta_r - \rho_r)s_r) > 0$$

$$Re(\mu - \beta + (t' - \zeta')R_1 + \cdots + (t^{(s)} - \zeta^{(s)})R_s + (t_1 - \zeta_1)s_1 + \cdots + (t_r - \zeta_r)s_r) > 0$$

$|argz_i| < \dfrac{1}{2}\Omega_i \pi$, where Ω_i is defined by (1.3)

Proof de (3.1) : Fisrt we use series representation (1.11) for $S_L^{h_1,\cdots,h_s}[.]$ and expressing the multivariable I -function defined by Prasad [4] involving in the left hand side of (3.1) in terms of Mellin-Barnes contour integral with the help of (1.1) and then interchanching the order of integration. We get L.H.S.

$$= \frac{1}{(2\pi\omega)^r}\left(\int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \sum_{R_1,\cdots,R_s=0}^{h_1 R_1 + \cdots h_s R_s \leqslant L} B_s y_1^{R_1} \cdots y_s^{R_s}\right.$$

$$\left(\int_0^1 x^{c + c_1 R_1 + \cdots + c_s R_s + \sigma_1 s_1 + \cdots + \sigma_r s_r - 1}(1 - x)^{-1/2} {}_2F_1(a, b; a + b + 1/2; x)\mathrm{d}x\right)$$

$$\times\left(\int_0^1\int_0^1 y^{\beta+\rho' R_1+\cdots+\rho^{(s)}R_s+\rho_1 s_1+\cdots+\rho_r s_r}z^{\alpha+\zeta' R_1+\cdots+\zeta^{(s)}R_s+\zeta_1 s_1+\cdots+\zeta_r s_r-1}\right.$$

$$(1-yzt)^{-(\lambda+\mu_1 R_1+\cdots+\mu_s R_s+\eta_1 s_1+\cdots+\eta_r s_r)}$$

$$\times(1-y)^{(\lambda+\mu_1 R_1+\cdots+\mu_s R_s+\eta_1 s_1+\cdots+\eta_r s_r)-(\beta+\rho' R_1+\cdots\rho^{(s)}R_s+\rho_1 s_1+\cdots+\rho_r s_r)-1}$$

$$\left.\times(1-z)^{(\lambda+\mu_1 R_1+\cdots+\mu_s R_s+\eta_1 s_1+\cdots+\eta_r s_r)-(\alpha+\zeta' R_1+\cdots+\zeta^{(s)}R_s+\zeta_1 s_1+\cdots+\zeta_r s_r)-1}\mathrm{d}y\mathrm{d}z\right)\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Now using the result (2.1), (2.2) and (1.1) we get right hand side of (3.1). Similarly we can prove (3.2) and (3.3) with help of the results (2.3) and (2.4).

4. Multivariable H-function

If $U_r = V_r = A = B = 0$, the multivariable I-function reduces to the multivariable H-function defined by Srivastava et al [7] and we obtain the following result.

Corollary 1

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c}\mathrm{y}_1 x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1}\\ \cdots\\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s}\end{array}\right)$$

$$H\left(\begin{array}{c}\mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1}\\ \cdots\\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r}\end{array}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k=0}^{\infty}\frac{t^k}{k!}B_s y_1^{R_1}\cdots y_s^{R_s}H^{0,n_r+6;X_r}_{p_r+6,q_r+4;W_r}\left(\begin{array}{c}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{array}\right|$$

$$\begin{array}{c}(1-c-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\quad(\tfrac{1}{2}-c+a+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\cdots\\ (\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),(\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\end{array}$$

$$\begin{array}{c}(1-\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),\\ \cdots\\ \cdots\end{array}$$

$$(1\text{-}\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1\text{-}\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r),$$

$$\left.\begin{matrix}(1\text{-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),\mathfrak{A}:A'\\ \cdots\\ \mathfrak{B}:B'\end{matrix}\right) \tag{4.1}$$

under the same conditions and notations that (3.1) with $U_r=V_r=A=B=0$

Corollary 2

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}$$

$$(1-uy-vz)^{-n}\,S_L^{h_1,\cdots,h_s}\left(\begin{matrix}\mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}}\end{matrix}\right)$$

$$H\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r}\end{matrix}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}\,B_sy_1^{R_1}\cdots y_s^{R_s}H^{0,n_r+7;X_r}_{p_r+7,q_r+5;W_r}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\right|$$

$$(1\text{ -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ \ (1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots\qquad\qquad\cdots$$
$$(\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ \ (1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r)$$
$$\cdots$$
$$\cdots$$

$$(1\text{- }\mu-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r)$$
$$\cdots$$
$$(1\text{-n-}\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),(1-\lambda-e'R_1-\cdots-e^{(s)}R_s-k;\eta_1,\cdots,\eta_r)$$

$$(1\text{ -}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),(1-n-\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),$$
$$\cdots$$
$$\cdots$$

$$\left.\begin{matrix} (1\text{ -}\alpha - m - \zeta' R_1 - \cdots - \zeta^{(s)} R_s; \zeta_1, \cdots, \zeta_r), \mathfrak{A} : A' \\ \cdots \\ (1\text{ -}\mu - m - t' R_1 - \cdots - t^{(s)} R_s; t_1, \cdots, t_r), \mathfrak{B} : B' \end{matrix}\right) \tag{4.2}$$

under the same conditions and notations that (3.2) with $U_r = V_r = A = B = 0$

Corollary 3

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{matrix} \mathrm{y}_1 x^{\sigma'} y^{\rho'} z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'} \\ \cdots \\ \mathrm{y}_s x^{c_s} y^{\rho^{(s)}} z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}} \end{matrix}\right)$$

$$H\left(\begin{matrix} \mathrm{z}_1 x^{\sigma_1} y^{\rho_1} z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r} y^{\rho_r} z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r} \end{matrix}\right)$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$= \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} \sum_{k,m=0}^{\infty} \frac{u^k(1-v)^k v^m(1-u)^m}{k!m!} B_s y_1^{R_1}\cdots y_s^{R_s} H^{0,n_r+6;X_r}_{p_r+6,q_r+4;W_r}\left(\begin{matrix} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{matrix}\right|$$

$$\begin{matrix} (1\text{ -c -}\sigma' R_1 - \cdots - \sigma^{(k)} R_s; \sigma_1, \cdots, \sigma_r), \quad (\tfrac{1}{2}\text{-c - }\sigma' R_1 - \cdots - \sigma^{(s)} R_s + a + b; \sigma_1, \cdots, \sigma_r), \\ \cdots \qquad\qquad \cdots \\ (\tfrac{1}{2}\text{ -c- }\sigma' R_1 - \cdots - \sigma^{(k)} R_s + a; \sigma_1, \cdots, \sigma_r), \quad (\tfrac{1}{2}\text{ -c - }\sigma' R_1 - \cdots - \sigma^{(s)} R_s + b; \sigma_1, \cdots, \sigma_r), \end{matrix}$$

$$\begin{matrix} (1\text{ -}\mu - (t'-\zeta')R_1 + \cdots + (t^{(s)}-\zeta^{(s)})R_s + \beta; t_1-\zeta_1, \cdots, t_r-\zeta_r), \\ \cdots \\ (1\text{-}\lambda - (\eta'-\rho')R_1 - \cdots - (\eta^{(s)}-\rho^{(s)})R_s - k; \eta_1, \cdots, \eta_r), \end{matrix}$$

$$\begin{matrix} (1\text{ -}\lambda + \alpha - (\eta'-\rho')R_1 - \cdots - (\eta^{(s)}-\rho^{(s)})R_s; \eta_1-\rho_1, \cdots, \eta_r-\rho_r), \\ \cdots \\ (1\text{ -}\mu - m - (t'-\zeta')R_1 - \cdots - (t^{(s)}-\zeta^{(s)})R_s - m; t_1, \cdots, t_r), \end{matrix}$$

$$\begin{matrix} (1\text{ -}\alpha - k - \rho' R_1 - \cdots - \rho^{(s)} R_s - m; \rho_1, \cdots, \rho_r), \\ \cdots \\ , \cdots\cdots \end{matrix}$$

$$(1 \text{ -}\beta - k - \zeta' R_1 - \cdots - \zeta^{(s)} R_s; \zeta_1, \cdots, \zeta_r), \mathfrak{A} : A' \\ \cdots \\ \mathfrak{B}: \text{B'} \Bigg) \tag{4.3}$$

under the same conditions and notations that (3.2) with $U_r = V_r = A = B = 0$

5. Srivastava-Daoust polynomial

$$\text{If } B(L; R_1, \cdots, R_s) = \frac{\prod_{j=1}^{\bar{A}} (a_j)_{R_1\theta'_j + \cdots + R_s\theta_j^{(s)}} \prod_{j=1}^{B'} (b'_j)_{R_1\phi'_j} \cdots \prod_{j=1}^{B^{(s)}} (b_j^{(s)})_{R_s\phi_j^{(s)}}}{\prod_{j=1}^{\bar{C}} (c_j)_{m_1\psi'_j + \cdots + m_s\psi_j^{(s)}} \prod_{j=1}^{D'} (d'_j)_{R_1\delta'_j} \cdots \prod_{j=1}^{D^{(s)}} (d_j^{(s)})_{R_s\delta_j^{(s)}}} \tag{5.1}$$

then the general class of multivariable polynomial $S_L^{h_1, \cdots, h_s}[z_1, \cdots, z_s]$ reduces to generalized Srivastava-Daoust polynomial defined by Srivastava et al [5].

$$F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}} \left(\begin{matrix} \text{z}_1 \\ \cdots \\ \cdots \\ \text{z}_s \end{matrix} \middle| \begin{matrix} [(\text{-L});\text{R}_1, \cdots, R_s][(a); \theta', \cdots, \theta^{(s)}] : [(b'); \phi']; \cdots; [(b^{(s)}); \phi^{(s)}] \\ [(\text{c});\psi', \cdots, \psi^{(s)}] : [(d'); \delta']; \cdots; [(d^{(s)}); \delta^{(s)}] \end{matrix} \right) \tag{5.2}$$

and we have the following formulas

Corollary 4

$$\int_0^1 \int_0^1 \int_0^1 x^{c-1}(1-x)^{-1/2} {}_2F_1(a, b; a+b+1/2; x) y^{\beta-1} z^{\alpha-1} (1-y)^{\lambda-\beta-1} (1-z)^{\lambda-\alpha-1} (1-yzt)^{-\lambda}$$

$$F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}} \left(\begin{matrix} \text{y}_1 x^{c_1} y^{\rho'} z^{\zeta'} (1-y)^{\mu_1-\rho'} (1-z)^{\mu_1-\zeta'} (1-yzt)^{-\mu_1} \\ \cdots \\ \text{y}_s x^{c_s} y^{\rho^{(s)}} z^{\zeta^{(s)}} (1-y)^{\mu_s-\rho^{(s)}} (1-z)^{\mu_s-\zeta^{(s)}} (1-yzt)^{-\mu_s} \end{matrix} \right|$$

$$\begin{matrix} [(\text{-L});\text{R}_1, \cdots, R_s][(a); \theta', \cdots, \theta^{(s)}] : [(b'); \phi']; \cdots; [(b^{(s)}); \phi^{(s)}] \\ [(\text{c});\psi', \cdots, \psi^{(s)}] : [(d'); \delta']; \cdots; [(d^{(s)}); \delta^{(s)}] \end{matrix} \Bigg)$$

$$I \begin{pmatrix} \text{z}_1 x^{\sigma_1} y^{\rho_1} z^{\zeta_1} (1-y)^{\eta_1-\rho_1} (1-z)^{\eta_1-\zeta_1} (1-yzt)^{-\eta_1} \\ \cdots \\ \text{z}_r x^{\sigma_r} y^{\rho_r} z^{\zeta_r} (1-y)^{\eta_r-\rho_r} (1-z)^{\eta_r-\zeta_r} (1-yzt)^{-\eta_r} \end{pmatrix} \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$= \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{R_1, \cdots, R_s=0}^{h_1R_1+\cdots h_sR_s \leqslant L} \sum_{k=0}^{\infty} \frac{t^k}{k!} B'_s y_1^{R_1} \cdots y_s^{R_s} I_{U_r:p_r+6,q_r+4;W_r}^{V_r;0,n_r+6;X_r} \left(\begin{matrix} \text{z}_1 \\ \cdots \\ \text{z}_r \end{matrix} \middle| \begin{matrix} \text{A} \\ \cdots \\ \text{B} \end{matrix} \right.$$

$$(1\text{- c -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\quad (\tfrac{1}{2}\text{- c +a+b-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots \qquad \cdots$$
$$(\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),(\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{-}\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),$$
$$\cdots$$
$$\cdots$$

$$(1\text{-}\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1\text{-}\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r),$$

$$\left.\begin{matrix}(1\text{-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),\mathfrak{A}:A'\\ \cdots \\ \mathfrak{B}:B'\end{matrix}\right) \tag{5.3}$$

under the same notations and conditions that (3.1) with $B'_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where

$B(E;R_1,\cdots,R_s)$ is defined by (5.1)

Corollary 5

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}(1-uy-vz)^{-n}$$

$$F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{matrix}\mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'}\\ \cdots \\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}}\end{matrix}\right|$$

$$\left.\begin{matrix}[(\text{-L});\text{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\text{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right)$$

$$I\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1}\\ \cdots \\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}B'_sy_1^{R_1}\cdots y_s^{R_s}I^{V_r;0,n_r+7;X_r}_{U_r:p_r+7,q_r+5;W_r}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\ \middle|\ \begin{matrix}\mathrm{A}\\ \cdots\\ \mathrm{B}\end{matrix}\right.$$

$$(1\text{ -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ \ (1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots \qquad\qquad \cdots$$
$$(\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ \ (1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -n -m- }\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),(1-\beta-k-\rho'R_1-\cdots-\rho^{(s)}R_s;\rho_1,\cdots,\rho_r),$$
$$\cdots$$
$$(1\text{ -n - }\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),$$

$$(1\text{ -}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1\text{ -}\mu+\alpha-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r),$$

$$(1\text{ -}\lambda-k-e'R_s-\cdots-e^{(s)}R_s;\eta_1,\cdots,\eta_r),$$
$$\cdots$$
$$\cdots$$

$$\left.\begin{matrix}(1\text{ -}\alpha-m-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),\mathfrak{A}:A'\\ \cdots \\ (1\text{ -}\mu-m-t'R_1-\cdots-t^{(s)}R_s;t_1,\cdots,t_r),\mathfrak{B}:B'\end{matrix}\right) \tag{5.4}$$

under the same notations and conditions that (3.2) with $B'_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where $B(E;R_1,\cdots,R_s)$ is defined by (5.1)

Corollary 6

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}$$

$$\left(\begin{matrix}\text{y}_1x^{\sigma'}y^{\rho'}z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'}\\ \cdots \\ \text{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}}\end{matrix}\right|$$

$$I\left(\begin{matrix}\text{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1}\\ \cdots \\ \text{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r}\end{matrix}\right)$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z = \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$\sum_{k,m=0}^{\infty} \frac{u^k(1-v)^k v^m(1-u)^m}{k!m!} B_s y_1^{R_1}\cdots y_s^{R_s} I_{U_r:p_r+6,q_r+4;W_r}^{V_r;0,n_r+6;X_r}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdot\cdot\cdot \\ \mathrm{z}_r \end{array}\middle| \begin{array}{c} \mathrm{A} \\ \cdot\cdot\cdot \\ \mathrm{B} \end{array}\right.$$

$$(1\text{ -c }-\sigma' R_1-\cdots-\sigma^{(k)}R_s;\sigma_1,\cdots,\sigma_r),\ \ (\tfrac{1}{2}\text{-c - }\sigma' R_1-\cdots-\sigma^{(s)}R_s+a+b;\sigma_1,\cdots,\sigma_r),$$

$$\cdots \qquad \cdots$$

$$(\tfrac{1}{2}\text{ -c- }\sigma' R_1-\cdots-\sigma^{(k)}R_s+a;\sigma_1,\cdots,\sigma_r),\ \ (\tfrac{1}{2}\text{ -c - }\sigma' R_1-\cdots-\sigma^{(s)}R_s+b;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -}\mu-(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+\beta;t_1-\zeta_1,\cdots,t_r-\zeta_r),$$

$$\cdots$$

$$(1\text{-}\lambda-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s-k;\eta_1,\cdots,\eta_r),$$

$$(1\text{ -}\lambda+\alpha-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$

$$\cdots$$

$$(1\text{ -}\mu-m-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s-m;t_1,\cdots,t_r),$$

$$(1\text{ -}\alpha-k-\rho' R_1-\cdots-\rho^{(s)}R_s-m;\rho_1,\cdots,\rho_r),$$

$$\cdots$$

$$,\cdots\cdots$$

$$\left.\begin{array}{c}(1\text{ -}\beta-k-\zeta' R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),\mathfrak{A}:A' \\ \cdots \\ \mathfrak{B}:\ \mathrm{B'}\end{array}\right) \tag{5.5}$$

under the same notations and conditions that (3.2) with $B'_s = \dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where

$B(E;R_1,\cdots,R_s)$ is defined by (5.1)

6. Conclusion

The I-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain the triple Eulerian integrals concerning various other special functions such as H-function of several variables defined by Srivastava et al [8], for more details, see Garg et al [3], and the H-function of two variables , see Srivastava et a[7].

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