Euler type triple integrals involving, general class of polynomials and multivariable I-function defined by Nambisan

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ABSTRACT
The aim of the present document is to evaluate three triple Euler type integrals involving general class of polynomials, special functions and multivariable I-function defined by Nambisan et al [5]. Importance of our findings lies in the fact that they involve the multivariable I-function, which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them. Further we establish some special cases.

KEYWORDS : I-function of several variables, triple Euler type integrals, special function, general class of polynomials,multivariable H-function Srivastava-Doust polynomial

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In this paper, we evaluate three triple Eulerian integrals involving the multivariable I-function and a class of multivariable polynomials with general arguments.
The multivariable I-function defined by Nambisan et al [5] is a extension of the multivariable H-function defined by Srivastava et al [9]. We will use the contracted form.

The I-function of r-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,\cdots,z_r) = I^{0,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;\alpha_j^{(1)},\cdots,\alpha_j^{(r)};A_j)_{1,p} : \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;\beta_j^{(1)},\cdots,\beta_j^{(r)};B_j)_{1,q} : \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)},\bar{\delta}_j^{(1)};D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},\bar{\delta_j}^{(r)};D_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

where $\phi(s_1,\cdots,s_r),\theta_i(s_i), i=1,\cdots,r$ are given by :

$$\psi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i} \Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\bar{\delta}_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}\left(1-d_j^{(i)}+\bar{\delta}_j^{(i)}s_i\right)} \tag{1.4}$$

where $i=1,\cdots,r$. Also $z_i \neq 0$ for $i=1,\cdots,r$

The parameters $m_j, n_j, p_j, q_j (j=1,\cdots,r), n, p, q$ are non negative integers (for more details, see Nambisan [6])

$\alpha_j^{(i)}(j=1,\cdots,p; i=1,\cdots,r), \beta_j^{(i)}(j=1,\cdots,q; i=1,\cdots,r), \gamma_j^{(i)}(j=1,\cdots,p_i; i=1,\cdots,r)$ and $\delta_j^{(i)}$

$(j=1,\cdots,q_i; i=1,\cdots,r)$ are assumed to be positive quantities for standardisation purpose.

$a_j(j=1,\cdots,p), b_j(j=1,\cdots,q), c_j^{(i)}(j=1,\cdots,p_i, i=1,\cdots,r), d_j^{(i)}(j=1,\cdots,q_i, i=1,\cdots,r)$ are complex numbers.

The exposants $A_j(j=1,\cdots,p), B_j(j=1,\cdots,q), C_j^{(i)}(j=1,\cdots,p_i; i=1,\cdots,r), D_j^{(i)}(j=1,\cdots,q_i; i=1,\cdots,r)$

of various gamma function involved in (2.2) and (2.3) may take non integer values.

The contour L_i in the complex s_i-plane is of Mellin Barnes type which runs from $c-i\infty$ to $c+i\infty$ (c real) with indentation, if necessary, in such a manner that all singularities of $\Gamma^{D_j^{(i)}}(d_j^{(i)}-\delta_j^{(i)}s_i), j=1,\cdots,m_i$lie to the right and $\Gamma^{C_j^{(i)}}(1-c_j^{(i)}-\gamma_j^{(i)}s_i), j=1,\cdots,n_i$ are to the left of L_i.

Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i = \sum_{j=1}^{p} A_j\alpha_j^{(i)} - \sum_{j=1}^{q} B_j\beta_j^{(i)} + \sum_{j=1}^{p_i} C_j^{(i)}\gamma_j^{(i)} - \sum_{j=1}^{q_i} D_j^{(i)}\bar{\delta}_j^{(i)}, i=1,\cdots,r \tag{1.5}$$

The integral (2.1) converges absolutely if

$|arg(z_k)| < \frac{1}{2}\Delta_k\pi, k=1,\cdots,r$ where

$$\Delta_k = -\sum_{j=n+1}^{p} A_j\alpha_j^{(k)} - \sum_{j=1}^{q} B_j\beta_j^{(k)} + \sum_{j=1}^{m_k} D_j^{(k)}\delta_j^{(k)} - \sum_{j=m_k+1}^{q_k} D_j^{(k)}\bar{\delta}_j^{(k)} + \sum_{j=1}^{n_k} C_j^{(k)}\gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)}\gamma_j^{(k)} > 0 \tag{1.6}$$

We will use these notations for this paper :

$$X = m_1, n_1; \cdots; m_r, n_r; V = p_1, q_1; \cdots; p_r, q_r \tag{1.7}$$

$$A = (a_j; \alpha_j^{(1)}, \cdots, \alpha_j^{(r)}; A_j)_{1,p} \tag{1.8}$$

$$B = (b_j; \beta_j^{(1)}, \cdots, \beta_j^{(r)}; B_j)_{1,q} \tag{1.9}$$

$$\mathrm{C} = (\mathrm{c}_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; \cdots; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r} \tag{1.10}$$

$$D = (\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r} \tag{1.11}$$

the contracted form is

$$I_{p,q;V}^{0,n;X}\left(\begin{array}{c|c} z_1 & \mathrm{A : C} \\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot & \\ \cdot & \\ z_r & \mathrm{B : D} \end{array}\right) \tag{1.12}$$

Srivastava and Garg [7] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]=\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)\frac{z_1^{R_1}\cdots z_s^{R_s}}{R_1!\cdots R_s!} \quad (1.11)$$

The coefficients $B(E;R_1,\cdots,R_s)$ are arbitrary constants, real or complex.

We will note : $B_s=\frac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ (1.12)

2 . Results required :

a) $$\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x=\frac{\pi\Gamma(c)\Gamma(a+b+1/2)\Gamma(c-a-b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma(c-a+1/2)\Gamma(c-b+1/2)} \quad (2.1)$$

Where Re(c) > 0, Re(2c-a-b) > -1 , see Vyas and Rathie [10].

Erdélyi [2] [p.78, eq.(2.4) (1), vol 1]

b) $$\int_0^1\int_0^1 t^{b-1}r^{a-1}(1-t)^{c-b-1}(1-r)^{c-a-1}(1-trz)^{-c}\,\mathrm{d}r\mathrm{d}t$$

$$=\frac{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}{[\Gamma(c)]^2}\,{}_2F_1(a,b;c;z) \quad (2.2)$$

$Re(a)>0, Re(b)>0, Re(c-a)>0, Re(c-b)>0$

Erdélyi [2] [p.230, eq.(5.8.1) (2), vol 1]

c) $$\int_0^1\int_0^1 u^{\beta-1}v^{\beta'-1}(1-u)^{\gamma-\beta-1}(1-v)^{\gamma'-\beta'-1}(1-ux-vy)^{-\alpha}\,\mathrm{d}u\mathrm{d}v$$

$$=\frac{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta)\Gamma(\gamma'-\beta')}{\Gamma(\gamma)\Gamma(\gamma')}\,F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y) \quad (2.3)$$

$Re(\beta)>0, Re(\beta')>0, Re(\gamma-\beta)>0, Re(\gamma'-\beta')>0$

Erdélyi [2] [p.230, eq.(5.8.1) (4), vol 1]

d) $$\int_0^1\int_0^1 u^{\alpha-1}v^{\beta-1}(1-u)^{\gamma-\alpha-1}(1-v)^{\gamma'-\beta-1}(1-ux)^{\alpha-\gamma-\gamma'+1}(1-vy)^{\beta-\gamma-\gamma'+1}$$

$$(1-ux-vy)^{\gamma+\gamma'-\alpha-\beta-1}\mathrm{d}u\mathrm{d}v$$

$$= \frac{\Gamma(\beta)\Gamma(\alpha)\Gamma(\gamma-\alpha)\Gamma(\gamma'-\beta)}{\Gamma(\gamma)\Gamma(\gamma')} F_4(\alpha,\beta,\gamma,\gamma';x(1-y),y(1-x)) \tag{2.4}$$

$$Re(\beta)>0, Re(\alpha)>0, Re(\gamma-\alpha)>0, Re(\gamma'-\beta)>0$$

3. Main results

Theorem 1

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c} \mathrm{y}_1 x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1} \\ \cdots \\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s} \end{array}\right)$$

$$I\left(\begin{array}{c} \mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r} \end{array}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$= \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k=0}^{\infty}\frac{t^k}{k!}B_s y_1^{R_1}\cdots y_s^{R_s} I_{p+6,q+4;V}^{0,n+6;X}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array}\right|$$

$$\begin{array}{c} (1\text{- c -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1), \quad (\tfrac{1}{2}\text{- c +a+b-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1), \\ \cdots \qquad\qquad \cdots \\ (\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),(\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1), \end{array}$$

$$\begin{array}{c} (1\text{-}\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r;1), \\ \cdots \\ \cdots \end{array}$$

$$\begin{array}{c} (1\text{-}\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r;1), \\ \cdots \\ (1\text{-}\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r;1),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r;1), \end{array}$$

$$\left.\begin{array}{c} (1\text{-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r;1),A:C \\ \cdots \\ \mathrm{B}:\mathrm{D} \end{array}\right) \tag{3.1}$$

Provided that :

$$Re(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$$

$Re(2(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$

$Re(\beta+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)>0$

$Re(\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)>0$

$Re(\lambda-\alpha+(\mu_1-\zeta')R_1+\cdots+(\mu_s-\zeta^{(s)})R_s+(\eta_1-\zeta_1)s_1+\cdots+(\eta_r-\zeta_r)s_r)>0$

$Re(\lambda-\beta+(\mu_1-\rho')R_1+\cdots+(\mu_s-\rho^{(s)})R_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$

$|argz_k|<\frac{1}{2}\Delta_k\pi, k=1,\cdots,r$, where Δ_k is given in (1.6)

Theorem 2

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}$$

$$(1-uy-vz)^{-n}\,S_L^{h_1,\cdots,h_s}\left(\begin{array}{c}\mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}}\end{array}\right)$$

$$I\left(\begin{array}{c}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r}\end{array}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}B_sy_1^{R_1}\cdots y_s^{R_s}I_{p+7,q+5;V}^{0,n+7;X}\left(\begin{array}{c}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{array}\right|$$

$$\begin{array}{cc}(1\text{-c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1), & (1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),\\ \cdots & \cdots\\ (\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1), & (1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),\end{array}$$

$$\begin{array}{c}(1\text{-}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r;1)\\ \cdots\\ \cdots\end{array}$$

$$\begin{array}{c}(1\text{-}\mu-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r;1)\\ \cdots\\ (1\text{-n-}\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r;1),(1-\lambda-e'R_1-\cdots-e^{(s)}R_s-k;\eta_1,\cdots,\eta_r;1)\end{array}$$

$$(1\text{ -}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r;1),(1-n-\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r;1),$$
$$\cdots$$
$$\cdots$$

$$\left.\begin{array}{c}(1\text{ -}\alpha-m-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r;1),A:C\\ \cdots\\ (1\text{ -}\mu-m-t'R_1-\cdots-t^{(s)}R_s;t_1,\cdots,t_r;1),B:D\end{array}\right) \tag{3.2}$$

Provided that :

$Re(c+c_1R_1+\cdots+C_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$

$Re(2(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$

$Re(\beta+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)>0$

$Re(\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)>0$

$Re(\lambda-\beta+(e'-\rho')R_1+\cdots+(e^{(s)}-\rho^{(s)})R_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$

$Re(\mu-\alpha+(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+(t_1-\zeta_1)s_1+\cdots+(t_r-\zeta_r)s_r)>0$

$|argz_k|<\frac{1}{2}\Delta_k\pi, k=1,\cdots,r$, where Δ_k is given in (1.6)

Theorem 3

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c}\mathrm{y}_1x^{\sigma'}y^{\rho'}z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}}\end{array}\right)$$

$$I\left(\begin{array}{c}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r}\end{array}\right)$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$\sum_{k,m=0}^{\infty}\frac{u^k(1-v)^kv^m(1-u)^m}{k!m!}B_sy_1^{R_1}\cdots y_s^{R_s}I_{p+6,q+4;V}^{0,n+6;X}\left(\begin{array}{c} z_1 \\ \cdot\cdot\cdot \\ z_r \end{array}\right|$$

$$(1\text{ -c }-\sigma'R_1-\cdots-\sigma^{(k)}R_s;\sigma_1,\cdots,\sigma_r;1),\quad(\tfrac{1}{2}\text{-c - }\sigma'R_1-\cdots-\sigma^{(s)}R_s+a+b;\sigma_1,\cdots,\sigma_r;1),$$
$$\cdots\qquad\qquad\cdots$$
$$(\tfrac{1}{2}\text{ -c- }\sigma'R_1-\cdots-\sigma^{(k)}R_s+a;\sigma_1,\cdots,\sigma_r;1),\ (\tfrac{1}{2}\text{ -c - }\sigma'R_1-\cdots-\sigma^{(s)}R_s+b;\sigma_1,\cdots,\sigma_r;1),$$

$$(1\text{ -}\mu-(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+\beta;t_1-\zeta_1,\cdots,t_r-\zeta_r;1),$$
$$\cdots$$
$$(1\text{-}\lambda-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s-k;\eta_1,\cdots,\eta_r;1),$$

$$(1\text{ -}\lambda+\alpha-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r;1),$$
$$\cdots$$
$$(1\text{ -}\mu-m-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s-m;t_1,\cdots,t_r;1),$$

$$(1\text{ -}\alpha-k-\rho'R_1-\cdots-\rho^{(s)}R_s-m;\rho_1,\cdots,\rho_r;1),$$
$$\cdots$$
$$,\cdots\cdots\cdot$$

$$\left.\begin{array}{c}(1\text{ -}\beta-k-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r;1),A:C \\ \cdots \\ \mathrm{B}:\mathrm{D}\end{array}\right) \tag{4.3}$$

Provided that :

$$Re(c+\sigma'R_1+\cdots+\sigma_s^{(s)}R_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$$

$$Re(2(c+\sigma_1'R_1+\cdots+\sigma^{(s)}R_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$$

$$Re(\alpha+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)>0$$

$$Re(\beta+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)>0$$

$$Re(\lambda-\alpha+(\eta'-\rho')R_1+\cdots+(\eta^{(s)}-\rho^{(s)})R_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$$

$$Re(\mu-\beta+(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+(t_1-\zeta_1)s_1+\cdots+(t_r-\zeta_r)s_r)>0$$

$|argz_k|<\frac{1}{2}\Delta_k\pi, k=1,\cdots,r$, where Δ_k is given in (1.6)

Proof de (3.1) : Fisrt we use series representation (1.11) for $S_L^{h_1,\cdots,h_s}[.]$ and expressing the multivariable I -function defined by Nambisan et al [5] involving in the left hand side of (3.1) in terms of Mellin-Barnes contour integral with the help of (1.1) and then interchanching the order of integration. We get L.H.S.

$$=\frac{1}{(2\pi\omega)^r}\left(\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_sy_1^{R_1}\cdots y_s^{R_s}\right.$$

$$\left(\int_0^1 x^{c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x\right)$$

$$\times\left(\int_0^1\int_0^1 y^{\beta+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r}z^{\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r-1}\right.$$

$$(1-yzt)^{-(\lambda+\mu_1R_1+\cdots+\mu_sR_s+\eta_1s_1+\cdots+\eta_rs_r)}$$

$$\times(1-y)^{(\lambda+\mu_1R_1+\cdots+\mu_sR_s+\eta_1s_1+\cdots+\eta_rs_r)-(\beta+\rho'R_1+\cdots\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)-1}$$

$$\left.\times(1-z)^{(\lambda+\mu_1R_1+\cdots+\mu_sR_s+\eta_1s_1+\cdots+\eta_rs_r)-(\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)-1}\mathrm{d}y\mathrm{d}z\right)\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Now using the result (2.1), (2.2) and (1.1) we get right hand side of (3.1). Similarly we can prove (3.2) and (3.3) with help of the results (2.3) and (2.4).

4. Multivariable H-function

If $A_j = B_j = C_j^{(i)} = D_j^{(i)} = 1$, the multivariable I-function defined by Nambisan et al [4] reduces to the multivariable H-funcction defined by Srivastava et al [9] and we have the following results.

Corollary1

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c}\mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s}\end{array}\right)$$

$$H\left(\begin{array}{c}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r}\end{array}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k=0}^{\infty}\frac{t^k}{k!}B_sy_1^{R_1}\cdots y_s^{R_s}H^{0,n+6;X}_{p+6,q+4;V}\left(\begin{array}{c}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{array}\right|$$

$$\begin{array}{c}(1-c-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\quad(\tfrac{1}{2}-c+a+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\\ \cdots\qquad\qquad\cdots\\ (\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),(\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\end{array}$$

$$(1\text{-}\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),$$
$$\cdots$$
$$\cdots$$

$$(1\text{-}\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1\text{-}\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r),$$

$$\left.\begin{matrix}(1\text{-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ \mathrm{B:D}\end{matrix}\right) \tag{4.1}$$

under the same conditions and notations that (3.1) with $A_j=B_j=C_j^{(i)}=D_j^{(i)}=1$

Corollary2

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}$$

$$(1-uy-vz)^{-n}\,S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}}\end{pmatrix}$$

$$H\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}B_sy_1^{R_1}\cdots y_s^{R_s}H^{0,n+7;X}_{p+7,q+5;V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\right|$$

$$(1\text{ -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ \ (1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots\qquad\qquad\cdots$$
$$(\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ \ (1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r)$$
$$\cdots$$
$$\cdots$$

$$(1\text{-}\ \mu-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r)$$
$$\cdots$$
$$(1\text{-n-}\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),(1-\lambda-e'R_1-\cdots-e^{(s)}R_s-k;\eta_1,\cdots,\eta_r)$$

$$(1\ \text{-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),(1-n-\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),$$
$$\cdots$$
$$\cdots$$

$$\left.\begin{matrix}(1\ \text{-}\alpha-m-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),A:C\\ \cdots\\ (1\ \text{-}\mu-m-t'R_1-\cdots-t^{(s)}R_s;t_1,\cdots,t_r),B:D\end{matrix}\right) \tag{4.2}$$

under the same conditions and notations that (3.2) with $A_j=B_j=C_j^{(i)}=D_j^{(i)}=1$

Corollary 3

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix}\mathrm{y}_1x^{\sigma'}y^{\rho'}z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}}\end{pmatrix}$$

$$H\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r}\end{pmatrix}$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$\sum_{k,m=0}^{\infty}\frac{u^k(1-v)^kv^m(1-u)^m}{k!m!}B_sy_1^{R_1}\cdots y_s^{R_s}H_{p+6,q+4;V}^{0,n+6;X}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\ \middle|\right.$$

$$(1\ \text{-c -}\sigma'R_1-\cdots-\sigma^{(k)}R_s;\sigma_1,\cdots,\sigma_r),\quad(\tfrac{1}{2}\text{-c - }\sigma'R_1-\cdots-\sigma^{(s)}R_s+a+b;\sigma_1,\cdots,\sigma_r),$$
$$\cdots\qquad\cdots$$
$$(\tfrac{1}{2}\ \text{-c- }\sigma'R_1-\cdots-\sigma^{(k)}R_s+a;\sigma_1,\cdots,\sigma_r),\quad(\tfrac{1}{2}\ \text{-c - }\sigma'R_1-\cdots-\sigma^{(s)}R_s+b;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -}\mu-(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+\beta;t_1-\zeta_1,\cdots,t_r-\zeta_r),$$
$$\cdots$$
$$(1\text{-}\lambda-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s-k;\eta_1,\cdots,\eta_r),$$

$$(1\text{ -}\lambda+\alpha-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1\text{ -}\mu-m-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s-m;t_1,\cdots,t_r),$$

$$(1\text{ -}\alpha-k-\rho'R_1-\cdots-\rho^{(s)}R_s-m;\rho_1,\cdots,\rho_r),$$
$$\cdots$$
$$,\cdots\cdots\cdot$$

$$\left.\begin{matrix}(1\text{ -}\beta-k-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),A:C\\ \cdots\\ \mathrm{B:D}\end{matrix}\right) \tag{4.3}$$

under the same conditions and notations that (3.3) with $A_j=B_j=C_j^{(i)}=D_j^{(i)}=1$

5. Srivastava-Daoust polynomial

If $B(L;R_1,\cdots,R_s)=\dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta_j'+\cdots+R_s\theta_j^{(s)}}\prod_{j=1}^{B'}(b_j')_{R_1\phi_j'}\cdots\prod_{j=1}^{B^{(s)}}(b_j^{(s)})_{R_s\phi_j^{(s)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi_j'+\cdots+m_s\psi_j^{(s)}}\prod_{j=1}^{D'}(d_j')_{R_1\delta_j'}\cdots\prod_{j=1}^{D^{(s)}}(d_j^{(s)})_{R_s\delta_j^{(s)}}}$ **(5.1)**

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]$ reduces to generalized Srivastava-Daoust polynomial defined by Srivastava et al [6].

$$F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \cdots\\ \mathrm{z}_s\end{matrix}\left|\begin{matrix}[(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right.\right) \tag{5.2}$$

and we have the following formulas

Corollary 4

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$

$$F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}\left(\begin{matrix}\mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s}\end{matrix}\right|$$

$$\left.\begin{matrix}[\text{(-L)};\text{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [\text{(c)};\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right)$$

$$I\begin{pmatrix}\text{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1}\\ \cdots\\ \text{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k=0}^{\infty}\frac{t^k}{k!}B'_sy_1^{R_1}\cdots y_s^{R_s}I^{0,n+6;X}_{p+6,q+4;V}\left(\begin{matrix}\text{z}_1\\ \cdots\\ \text{z}_r\end{matrix}\,\middle|\right.$$

$$\begin{matrix}(\text{1- c -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),\quad(\tfrac{1}{2}\text{- c +a+b-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),\\ \cdots\qquad\qquad\qquad\qquad\cdots\\ (\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),(\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),\end{matrix}$$

$$\begin{matrix}(\text{1-}\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r;1),\\ \cdots\\ \cdots\end{matrix}$$

$$\begin{matrix}(\text{1-}\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r;1),\\ \cdots\\ (\text{1-}\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r;1),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r;1),\end{matrix}$$

$$\left.\begin{matrix}(\text{1-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r;1),A:C\\ \cdots\\ \text{B :D}\end{matrix}\right)\tag{5.3}$$

under the same notations and conditions that (3.1) with $B'_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where

$B(E;R_1,\cdots,R_s)$ is defined by (5.1)

Corollary 5

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}(1-uy-vz)^{-n}$$

$$F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{matrix}\text{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'}\\ \cdots\\ \text{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}}\end{matrix}\,\middle|\right.$$

$$\left.\begin{matrix}[(\text{-L});\text{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\text{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right)$$

$$I\begin{pmatrix}\text{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1}\\ \cdot\cdot\cdot\\ \text{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r}\end{pmatrix}\text{d}x\text{d}y\text{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}B'_sy_1^{R_1}\cdots y_s^{R_s}I^{0,n+7;X}_{p+7,q+5;V}\left(\begin{matrix}\text{z}_1\\ \cdot\cdot\cdot\\ \text{z}_r\end{matrix}\right|$$

$$\begin{matrix}(1\text{ -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1), & (1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),\\ \cdot\cdot\cdot & \cdot\cdot\cdot\\ (\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1), & (1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r;1),\end{matrix}$$

$$\begin{matrix}(1\text{ -}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r;1)\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\end{matrix}$$

$$\begin{matrix}(1\text{- }\mu-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r;1)\\ \cdot\cdot\cdot\\ (1\text{-n-}\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r;1),(1-\lambda-e'R_1-\cdots-e^{(s)}R_s-k;\eta_1,\cdots,\eta_r;1)\end{matrix}$$

$$\begin{matrix}(1\text{ -}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r;1),(1-n-\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r;1),\\ \cdot\cdot\cdot\\ \cdot\cdot\cdot\end{matrix}$$

$$\left.\begin{matrix}(1\text{ -}\alpha-m-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r;1),A:C\\ \cdot\cdot\cdot\\ (1\text{ -}\mu-m-t'R_1-\cdots-t^{(s)}R_s;t_1,\cdots,t_r;1),B:D\end{matrix}\right) \tag{5.4}$$

under the same notations and conditions that (3.2) with $B'_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where $B(E;R_1,\cdots,R_s)$ is defined by (5.1)

Corollary 6

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}F_{\bar{C}:D';\cdots;D^{(s)}}^{1+\bar{A}:B';\cdots;B^{(s)}}$$

$$\left(\begin{array}{c} \mathrm{y}_1 x^{\sigma'}y^{\rho'}z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'} \\ \cdots \\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}} \end{array}\right|$$

$$\left.\begin{array}{c} [(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}] \\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}] \end{array}\right)$$

$$I\begin{pmatrix} \mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r} \end{pmatrix}$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z = \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$\sum_{k,m=0}^{\infty}\frac{u^k(1-v)^k v^m(1-u)^m}{k!m!}B_s y_1^{R_1}\cdots y_s^{R_s} I_{p+6,q+4;V}^{0,n+6;X}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array}\right|$$

$$(1\text{ -c }-\sigma'R_1-\cdots-\sigma^{(k)}R_s;\sigma_1,\cdots,\sigma_r;1),\quad (\tfrac{1}{2}\text{-c - }\sigma'R_1-\cdots-\sigma^{(s)}R_s+a+b;\sigma_1,\cdots,\sigma_r;1),$$
$$\cdots \qquad \cdots$$
$$(\tfrac{1}{2}\text{ -c- }\sigma'R_1-\cdots-\sigma^{(k)}R_s+a;\sigma_1,\cdots,\sigma_r;1),\quad (\tfrac{1}{2}\text{ -c - }\sigma'R_1-\cdots-\sigma^{(s)}R_s+b;\sigma_1,\cdots,\sigma_r;1),$$

$$(1\text{ -}\mu-(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+\beta;t_1-\zeta_1,\cdots,t_r-\zeta_r;1),$$
$$\cdots$$
$$(1\text{-}\lambda-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s-k;\eta_1,\cdots,\eta_r;1),$$

$$(1\text{ -}\lambda+\alpha-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r;1),$$
$$\cdots$$
$$(1\text{ -}\mu-m-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s-m;t_1,\cdots,t_r;1),$$

$$(1\text{ -}\alpha-k-\rho'R_1-\cdots-\rho^{(s)}R_s-m;\rho_1,\cdots,\rho_r;1),$$
$$\cdots$$
$$,\cdots\cdots\cdots$$

$$\left.\begin{array}{c} (1\text{ -}\beta-k-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r;1),A:C \\ \cdots \\ \mathrm{B}:\mathrm{D} \end{array}\right) \tag{5.5}$$

under the same notations and conditions that (3.3) with $B'_s = \frac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where

$B(E;R_1,\cdots,R_s)$ is defined by (5.1)

6. Conclusion

The I-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain the triple Eulerian integrals concerning various other special functions such as H-function of several variables defined by Srivastava et al [9], for more details, see Garg et al [4], and the H-function of two variables , see Srivastava et a[8].

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