Euler type triple integrals involving, general class of polynomials and multivariable A-function

$F.Y.\ AYANT^1$

1 Teacher in High School , France

ABSTRACT
The aim of the present document is to evaluate three triple Euler type integrals involving general class of polynomials, special functions and multivariable A-function defined by Gautam et al [4]. Importance of our findings lies in the fact that they involve the multivariable A-function, which are the sufficiently general in nature and are capable of yielding a large number of simpler and useful results merely by specializing the parameters in them. Further we establish some special cases.

KEYWORDS : A-function of several variables, triple Euler type integrals, special function, general class of polynomials, multivariable H-function Srivastava-Doust polynomial

2010 Mathematics Subject Classification :33C05, 33C60

1. Introduction

In this paper, we evaluate three triple Eulerian integrals involving the multivariable A-function and class of multivariable polynomials with general arguments.
The multivariable A-function defined by Gautam et al [4] is a extension of the multivariable H-function defined by Srivastava et al [8]. We will use the contracted form.

The A-function is defined and represented in the following manner.

$$A(z_1,\cdots,z_r) = A^{m,n:m_1,n_1;\cdots;m_r,n_r}_{p,q:p_1,q_1;\cdots;p_r,q_r}\left(\begin{array}{c|c} \mathrm{z}_1 & (\mathrm{a}_j;A_j^{(1)},\cdots,A_j^{(r)})_{1,p}: \\ \cdot & \\ \cdot & \\ \cdot & \\ \mathrm{z}_r & (\mathrm{b}_j;B_j^{(1)},\cdots,B_j^{(r)})_{1,q}: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j^{(1)},C_j^{(1)})_{1,p_1};\cdots;(c_j^{(r)},C_j^{(r)})_{1,p_r} \\ \\ (\mathrm{d}_j^{(1)},D_j^{(1)})_{1,q_1};\cdots;(d_j^{(r)},D_j^{(r)})_{1,q_r} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.2}$$

where $\phi(s_1,\cdots,s_r),\theta_i(s_i),i=1,\cdots,r$ are given by :

$$\psi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{m'}\Gamma(b_j-\sum_{i=1}^{r}B_j^{(i)}s_i)\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{r}A_j^{(i)}s_j)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{r}A_j^{(i)}s_j)\prod_{j=m+1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{r}B_j^{(i)}s_j\right)} \tag{1.3}$$

$$\theta_i(s_i) = \frac{\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+C_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma(d_j^{(i)}-D_j^{(i)}s_i)}{\prod_{j=n_i+1}^{p_i}\Gamma(c_j^{(i)}-C_j^{(i)}s_i)\prod_{j=m_i+1}^{q_i}\Gamma(1-d_j^{(i)}+D_j^{(i)}s_i)} \tag{1.4}$$

Here $m', n, p, m_i, n_i, p_i, c_i \in \mathbb{N}^*; i = 1, \cdots, r$; $a_j, b_j, c_j^{(i)}, d_j^{(i)}, A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{C}$

The multiple integral defining the A-function of r variables converges absolutely if :

$$|arg(\Omega_i)z_k| < \frac{1}{2}\eta_k\pi, \xi^* = 0, \eta_i > 0 \tag{1.5}$$

$$\Omega_i = \prod_{j=1}^{p}\{A_j^{(i)}\}^{A_j^{(i)}}\prod_{j=1}^{q}\{B_j^{(i)}\}^{-B_j^{(i)}}\prod_{j=1}^{q_i}\{D_j^{(i)}\}^{D_j^{(i)}}\prod_{j=1}^{p_i}\{C_j^{(i)}\}^{-C_j^{(i)}}; i = 1, \cdots, r \tag{1.6}$$

$$\xi_i^* = Im(\sum_{j=1}^{p}A_j^{(i)} - \sum_{j=1}^{q}B_j^{(i)} + \sum_{j=1}^{q_i}D_j^{(i)} - \sum_{j=1}^{p_i}C_j^{(i)}); i = 1, \cdots, r \tag{1.7}$$

$$\eta_i = Re\left(\sum_{j=1}^{n}A_j^{(i)} - \sum_{j=n+1}^{p}A_j^{(i)} + \sum_{j=1}^{m'}B_j^{(i)} - \sum_{j=m'+1}^{q}B_j^{(i)} + \sum_{j=1}^{m_i}D_j^{(i)} - \sum_{j=m_i+1}^{q_i}D_j^{(i)} + \sum_{j=1}^{n_i}C_j^{(i)} - \sum_{j=n_i+1}^{p_i}C_j^{(i)}\right) \text{ with}$$

$$i = 1, \cdots, r \tag{1.8}$$

Let

$$X = m_1, n_1; \cdots; m_r, n_r; Y = p'_1, q'_1; \cdots; p'_s, q'_s \tag{1.9}$$

$$A = (a_j; A_j^{(1)}, \cdots, A_j^{(r)})_{1,p} \quad ; B = (b_j; B_j^{(1)}, \cdots, B_j^{(r)})_{1,q} \tag{1.10}$$

$$\mathrm{C} = (\mathrm{c}_j^{(1)}, C_j^{(1)})_{1,p_1}; \cdots; (c_j^{(s)}, C_j^{(r)})_{1,p_r}; D = (\mathrm{d}_j^{(1)}, D_j^{(1)})_{1,q_1}; \cdots; (d_j^{(r)}, D_j^{(r)})_{1,q_r} \tag{1.11}$$

the contracted form is

$$A(z_1, \cdots, z_r) = A_{p,q:Y}^{m',n:X}\left(\begin{array}{c|c} z_1 & \mathrm{A} : \mathrm{C} \\ \cdot & \cdot\ \cdot\ \cdot \\ \cdot & \\ z_r & \mathrm{B} : \mathrm{D} \end{array}\right) \tag{1.12}$$

Srivastava and Garg [6] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,\cdots,h_s}[z_1, \cdots, z_s] = \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s \leqslant L}(-L)_{h_1R_1+\cdots+h_sR_s}B(E; R_1, \cdots, R_s)\frac{z_1^{R_1}\cdots z_s^{R_s}}{R_1!\cdots R_s!} \tag{1.11}$$

The coefficients $B(E; R_1, \cdots, R_s)$ are arbitrary constants, real or complex.

We will note $:B_s = \dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E; R_1, \cdots, R_s)}{R_1!\cdots R_s!}$ (1.12)

2 . Results required :

a) $$\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x = \frac{\pi\Gamma(c)\Gamma(a+b+1/2)\Gamma(c-a-b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)\Gamma(c-a+1/2)\Gamma(c-b+1/2)} \quad (2.1)$$

Where Re(c) > 0, Re(2c-a-b) > -1 , see Vyas and Rathie [9].

Erdélyi [1] [p.78, eq.(2.4) (1), vol 1]

b) $$\int_0^1\int_0^1 t^{b-1}r^{a-1}(1-t)^{c-b-1}(1-r)^{c-a-1}(1-trz)^{-c}\,\mathrm{d}r\mathrm{d}t$$

$$= \frac{\Gamma(a)\Gamma(b)\Gamma(c-a)\Gamma(c-b)}{[\Gamma(c)]^2}\,{}_2F_1(a,b;c;z) \quad (2.2)$$

$Re(a) > 0, Re(b) > 0, Re(c-a) > 0, Re(c-b) > 0$

Erdélyi [1] [p.230, eq.(5.8.1) (2), vol 1]

c) $$\int_0^1\int_0^1 u^{\beta-1}v^{\beta'-1}(1-u)^{\gamma-\beta-1}(1-v)^{\gamma'-\beta'-1}(1-ux-vy)^{-\alpha}\,\mathrm{d}u\mathrm{d}v$$

$$= \frac{\Gamma(\beta)\Gamma(\beta')\Gamma(\gamma-\beta)\Gamma(\gamma'-\beta')}{\Gamma(\gamma)\Gamma(\gamma')}\,F_2(\alpha,\beta,\beta',\gamma,\gamma';x,y) \quad (2.3)$$

$Re(\beta) > 0, Re(\beta') > 0, Re(\gamma-\beta) > 0, Re(\gamma'-\beta') > 0$

Erdélyi [1] [p.230, eq.(5.8.1) (4), vol 1]

d) $$\int_0^1\int_0^1 u^{\alpha-1}v^{\beta-1}(1-u)^{\gamma-\alpha-1}(1-v)^{\gamma'-\beta-1}(1-ux)^{\alpha-\gamma-\gamma'+1}(1-vy)^{\beta-\gamma-\gamma'+1}$$

$$(1-ux-vy)^{\gamma+\gamma'-\alpha-\beta-1}\mathrm{d}u\mathrm{d}v$$

$$= \frac{\Gamma(\beta)\Gamma(\alpha)\Gamma(\gamma-\alpha)\Gamma(\gamma'-\beta)}{\Gamma(\gamma)\Gamma(\gamma')}\,F_4(\alpha,\beta,\gamma,\gamma';x(1-y),y(1-x)) \quad (2.4)$$

$Re(\beta) > 0, Re(\alpha) > 0, Re(\gamma-\alpha) > 0, Re(\gamma'-\beta) > 0$

3. Main results

Theorem 1

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c} \mathrm{y}_1 x^{c_1} y^{\rho'} z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1} \\ \cdots \\ \mathrm{y}_s x^{c_s} y^{\rho^{(s)}} z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s} \end{array}\right)$$

$$A\left(\begin{array}{c} \mathrm{z}_1 x^{\sigma_1} y^{\rho_1} z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r} y^{\rho_r} z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r} \end{array}\right) \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$= \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} \sum_{k=0}^{\infty} \frac{t^k}{k!} B_s y_1^{R_1}\cdots y_s^{R_s} A_{p+6,q+4;V}^{m',n+6;X}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array}\right|$$

$$(1-c-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r), \quad (\tfrac{1}{2}-c+a+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots \qquad\qquad \cdots$$
$$(\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),(\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1-\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),$$
$$\cdots$$
$$\cdots$$

$$(1-\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r),$$

$$\left.\begin{array}{c}(1-\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),A:C \\ \cdots \\ \mathrm{B}:\mathrm{D}\end{array}\right) \tag{3.1}$$

Provided that :

$Re(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$

$Re(2(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$

$Re(\beta+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)>0$

$Re(\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)>0$

$Re(\lambda-\alpha+(\mu_1-\zeta')R_1+\cdots+(\mu_s-\zeta^{(s)})R_s+(\eta_1-\zeta_1)s_1+\cdots+(\eta_r-\zeta_r)s_r)>0$

$Re(\lambda-\beta+(\mu_1-\rho')R_1+\cdots+(\mu_s-\rho^{(s)})R_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$

$|argz_k|<\frac{1}{2}\eta_k\pi,\xi^*=0,\eta_i>0$, η_k is defined by (1.8)

Theorem 2

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}$$

$$(1-uy-vz)^{-n}\,S_L^{h_1,\cdots,h_s}\begin{pmatrix} \mathrm{y}_1 x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'} \\ \cdots \\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}} \end{pmatrix}$$

$$A\begin{pmatrix} \mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r} \end{pmatrix}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^k v^m}{k!m!}B_s y_1^{R_1}\cdots y_s^{R_s}A_{p+7,q+5;V}^{m',n+7;X}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array}\right|$$

$$(1\text{-c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ (1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots \qquad \cdots$$
$$(\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ (1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r)$$
$$\cdots$$
$$\cdots$$

$$(1\text{- }\mu-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r)$$
$$\cdots$$
$$(1\text{-n-}\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),(1-\lambda-e'R_1-\cdots-e^{(s)}R_s-k;\eta_1,\cdots,\eta_r)$$

$$(1\text{ -}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),(1-n-\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),$$
$$\cdots$$
$$\cdots$$

$$\left.\begin{array}{c}(1\text{ -}\alpha-m-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),A:C \\ \cdots \\ (1\text{ -}\mu-m-t'R_1-\cdots-t^{(s)}R_s;t_1,\cdots,t_r),B:D\end{array}\right) \tag{3.2}$$

Provided that :

$Re(c+c_1R_1+\cdots+C_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$

$Re(2(c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$

$$Re(\beta+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1 s_1+\cdots+\rho_r s_r)>0$$

$$Re(\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1 s_1+\cdots+\zeta_r s_r)>0$$

$$Re(\lambda-\beta+(e'-\rho')R_1+\cdots+(e^{(s)}-\rho^{(s)})R_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$$

$$Re(\mu-\alpha+(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+(t_1-\zeta_1)s_1+\cdots+(t_r-\zeta_r)s_r)>0$$

$|argz_k|<\frac{1}{2}\eta_k\pi, \xi^*=0, \eta_i>0$, η_k is defined by (1.8)

Theorem 3

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c} \mathrm{y}_1 x^{\sigma'}y^{\rho'}z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'} \\ \cdots \\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}} \end{array}\right)$$

$$A\left(\begin{array}{c} \mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r} \end{array}\right)$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$\sum_{k,m=0}^{\infty}\frac{u^k(1-v)^k v^m(1-u)^m}{k!m!}B_s y_1^{R_1}\cdots y_s^{R_s}A_{p+6,q+4;V}^{m',n+6;X}\left(\begin{array}{c}\mathrm{z}_1\\ \cdots \\ \mathrm{z}_r\end{array}\right|$$

$$(1\text{ -c }-\sigma'R_1-\cdots-\sigma^{(k)}R_s;\sigma_1,\cdots,\sigma_r),\quad (\tfrac{1}{2}\text{-c - }\sigma'R_1-\cdots-\sigma^{(s)}R_s+a+b;\sigma_1,\cdots,\sigma_r),$$
$$\cdots \qquad\qquad \cdots$$
$$(\tfrac{1}{2}\text{ -c- }\sigma'R_1-\cdots-\sigma^{(k)}R_s+a;\sigma_1,\cdots,\sigma_r),\quad (\tfrac{1}{2}\text{ -c - }\sigma'R_1-\cdots-\sigma^{(s)}R_s+b;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -}\mu-(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+\beta;t_1-\zeta_1,\cdots,t_r-\zeta_r),$$
$$\cdots$$
$$(1\text{-}\lambda-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s-k;\eta_1,\cdots,\eta_r),$$

$$(1\text{ -}\lambda+\alpha-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1\text{ -}\mu-m-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s-m;t_1,\cdots,t_r),$$

$$(1\text{ -}\alpha-k-\rho'R_1-\cdots-\rho^{(s)}R_s-m;\rho_1,\cdots,\rho_r),$$
$$\cdots$$
$$,\cdots\cdots$$

$$\left.\begin{matrix}(1\text{ -}\beta-k-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),A:C\\ \cdots \\ \text{B : D}\end{matrix}\right) \tag{3.3}$$

Provided that :

$Re(c+\sigma'R_1+\cdots+\sigma_s^{(s)}R_s+\sigma_1s_1+\cdots+\sigma_rs_r)>0;$

$Re(2(c+\sigma_1'R_1+\cdots+\sigma^{(s)}R_s+\sigma_1s_1+\cdots+\sigma_rs_r)-a-b)>-1$

$Re(\alpha+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)>0$

$Re(\beta+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)>0$

$Re(\lambda-\alpha+(\eta'-\rho')R_1+\cdots+(\eta^{(s)}-\rho^{(s)})R_s+(\eta_1-\rho_1)s_1+\cdots+(\eta_r-\rho_r)s_r)>0$

$Re(\mu-\beta+(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+(t_1-\zeta_1)s_1+\cdots+(t_r-\zeta_r)s_r)>0$

$|argz_k|<\frac{1}{2}\eta_k\pi,\xi^*=0,\eta_i>0$, η_k is defined by (1.8)

Proof de (3.1) : Fisrt we use series representation (1.11) for $S_L^{h_1,\cdots,h_s}[.]$ and expressing the multivariable A -function defined by Gautam et al [4] involving in the left hand side of (3.1) in terms of Mellin-Barnes contour integral with the help of (1.1) and then interchanching the order of integration. We get L.H.S.

$$=\frac{1}{(2\pi\omega)^r}\left(\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^r\theta_k(s_k)z_k^{s_k}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}B_sy_1^{R_1}\cdots y_s^{R_s}\right.$$

$$\left(\int_0^1 x^{c+c_1R_1+\cdots+c_sR_s+\sigma_1s_1+\cdots+\sigma_rs_r-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)\mathrm{d}x\right)$$

$$\times\left(\int_0^1\int_0^1 y^{\beta+\rho'R_1+\cdots+\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r}z^{\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r-1}\right.$$

$$(1-yzt)^{-(\lambda+\mu_1R_1+\cdots+\mu_sR_s+\eta_1s_1+\cdots+\eta_rs_r)}$$

$$\times(1-y)^{(\lambda+\mu_1R_1+\cdots+\mu_sR_s+\eta_1s_1+\cdots+\eta_rs_r)-(\beta+\rho'R_1+\cdots\rho^{(s)}R_s+\rho_1s_1+\cdots+\rho_rs_r)-1}$$

$$\left.\times(1-z)^{(\lambda+\mu_1R_1+\cdots+\mu_sR_s+\eta_1s_1+\cdots+\eta_rs_r)-(\alpha+\zeta'R_1+\cdots+\zeta^{(s)}R_s+\zeta_1s_1+\cdots+\zeta_rs_r)-1}\mathrm{d}y\mathrm{d}z\right)\mathrm{d}s_1\cdots\mathrm{d}s_r$$

Now using the result (2.1), (2.2) and (1.1) we get right hand side of (3.1). Similarly we can prove (3.2) and (3.3) with help of the results (2.3) and (2.4).

4. Multivariable H-function

If $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$ and $m' = 0$, the multivariable A-function reduces to multivariable H-function defined by Srivastava et al [6], we obtain the following formulas.

Corollary1

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$

$$S_L^{h_1,\cdots,h_s}\left(\begin{array}{c} \mathrm{y}_1 x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1} \\ \cdots \\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s} \end{array}\right)$$

$$H\left(\begin{array}{c} \mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r} \end{array}\right) \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$= \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)} \sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L} \sum_{k=0}^{\infty} \frac{t^k}{k!} B_s y_1^{R_1}\cdots y_s^{R_s} H^{0,n+6;X}_{p+6,q+4;V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array}\right|$$

$$(1\text{- c -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r), \quad (\tfrac{1}{2}\text{- c +a+b-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots \qquad \cdots$$
$$(\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),(\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{-}\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),$$
$$\cdots$$
$$\cdots$$

$$(1\text{-}\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1\text{-}\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r),$$

$$\left.\begin{array}{c}(1\text{-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),A:C \\ \cdots \\ \mathrm{B}:\mathrm{D}\end{array}\right) \tag{4.1}$$

under the same conditions and notations that (3.1) with $A_j^{(i)}, B_j^{(i)}, C_j^{(i)}, D_j^{(i)} \in \mathbb{R}$ and $m' = 0$

Corollary2

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}$$

$$(1-uy-vz)^{-n}\, S_L^{h_1,\cdots,h_s}\left(\begin{array}{c} \mathrm{y}_1 x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'} \\ \cdots \\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}} \end{array}\right)$$

$$H\left(\begin{array}{c} \mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r} \end{array}\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}B_s y_1^{R_1}\cdots y_s^{R_s}H^{0,n+7;X}_{p+7,q+5;V}\left(\begin{array}{c} \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_r \end{array}\right|$$

$$\begin{array}{c} (1\text{ -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ (1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r), \\ \cdots \qquad\qquad \cdots \\ (\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\ (1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r), \end{array}$$

$$\begin{array}{c} (1\text{ -}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r) \\ \cdots \\ \cdots \end{array}$$

$$\begin{array}{c} (1\text{- }\mu-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r) \\ \cdots \\ (1\text{-n-}\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),(1-\lambda-e'R_1-\cdots-e^{(s)}R_s-k;\eta_1,\cdots,\eta_r) \end{array}$$

$$\begin{array}{c} (1\text{ -}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),(1-n-\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r), \\ \cdots \\ \cdots \end{array}$$

$$\left.\begin{array}{c} (1\text{ -}\alpha-m-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),A:C \\ \cdots \\ (1\text{ -}\mu-m-t'R_1-\cdots-t^{(s)}R_s;t_1,\cdots,t_r),B:D \end{array}\right) \quad (4.2)$$

under the same conditions and notations that (3.2) with $A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{R}$ and $m'=0$

Corollary 3

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}$$

$$S_L^{h_1,\cdots,h_s}\begin{pmatrix} \mathrm{y}_1 x^{\sigma'}y^{\rho'}z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'} \\ \cdots \\ \mathrm{y}_s x^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}} \end{pmatrix}$$

$$H\begin{pmatrix} \mathrm{z}_1 x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1} \\ \cdots \\ \mathrm{z}_r x^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r} \end{pmatrix}$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z = \frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$\sum_{k,m=0}^{\infty}\frac{u^k(1-v)^k v^m(1-u)^m}{k!m!}B_s y_1^{R_1}\cdots y_s^{R_s}H_{p+6,q+4;V}^{0,n+6;X}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\right|$$

$$(1-c-\sigma' R_1-\cdots-\sigma^{(k)}R_s;\sigma_1,\cdots,\sigma_r),\ (\tfrac{1}{2}-c-\sigma' R_1-\cdots-\sigma^{(s)}R_s+a+b;\sigma_1,\cdots,\sigma_r),$$
$$\cdots \qquad \cdots$$
$$(\tfrac{1}{2}-c-\sigma' R_1-\cdots-\sigma^{(k)}R_s+a;\sigma_1,\cdots,\sigma_r),\ (\tfrac{1}{2}-c-\sigma' R_1-\cdots-\sigma^{(s)}R_s+b;\sigma_1,\cdots,\sigma_r),$$

$$(1-\mu-(t'-\zeta')R_1+\cdots+(t^{(s)}-\zeta^{(s)})R_s+\beta;t_1-\zeta_1,\cdots,t_r-\zeta_r),$$
$$\cdots$$
$$(1-\lambda-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s-k;\eta_1,\cdots,\eta_r),$$

$$(1-\lambda+\alpha-(\eta'-\rho')R_1-\cdots-(\eta^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1-\mu-m-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s-m;t_1,\cdots,t_r),$$

$$(1-\alpha-k-\rho' R_1-\cdots-\rho^{(s)}R_s-m;\rho_1,\cdots,\rho_r),$$
$$\cdots$$
$$,\cdots\cdots$$

$$\left.\begin{matrix}(1-\beta-k-\zeta' R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),A:C\\ \cdots\\ \mathrm{B}:\mathrm{D}\end{matrix}\right) \tag{4.3}$$

under the same conditions and notations that (3.3) with $A_j^{(i)},B_j^{(i)},C_j^{(i)},D_j^{(i)}\in\mathbb{R}$ and $m'=0$

5. Srivastava-Daoust polynomial

If $B(L;R_1,\cdots,R_s)=\dfrac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+\cdots+R_s\theta_j^{(s)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(s)}}(b_j^{(s)})_{R_s\phi_j^{(s)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+\cdots+m_s\psi_j^{(s)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(s)}}(d_j^{(s)})_{R_s\delta_j^{(s)}}}$ **(5.1)**

then the general class of multivariable polynomial $S_L^{h_1,\cdots,h_s}[z_1,\cdots,z_s]$ reduces to generalized Srivastava-Daoust polynomial defined by Srivastava et al [5].

$$F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{array}{c}\mathrm{z}_1\\ \cdots\\ \cdots\\ \mathrm{z}_s\end{array}\middle|\begin{array}{c}[(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{array}\right)\quad(5.2)$$

and we have the following formulas

Corollary 4

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\lambda-\alpha-1}(1-yzt)^{-\lambda}$$

$$F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{array}{c}\mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{\mu_1-\rho'}(1-z)^{\mu_1-\zeta'}(1-yzt)^{-\mu_1}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{\mu_s-\rho^{(s)}}(1-z)^{\mu_s-\zeta^{(s)}}(1-yzt)^{-\mu_s}\end{array}\right|$$

$$\left.\begin{array}{c}[(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{array}\right)$$

$$A\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{\eta_1-\zeta_1}(1-yzt)^{-\eta_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{\eta_r-\zeta_r}(1-yzt)^{-\eta_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k=0}^{\infty}\frac{t^k}{k!}B'_sy_1^{R_1}\cdots y_s^{R_s}A^{m',n+6;X}_{p+6,q+4;V}\left(\begin{array}{c}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{array}\right|$$

$$\begin{array}{cc}(1\text{- c -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r), & (\tfrac{1}{2}\text{- c +a+b-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\\ \cdots & \cdots\\ (\tfrac{1}{2}-c+a-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r), & (\tfrac{1}{2}-c+b-c_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\end{array}$$

$$(1\text{-}\lambda+\alpha-(\mu'-\zeta')R_1-\cdots-(\mu^{(s)}-\zeta^{(s)})R_s;\eta_1-\zeta_1,\cdots,\eta_r-\zeta_r),$$
$$\cdots$$
$$\cdots$$

$$(1\text{-}\lambda+\beta-(\mu'-\rho')R_1-\cdots-(\mu^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r),$$
$$\cdots$$
$$(1\text{-}\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s-k;\eta_1,\cdots,\eta_r),(1-\lambda-\mu'R_1-\cdots-\mu^{(s)}R_s;\eta_1,\cdots,\eta_r),$$

$$\left.\begin{matrix}(1\text{-}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),A:C\\ \cdots\\ \mathrm{B:D}\end{matrix}\right) \tag{5.3}$$

under the same notations and conditions that (3.1) with $B'_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where

$B(E;R_1,\cdots,R_s)$ is defined by (5.1)

Corollary 5

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\beta-1}z^{\alpha-1}(1-y)^{\lambda-\beta-1}(1-z)^{\mu-\alpha-1}(1-uy-vz)^{-n}$$

$$F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}\left(\begin{matrix}\mathrm{y}_1x^{c_1}y^{\rho'}z^{\zeta'}(1-y)^{e'-\rho'}(1-z)^{t'-\zeta'}(1-uy-vz)^{-\omega'}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy-vz)^{-\omega^{(s)}}\end{matrix}\right|$$

$$\left.\begin{matrix}[(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right)$$

$$A\begin{pmatrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-z)^{t_1-\zeta_1}(1-uy-vz)^{-n_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-z)^{t_r-\zeta_r}(1-uy-vz)^{-n_r}\end{pmatrix}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}\sum_{k,m=0}^{\infty}\frac{u^kv^m}{k!m!}B'_sy_1^{R_1}\cdots y_s^{R_s}A^{m',n+7;X}_{p+7,q+5;V}\left(\begin{matrix}\mathrm{z}_1\\ \cdots\\ \mathrm{z}_r\end{matrix}\right|$$

$$(1\text{ -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\quad(1/2\text{ - c +a+b -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$
$$\cdots\qquad\qquad\cdots$$
$$(\tfrac{1}{2}\text{+b -c-c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),\quad(1/2\text{ - c+a -c}_1R_1-\cdots-c_sR_s;\sigma_1,\cdots,\sigma_r),$$

$$(1\text{ -}\lambda-(e'-\rho')R_1-\cdots-(e^{(s)}-\rho^{(s)})R_s;\eta_1-\rho_1,\cdots,\eta_r-\rho_r)$$
$$\cdots$$
$$\cdots$$

$$(1\text{- }\mu-(t'-\zeta')R_1-\cdots-(t^{(s)}-\zeta^{(s)})R_s;t_1-\zeta_1,\cdots,t_r-\zeta_r)$$
$$\cdots$$
$$(1\text{-n-}\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),(1-\lambda-e'R_1-\cdots-e^{(s)}R_s-k;\eta_1,\cdots,\eta_r)$$

$$(1\text{ -}\beta-\rho'R_1-\cdots-\rho^{(s)}R_s-k;\rho_1,\cdots,\rho_r),(1-n-\omega'R_1-\cdots-\omega^{(s)}R_s;\eta_1,\cdots,\eta_r),$$
$$\cdots$$
$$\cdots$$

$$\left.\begin{matrix}(1\text{ -}\alpha-m-\zeta'R_1-\cdots-\zeta^{(s)}R_s;\zeta_1,\cdots,\zeta_r),A:C\\ \cdots\\ (1\text{ -}\mu-m-t'R_1-\cdots-t^{(s)}R_s;t_1,\cdots,t_r),B:D\end{matrix}\right) \tag{5.4}$$

under the same notations and conditions that (3.2) with $B'_s=\dfrac{(-L)_{h_1R_1+\cdots+h_sR_s}B(E;R_1,\cdots,R_s)}{R_1!\cdots R_s!}$ where

$B(E;R_1,\cdots,R_s)$ is defined by (5.1)

Corollary 6

$$\int_0^1\int_0^1\int_0^1 x^{c-1}(1-x)^{-1/2}{}_2F_1(a,b;a+b+1/2;x)y^{\alpha-1}z^{\beta-1}(1-y)^{\lambda-\alpha-1}(1-z)^{\mu-\beta-1}$$

$$(1-uy)^{\alpha-\lambda-\mu+1}(1-vz)^{\beta-\lambda-\mu+1}(1-ux-vy)^{\lambda+\mu-\alpha-\beta-1}F^{1+\bar{A}:B';\cdots;B^{(s)}}_{\bar{C}:D';\cdots;D^{(s)}}$$

$$\left(\begin{matrix}\mathrm{y}_1x^{\sigma'}y^{\rho'}z^{\zeta'}(1-y)^{\eta'-\rho'}(1-z)^{t'-\zeta'}(1-uy)^{\rho'-\eta'-t'}(1-vz)^{\zeta'-\eta'-t'}(1-uy-vz)^{\eta'+t'\rho'-\zeta'}\\ \cdots\\ \mathrm{y}_sx^{c_s}y^{\rho^{(s)}}z^{\zeta^{(s)}}(1-y)^{e^{(s)}-\rho^{(s)}}(1-z)^{t^{(s)}-\zeta^{(s)}}(1-uy)^{\rho^{(s)}-\eta^{(s)}-t^{(s)}}(1-vz)^{\zeta^{(s)}-\eta^{(s)}-t^{(s)}}(1-uy-vz)^{\eta^{(s)}+t^{(s)}-\rho^{(s)}-\zeta^{(s)}}\end{matrix}\right|$$

$$\left.\begin{matrix}[(\text{-L});\mathrm{R}_1,\cdots,R_s][(a);\theta',\cdots,\theta^{(s)}]:[(b');\phi'];\cdots;[(b^{(s)});\phi^{(s)}]\\ [(\mathrm{c});\psi',\cdots,\psi^{(s)}]:[(d');\delta'];\cdots;[(d^{(s)});\delta^{(s)}]\end{matrix}\right)$$

$$A\left(\begin{matrix}\mathrm{z}_1x^{\sigma_1}y^{\rho_1}z^{\zeta_1}(1-y)^{\eta_1-\rho_1}(1-uy)^{\rho_1-\eta_1-t_1}(1-vz)^{\zeta_1-\eta_1-t_1}(1-uy-vz)^{\eta_1+t_1-\zeta_1-\rho_1}\\ \cdots\\ \mathrm{z}_rx^{\sigma_r}y^{\rho_r}z^{\zeta_r}(1-y)^{\eta_r-\rho_r}(1-uy)^{\rho_r-\eta_r-t_r}(1-vz)^{\zeta_r-\eta_r-t_r}(1-uy-vz)^{\eta_r+t_r-\zeta_r-\rho_r}\end{matrix}\right)$$

$$\mathrm{d}x\mathrm{d}y\mathrm{d}z=\frac{\pi\Gamma(a+b+1/2)}{\Gamma(a+1/2)\Gamma(b+1/2)}\sum_{R_1,\cdots,R_s=0}^{h_1R_1+\cdots h_sR_s\leqslant L}$$

$$\sum_{k,m=0}^{\infty} \frac{u^k(1-v)^k v^m(1-u)^m}{k!m!} B_s y_1^{R_1} \cdots y_s^{R_s} A_{p+6,q+4;V}^{m',n+6;X} \left(\begin{array}{c} \mathrm{z}_1 \\ \cdot \cdot \cdot \\ \mathrm{z}_r \end{array} \right|$$

$$(1\text{ -c -}\sigma' R_1 - \cdots - \sigma^{(k)} R_s; \sigma_1, \cdots, \sigma_r), \quad (\tfrac{1}{2}\text{-c - } \sigma' R_1 - \cdots - \sigma^{(s)} R_s + a + b; \sigma_1, \cdots, \sigma_r),$$
$$\cdots \qquad \cdots$$
$$(\tfrac{1}{2}\text{ -c- } \sigma' R_1 - \cdots - \sigma^{(k)} R_s + a; \sigma_1, \cdots, \sigma_r), \quad (\tfrac{1}{2}\text{ -c - } \sigma' R_1 - \cdots - \sigma^{(s)} R_s + b; \sigma_1, \cdots, \sigma_r),$$

$$(1\text{ -}\mu - (t' - \zeta')R_1 + \cdots + (t^{(s)} - \zeta^{(s)})R_s + \beta; t_1 - \zeta_1, \cdots, t_r - \zeta_r),$$
$$\cdots$$
$$(1\text{-}\lambda - (\eta' - \rho')R_1 - \cdots - (\eta^{(s)} - \rho^{(s)})R_s - k; \eta_1, \cdots, \eta_r),$$

$$(1\text{ -}\lambda + \alpha - (\eta' - \rho')R_1 - \cdots - (\eta^{(s)} - \rho^{(s)})R_s; \eta_1 - \rho_1, \cdots, \eta_r - \rho_r),$$
$$\cdots$$
$$(1\text{ -}\mu - m - (t' - \zeta')R_1 - \cdots - (t^{(s)} - \zeta^{(s)})R_s - m; t_1, \cdots, t_r),$$

$$(1\text{ -}\alpha - k - \rho' R_1 - \cdots - \rho^{(s)} R_s - m; \rho_1, \cdots, \rho_r),$$
$$\cdots$$
$$, \cdots \cdots$$

$$\left. \begin{array}{c} (1\text{ -}\beta - k - \zeta' R_1 - \cdots - \zeta^{(s)} R_s; \zeta_1, \cdots, \zeta_r), A : C \\ \cdots \\ \mathrm{B} : \mathrm{D} \end{array} \right) \quad \textbf{(5.5)}$$

under the same notations and conditions that (3.3) with $B_s' = \frac{(-L)_{h_1 R_1 + \cdots + h_s R_s} B(E; R_1, \cdots, R_s)}{R_1! \cdots R_s!}$ where

$B(E; R_1, \cdots, R_s)$ is defined by (5.1)

6. Conclusion

The A-function of several variables presented in this paper, is quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain the triple Eulerian integrals concerning various other special functions such as H-function of several variables defined by Srivastava et al [8], for more details, see Garg et al [3], and the H-function of two variables , see Srivastava et a[7].

References :

[1] Erdelyi, A., Higher Transcendental function, McGraw-Hill, New York, Vol 1 (1953).

[2] Exton, H, Handbook of hypergeometric integrals, Ellis Horwood Ltd, Chichester (1978)

[3] Garg O.P., Kumar V. and Shakeeluddin : Some Euler triple integrals involving general class of polynomials and multivariable H-function. Acta. Ciencia. Indica. Math. 34(2008), no 4, page 1697-1702.

[4] Gautam B.P., Asgar A.S. and Goyal A.N. On the multivariable A-function. Vijnana Parishas Anusandhan Patrika Vol 29(4) 1986, page 67-81.

[5] Srivastava H.M. and Daoust M.C. Certain generalized Neumann expansions associated with Kampé de Fériet

function. Nederl. Akad. Wetensch. Proc. Ser A72 = Indag Math 31(1969) page 449-457.

[6] Srivastava H.M. And Garg M. Some integral involving a general class of polynomials and multivariable H-function.Rev. Roumaine Phys. 32(1987), page 685-692.

[7] Srivastava H.M., Gupta K.C. and Goyal S.P., the H-function of one and two variables with applications, South Asian Publications, NewDelhi (1982).

[8] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

[9] Vyas V.M. and Rathie K., An integral involving hypergeometric function. The mathematics education 31(1997) page33

Personal adress : 411 Avenue Joseph Raynaud

Le parc Fleuri , Bat B

83140 , Six-Fours les plages

Tel : 06-83-12-49-68

Department : VAR

Country : FRANCE