Solving Transportation Problem by Various Methods and Their Comaprison

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Abstract: *The most important and successful applications in the optimization refers to transportation problem (tp), that is a special class of the linear programming (lp) in the operation research (or).*

Transportation problem is considered a vitally important aspect that has been studied in a wide range of operations including research domains. As such, it has been used in simulation of several real life problems. The main objective of transportation problem solution methods is to minimize the cost or the time of transportation.

An Initial Basic Feasible Solution(IBFS) for the transportation problem can be obtained by using the North-West corner rule, Miinimum Cost Method and Vogel's Approximation Method. In this paper the best optimality condition has been checked.

Thus, optimizing transportation problem of variables has remarkably been significant to various disciplines.

Key words:

Transportation problem, Linear Programming (LP),

1. Introduction

The first main purpose is solving transportation

problem using three methods of transportation model by linear programming (LP).The three methods for solving Transportation problem are:

1. North West Corner Method

2.Minimum Cost Method

3. Vogel's approximation Method

Trannsportation Model

Transportation model is a special type of networks problems that for shipping a commodity from source (e.g., factories) to destinations (e.g., warehouse).

Transportation model deal with get the minimum-cost plan to transport a commodity from a number of sources (m) to number of destination (n).

Let s_i is the number of supply units required at source i (i=1, 2, 3....... m), d_j is the number of demand units required at destination j (j=1, 2,3..... n) and c_{ij} represent the unit transportation cost for transporting the units from sources i to destination j.

Using linear programming method to solve transportation problem, we determine the value of

objective function which minimize the cost for transporting and also determine the number of unit can be transported from source i to destination j.

If x_{ij} is number of units shipped from source i to

destination j.

The objective function

$$\text{minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = s_i \quad \text{for } i=1,2,...m.$$

$$\sum_{i=1}^{m} x_{ij} = d_j \quad \text{for } j=1,2,....n.$$

And $x_{ij} \geq 0$ for all i to j.

A transportation problem said to be balanced if the supply from all sources equals the total demand in all destinations

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

Otherwise it is called unbalanced.

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METHODS FOR SOLVING

TRANSPORTATION PROBLEM

There are three methods to determine the solution for balanced transportation problem:

1. Northwest Corner method

2. Minimum cost method

3. Vogel's approximation method

The three methods differ in the "quality" of the starting basic solution they produce and better starting solution yields a smaller objective value.

We present the three methods and an illustrative example is solved by these three methods.

1. **North- West Corner Method**

The method starts at the Northwest-corner cell (route) of the tableau (variable x_{11})

(i) Allocate as much as possible to the selected cell and adjust the associated a mounts of supply and demand by subtracting the allocated amount.

(ii) Cross out the row or Column with zero supply or demand to indicate that no further assignments can be made in that row or column. If both a row and a column net to zero simultaneously, cross out one only and leave a zero supply (demand in the uncrossed-out row (column).

(iii) If exactly one row or column is left uncrossed out,

stop .otherwise, move to the cell to the right if a column has just been crossed out or below if a row has been crossed out .Go to step (i).

2. **Minimum-Cost Method**

The minimum-cost method finds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost .Next, the satisfied row or column is crossed out and the amounts of supply and demand are adjusted accordingly. If both a row and a column are satisfied simultaneously, only one is crossed out, the same as in the northwest –corner method .Next ,look for the uncrossed-out cell with the smallest unit cost and repeat the process until exactly one row or column is left uncrossed out .

3. **Vogel's Approximation Method (VAM)**

Vogel's Approximation Method is an improved version of the minimum-cost method that generally produces better starting solutions.

(i) For each row (column) determine a penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

(ii) Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column satisfied row or column. If a row and column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column) is assigned zero supply (demand).

(iii) (a) If exactly one row or column with zero supply or demand remains uncrossed out, stop.

(b) If one row (column) with positive supply (demand) remains uncrossed out, determine the basic variables in the row (column) by the least-cost method .stop.

(c) If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least-cost method .stop.).

(d) Otherwise, go to step (i).

ILLUSTRATIVE EXAMPLE

Millennium Herbal Company ships truckloads of grain from three silos to four mills . The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckload on the different routes are summarized in the transportation model in table.1.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

Table 1. Transportation model of example (Millennium Herbal Company Transportation)

The model seeks the minimum-cost shipping schedule between the silos and the mills. This is equivalent to determining the quantity xij shipped from silo i to mill j (i=1, 2, 3; j=1, 2, 3, 4)

1. **North West-Corner method**

The application of the procedure to the model of the example gives the starting basic solution in table.2.

Table 2. The starting solution using Northwest-corner method

Since $\sum a_i = \sum b_j = 950$

The Starting basic Solution is given as follows :

The first allocation is made in the cell (1,1), the magnitude being x_{11}=min(250,200) = 200. The

Second allocation is made in the cell (1,2) and the magnitude of the allocation is given by

X_{12} =min(250-200,225) =50

Third allocation is made in the cell (2,2) and the magnitude of the allocation is given by

X_{22} =min(300,225-50) =175

Fourth allocation is made in the cell (2,3) and the magnitude of the allocation is given by

X_{23} =min(300-175,275) =125.

Fifth allocation is made in the cell (3,3) and the magnitude of the allocation is given by

X_{33} =min(400,275-125) =150.

Sixth allocation is made in the cell (3,4) and the magnitude of the allocation is given by

X_{34} =min(400-150,250) =250.

Table 2 T.P. solution using North West corner method

Hence an IBFS to the given TP has been obtained and is displayed in the Table 1.1

The Transportation cost according to the above route is given by

Z=200×11 + 50×13 + 175×18 + 125×14 + 150×13 +2 50×10=12200.

2. Minimum Cost Method

The minimum-cost method is applied to Example

(*Millennium Herbal Company*) in the following manner:

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

1. Cell (3,4) has the least unit cost in the tableau (=10).the most that can be shipped through (3,4) is x_{12}=min (250.,300) =250. which happens to satisfy

column 4 simultaneously, we arbitrarily cross out column 4 and adjust in the availability 400-250=50.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	250 10	400-250=150
Requirement	200	225	275	250-250=0	

2.Cell(1,1) has the least unit cost in the tableau (=11).

	D	E	F	G	Available

	D	E	F	G	Available
A	200 11	50 13	17	14	250
B	16	175 18	125 14	10	300
C	21	24	150 13	250 10	400
Requirement	200	225	275	250	
A	200 11	13	17	14	250-200=50
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

the most that can be shipped though (1,1) is x_{11}=min (200.,250) =200. which happens to satisfy

column 1 simultaneously, we arbitrarily cross out column 1 and adjust the in availablility 250-200=50.

3.Continuing in the same manner ,we successively assign Cell (1,2) has the least unit cost in the tableau (=13).the most that can be shipped through (1,2) is x_{12}=min (225.,50) =50. which happens to satisfy row 1 simultaneously, we arbitrarily cross out row 1 and adjust the in requirement 225-50=175.

	D	E	F	G	Available
A	11	50 13	17	14	50-50=0
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225-50=175	275	250	

4. Continuing in the same manner ,we successively assign Cell (3,3) has the least unit cost in the tableau (=13).the most that can be shipped through (3,3) is x_{33}=min (275.,400) =275. which happens to satisfy column 3 simultaneously, we arbitrarily cross out column 3 and adjust the in availability 400-275=125.

	D	E	F	G	Available
A	11	13	17	14	50
B	16	18	14	10	50
C	21	24	150 13	10	150-150=0
Requirement	200	175	275-150=125	250	

5.Continuing in the same manner ,we successively assign Cell (2,2) has the least unit cost in the tableau (=18).the most that can be shipped through (2,2) is x_{22}=min (50.,175) =50. which happens to satisfy row 2 simultaneously, we arbitrarily cross out row 2 and adjust the in requirement 175-50=125..

	D	E	F	G	Available
A	11	13	17	14	50
B	16	175 18	14	10	300-175=125
C	21	24	13	10	125
Requirement	200	175-50=125	275	250	

6.. Continuing in the same manner ,we successively assign Cell (3,2) has the least unit cost in the tableau (=24).the most that can be shipped through (3,2) is x_{32}=min (125.,125) =125. which happens to satisfy row 3 simultaneously, we arbitrarily cross out row 3 and balance the availability and requirement.

	D	E	F	G	Available
A	11	13	17	14	50
B	16	175 18	14	10	175
C	21	24	13	10	125
Requaireme nt	200	175-175=0	275	250	

The final T.P. rout

	D	E	F	G	Available
A	200 11	50 13	17	14	250
B	16	175 18	125 14	10	300
C	21	24	150 13	250 10	400
Requaireme nt	200	225	275	250	

The Transportation cost according to the above route is given by

Z=200×11 + 50×13 +175×18 + 125×14 + 150×13 +250×10 =12200.

3. Vogel's Approximation Method (VAM)

VAM is applied to Example in the following manner:- We computes the difference between the smallest and next-to- smallest cost in each row and each column are computeed and displayed inside the parenthesis against the respective rows and columns. The largest of these differences is (5) and is associated with the first column is c_{11} , we allocate x_{11}=min(250,200)=200 in the cell (1,1).This exhausts the requirement of the first column and , therefore ,we cross off the first column. The row and column differences are now computed for the resulting reduced transportation Table (3.1), the largest of these is (5) which is associated with the second column. Since c_{12}(=13) is the minimum cost we allocate x_{12} = min(50,225)=50.

	D	E	F	G	Available
A	200 11	13	17	14	250-200=50 (2)
B	16	18	14	10	300 (4)
C	21	24	13	10	400 (3)
Requirement	200-200=0 (5)	225 (5)	275 (1)	250 (0)	

Table 3.1

	E	F	G	Available
A	50 13	17	14	50-50=0 (1)
B	18	14	10	300 (4)
C	24	13	10	400 (3)
Requirement	225-50=175 (5)	275 (1)	250 (0)	

Table 3.2

This exhausts the availability of first row and therefore, we cross off the first row. Continuing in this manner , the subsequent reduced transportation tables and the differences of the surviving rows and columns are shown below :

	E	F	G	Available
B	175 18	14	10	300 - 175=125 (4)
C	24	13	10	400 (3)
Requirement	175-175=0(6)	275 (1)	250 (0)	

Table 3.3

	F	G	Available
B	14	125 10	125-125=0 (4)
C	13	10	400 (3)
Requirement	275 (1)	250-125=0 (0)	

Table 3.4

	F	G	Available
C	275 13	10	400 (3)
Requirement	275 (1)	250 (0)	

Table 3.5

Eventually, the basic feasible solution shown in Table 3.7 is obtained

	D	E	F	G	Available
A	200 11	50 13	17	14	250
B	16	175 18	14	125 10	300
C	21	24	275 13	125 10	400
Requirement	200	225	275	250	

The transportation cost according to this route is given by

Z=200×11 + 50×13 + 175×18 + 125×10 + 275×13 +12 5×10=12075.

VAM produces a better starting Solution.

This cost is less than northwest-corner method

COMPARISON BETWEEN THE THREE METHODS

North-west corner method is used when the purpose of completing demand No. 1 and then the next and is used when the purpose of completing the warehouse No. 1 and then the next. Advantage of North-West corner method is quick solution because computations take short time but yields a bad solution because it is very far from optimal solution.

Vogel's approximation method and Minimum-cost method is used to obtain the shortest road. Advantage of Vogel’s approximation method and Minimum-cost method yields the best starting basic solution because gives initial solution very near to optimal solution but the solution of Vogel’s approximation methods is slow because computations take long time. The cost of transportation with Vogel's approximation method and Minimum-cost method is less than North-West corner method.

The cost of transportation is less than North-west

corner method.

CONCLUSION

The result in three methods are defferent. The decision maker may choose the optimal result of the running of the three program (minimum) and determined the number of units transported from source i to destination j.

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