Derivation of Conditions for Derivative of Area equal Perimeter

[1]A.Pakapongpun

[1]Assistant Professor, Department of Mathematics, Faculty of Science, Burapha University
169 Muang Chonburi, Chonburi, Thailand 20131

Abstract—*Normally, area and perimeter of geometry are rarely related. In this research, the conditions of the relationship between the length of the perimeter and the derivative of area of the rhombus has been found. Moreover, the relationships for the polygon, rectangle, right-angled triangle and equilateral triangle have been derived.*

Keywords—*Perimeter, Area, Derivative, Geometry.*

1. INTRODUCTION

Finding the relationship between the length of the perimeter and the derivative of the area.In 2013, Rina Zazkis, llya Sinitsky and Roza Leikin also studied the relationship between the length of the perimeter and the derivative of the area of the circle, the square and the hexagon.

Circle

The relationship between the area A of a circle with radius r and its circumference P can be expressed by $A'(r) = (\pi r^2)' = 2\pi r = P(r)$.

Square

A square with size a the derivative relationship between the area A and the perimeter $P: A'(a) = \frac{P(a)}{2}$

Hexagon

A hexagon with size a the derivative relationship between the area A and the perimeter $P: A'(k) = P(k)$ where k is the radius of the inscribed circle.See more detail in [1].

In this paper, I have extended to find the relationship of the area of a rhombus and its perimeter with size a. We then extend this relationship to the rectangle, right-angled triangle and equilateral triangle.

2. MAIN RESULTS

2.1 Rhombus

Let n, m and i be given positive real numbers.Consider, a rhombus with size $(2n+i)ma$ and h is the height of rhombus, as shown in figure 1.

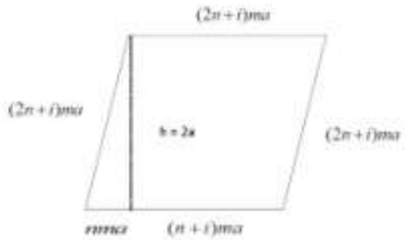

Fig. 1 A rhombus with dimensions.

Since

$$A'(a) = P(a) = 4(2n+i)ma.$$

Hence

$$\int P(a)da = \int 4(2n+i)mada$$
$$= 2(2n+i)ma^2$$
$$= A(a)$$

and we know that,

$$A(a) = (2n+i)ma \cdot h$$

Thus

$$h \cdot (2n+i)ma = 2(2n+i)ma^2,$$

so,

$$h = 2a.$$

Since

$$h = \sqrt{((2n+i)ma)^2 - (nma)^2} = 2a,$$

thus

$$\sqrt{(((2n+i)^2 - n^2)m^2a^2))} = 2a,$$
$$(((2n+i)^2 - n^2)m^2a^2) = 4a^2,$$
$$m^2 = \frac{4}{(2n+i)^2 - n^2}.$$

Hence

$$m=\frac{2}{\sqrt{(2n+i)^2-n^2}}.$$

Therefore, the derivative relativeship between the area A of a rhombus and its perimeter P is expressed. Given fixed positive real numbers n and i. We then get $m=\frac{2}{\sqrt{(2n+1)^2-n^2}}$ and $h=2a$.

Hence,

$$A'(a)=P(a).$$

Example 2.1. Given $n=2$ and $i=3$.

We then get $m=\frac{2}{\sqrt{45}}$ and $h=2a$ as shown in figure 2.

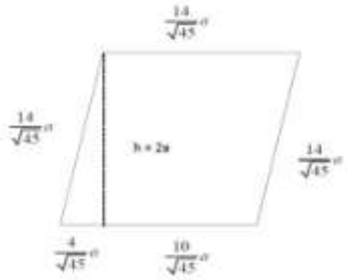

Fig. 2 An example of rhombus.

Let P is represented by the perimeter of rhombus. So, the perimeter

$$P(a)=4\left(\frac{14a}{\sqrt{45}}\right)(a)=\frac{56a}{\sqrt{45}},$$

and the area

$$A(a)=\frac{14a}{\sqrt{45}}\times 2a=\frac{28a^2}{\sqrt{45}}.$$

Therefore,

$$A'(a)=\frac{56a}{\sqrt{45}}=P(a).$$

2.2 Rectangle

Given fixed positive real numbers for m and n, and the rectangle with size as shown in figure 3.

Fig. 3 A rectangle of size shown.

We are going to find the relationship of m and n. The area of the rectangle $A(a)=mna^2$ and the perimeter $P(a)=2a(m+n)$.

So,

$$A'(a)=2mna=2a(m+n)=P(a).$$

Then,

$$m=\frac{n}{n-1}.$$

Therefore, given fixed positive real number $n>2$. The rectangle with size na and another $\frac{na}{n-1}$ as shown in figure 4.

Fig. 4 A further rectangle with dimensions shown.

Example 2.2. Given $n=3$.

Taking the rectangle with size $3a$ and $\frac{3a}{2}$ as shown in figure 5.

Fig. 5 A example of rectangle.

We know that $P(a)=9a$ and $A(a)=\frac{9a^2}{2}$.

Hence,

$$A'(a)=9a=P(a).$$

2.3 Right-Angled Triangle

Given fixed positive real numbers for m and n, the right-angled triangle with size as shown in figure 6.

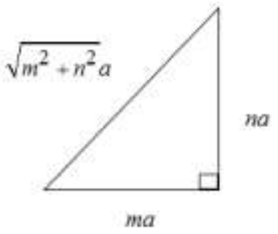

Fig. 6A right-angled triangle of size shown.

We are going to find the relationship of m and n.The area of the right-angled triangle:

$$A(a)=\frac{mna^2}{2},$$

And the perimeter:

$$P(a)=(m+n+\sqrt{m^2+n^2})a.$$

So,

$$A'(a)=mna=(m+n+\sqrt{m^2+n^2})a=P(a).$$

Hence,

$$m=\frac{2n-2}{n-2}.$$

Therefore, given fixed positive real number n and greater than 2, the right- angled triangle with the size is as shown in figure 7.

Hence,

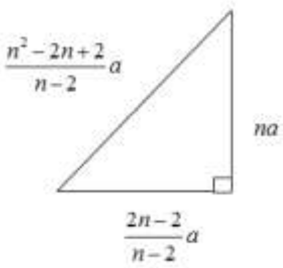

Fig. 7A right-angled triangle with dimension shown.

$$A'(a)=\frac{2(n^2-n)a}{n-2}=P(a).$$

Example 2.3. Given $n=3$.

The right-angled triangle with size as shown in figure 8.

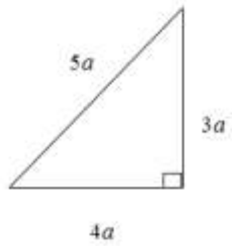

Fig. 8 An example ofright-angled triangle.

Since $P(a)=12a$ and $A(a)=6a^2$ hence,

$$A'(a)=12a=P(a).$$

2.4 Equilateral Trianble

The equilateral triangle with size $2\sqrt{3}a$ as shown in figure 9.

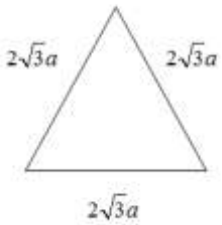

Fig. 9 Anequilateral triangle with dimension shown.

The relationship between the derivative of area of a equilateral triangle and its perimeter is expressed by:

$$P(a)=6\sqrt{3}a,$$

and

$$A(a)=\frac{1}{2}2\sqrt{3}a\cdot\sqrt{(2\sqrt{3}a)^2-(\sqrt{3}a)^2}=3\sqrt{3}a^2.$$

Hence,

$$A'(a)=6\sqrt{3}a=P(a).$$

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References

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