On rgwα-Continuous and rgwα-Irresolute Maps in Topological Spaces

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Abstract: *The aim of this paper is to introduce a new type of functions called the rgwα- continuous map, rgwα- irresolute maps, strongly rgwα-continuous maps, perfectly rgwα-continuous maps and study some of these properties.*

Keywords: *rgwα-open sets, rgwα-closed sets, rgwα-continuous map, rgwα-irresolute maps, strongly rgwα-continuous maps, perfectly rgwα-continuous.*

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I. Introduction:

The continuous functions plays very important role in Topology. Balachandran et.al [5], Levine [18], Mashhour et.al [16], Gnanmbal et.al [11], S. P. Arya and Gupta. R [24] have introduced g-continuity, Semi-continuity, pre- continuity, gpr-continuity, regular and completely-continuous respectively. In 1972 Crossley and Hiledebrand [6] introduced the notion of irresoluteness. In 1981, Munshi and Bassan [17] introduced the notion of generalized continuous (briefly g - continuous) functions which are called in [5] as g-irresolute functions. Furthermore, the notion of gs-irresolute [10] (resp.gp-irresolute [2], αg-irresolute [9], gb- irresolute [3], gsp-irresolute [32]) functions is introduced. Also, the concept of wα-continuous functions was introduced by S S Benchalli et al [30]. Recently R S Wali and Vijayalaxmi R.Patil [23] introduced and studied the properties of rgwα-closed set. The purpose of this paper is to introduce a new class of functions, namely, rgwα-continuous functions and rgwα-irresolute functions strongly rgwα-continuous maps, perfectly rgwα-continuous maps. Also, we study some of the characterization and basic properties of rgwα-continuous functions.

II. Preliminaries:

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) represent a topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A or A^c denotes the complement of A in X. We recall the following definitions and results.

Definition 2.1: A subset A of a topological space (X, τ) is called.

(1) semi-open set [19] if A ⊆ cl(int(A)) and semi-closed set if int(cl(A)) ⊆ A.

(2) pre-open set [1] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.

(3) α-open set [21] if A ⊆ int(cl(int(A))) and α -closed set if cl(int(cl(A)))⊆ A.

(4) semi-pre open set [7] (=β-open) if A⊆cl(int(cl(A)))) and a semi-pre closed set (=β-closed) if int(cl(int(A)))⊆A.

(5) regular open set [15] if A = int(clA)) and a regular closed set if A = cl(int(A)).

(6) Regular semi open set [11] if there is a regular open set U such that U ⊆ A ⊆ cl(U).

(7) Regular α-open set [8] (briefly, rα-open) if there is a regular open set U s.t U ⊆ A ⊆ αcl(U).

Definition 2.2: A subset A of a topological space (X, τ) is called

(1) w-closed set [23] if cl(A) ⊆ U whenever A⊆ U and U is semi-open in X.

(2) wα- closed set [30] if αcl(A) ⊆ U whenever A⊆ U and U is w-open in X.

(3) generalized closed set(briefly g-closed) [18] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

(4) generalized semi-closed set(briefly gs-closed)[27] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.

(5) generalized pre regular closed set(briefly gpr-closed)[33]if pcl(A)⊆U whenever A⊆U andU is regular open in X.

(6) regular generalized α-closed set (briefly, rgα-closed) [2] if αcl (A) ⊆ U whenever A⊆ U and U is regular α-open in X.

(7) α-generalized closed set (briefly αg -closed) [13] if αcl(A)⊆U whenever A⊆U and U is open in X.

(8) generalized α-closed set (briefly gα-closed) (20), if αcl(A) ⊆U whenever A⊆U and U is α-open in X.
(9) weakely generalized closed set (briefly, wg-closed)[20] if cl(int(A))⊆U whenever A⊆U and U is open in X.
(10) regular weakly generalized closed set (briefly, rwg-closed) [20] if cl(int(A)) ⊆ U whenever A⊆U and U is regular open in X.
(11) generalized pre closed (briefly gp-closed) set [12] if pcl(A) ⊆U whenever A⊆U and U is open in X.
(12) regular w-closed (briefly rw -closed) set [31] if cl(A) ⊆ U whenever A⊆U and U is regular semi-open in X.
(13) generalized regular closed (briefly gr–closed) set [24] if rcl(A)⊆ U whenever A⊆ U and U is open in X.
(14) regular generalized weak (briefly rgw-closed) set[25] if cl(int(A)) ⊆ U whenever A⊆ U and U is regular semi open in X.
(15) generalized weak α-closed (briefly gwα-closed) set [29] if αcl(A) ⊆U whenever A⊆U & U is wα- open in X.
(16) generalized star weakly α-closed set (briefly g*wα-closed) [28] if cl(A) ⊆U whenever A⊆U & U is wα-open in X.

The compliment of the above mentioned closed sets are their open sets respectively.

Definition 2.3: A map f: (X, τ) → (Y, σ) is said to be
(i) regular-continuous(r-continuous) [24] if $f^{-1}(V)$ is r-closed in X for every closed subset V of Y.
(ii) completely-continuous [24] if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.
(iii) strongly-continuous [15] if $f^{-1}(V)$ is clopen (both open and closed) in X for every subset V of Y.
(iv) g-continuous [30] if $f^{-1}(V)$ is g-closed in X for every closed subset V of Y
(v) w-continuous [23] if $f^{-1}(V)$ is w-closed in X for every closed subset V of Y
(vi) α-continuous [21] if $f^{-1}(V)$ is α-closed in X for every closed subset V of Y.
(vii) wα-continuous [30] if $f^{-1}(V)$ is wα-closed in X for every closed subset V of Y.
(viii) strongly α-continuous [34] if $f^{-1}(V)$ is α-closed in X for every semi-closed subset V of Y.
(ix) αg-continuous [13] if $f^{-1}(V)$ is αg-closed in X for every closed subset V of Y.
(x) wg-continuous [20] if $f^{-1}(V)$ is wg-closed in X for every closed subset V of Y.
(xi) rwg-continuous [20] if $f^{-1}(V)$ is rwg-closed in X for every closed subset V of Y.
(xii) gs-continuous [18] if $f^{-1}(V)$ is gs-closed in X for every closed subset V of Y.
(xiii) gpr-continuous [33] if $f^{-1}(V)$ is gpr-closed in X for every closed subset V of Y.
(xiv) rgα-continuous [2] if $f^{-1}(V)$ is rgα-closed in X for every closed subset V of Y.
(xv) gr-continuous [24] if $f^{-1}(V)$ is gr-closed in X for every closed subset V of Y.
(xvi) rw-continuous [31] if $f^{-1}(V)$ is rw-closed in X for every closed subset V of Y.
(xvii) rgw-continuous [25] if $f^{-1}(V)$ is rgw-closed in X for every closed subset V of Y.

Definition 2.4: A map f: (X,τ)→(Y,σ) is said to be
(i) irresolute [30] if $f^{-1}(V)$ is semi- closed in X for every semi-closed subset V of Y
(ii) α-irresolute [21] if $f^{-1}(V)$ is α-closed in X for every α-closed subset V of Y.
(iii) contra irresolute [21] if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V of Y.
(iv) contra w-irresolute [23] if $f^{-1}(V)$ is w-open in X for every w-closed subset V of Y.
(v) contra r-irresolute [24] if $f^{-1}(V)$ is regular-open in X for every regular-closed subset V of Y.
(vi) contra continuous [4] if $f^{-1}(V)$ is open in X for every closed subset V of Y .
(vii) rw*-open(resp rw*-closed) [31] map if f(U) is rw-open (resp. rw-closed) in Y for every rw-open (resp rw-closed) subset U of X.

Lemma 2.5: see [23]
1) Every closed (resp. regular-closed, w-closed, α-closed and β-closed) set is rgwα-closed set in X.

2) Every rw-closed (resp. rs-closed, rα-closed, wα-closed, gα-closed, rgα-closed, gwα-closed and g*wα-closed) set is rgwα-closed set in X.

3) Every rgwα-closed set is gβ-closed set

4) The set g-closed (resp. wg-closed, rg-closed, gr-closd, gpr-closed, rgw-closed, rwg-closed and αg closed) set is independent with rgwα-closed set.

Lemma 2.6: see [23] If a subset A of a topological space X, and

1) If A is weak-open and rgwα-closed then A is α-closed set in X.

2) If A is both weak α-open and rgwα-closed then it is rα-closed set in X

3) If A is weak-open and rα-closed then A is rgwα-closed set in X

4) If A is both open and g-closed then A is rgwα-closed set in

Definition 2.7: A topological space (X, τ) is called

(1) an α-space if every α-closed subset of X is closed in X.

3. rgwα- Continuous Functions.

Definition 3.1: A function f from a topological space X into a topological space Y is called regular generalized weakly α- continuous (briefly rgwα- continuous) if $f^{-1}(V)$ is rgwα- closed set in X for every closed set V in Y.

Theorem 3.2: If a map f is continuous, then it is rgwα-continuous but not conversely.

Proof: Let f: X→Y be continuous. Let F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is closed set in X. Since every closed set is rgwα- closed, by Lemma 2.5, $f^{-1}(F)$ is rgwα- closed in X. Therefore f is rgwα-continuous.

Theorem 3.3: If a map f: X→Y is α-continuous, then it is rgwα-continuous but not conversely.

Proof: Let f: X→Y be α-continuous. Let F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is α-closed set in X. Since every α-closed set is rgwα- closed by Lemma 2.5, $f^{-1}(F)$ is rgwα-closed in X. Therefore f is rgwα-continuous.

The converse need not be true as seen from the following example.

Example 3.4: Let X=Y={a,b,c,d,e}, τ ={X,ϕ,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}} and σ ={Y, ϕ, {a},{d},{e}, {a,d}, {a,e},{d,e},{a,d,e }. Let map f: X→Y defined by f(a)=c , f(b)= a , f(c)=b, f(d)=d ,f(e)=e, then f is rgwα- continuous but not continuous and not α-continuous, as closed set F= {a,c,e} in Y, then $f^{-1}(F)$={a,b,e} in X which is not closed and also not α-closed set in X.

Theorem 3.5: If a map f: X→Y is continuous, then the following holds.

(i) if f is r-continuous, then f is rgwα-continuous.

(ii) if f is w-continuous, β- continuous, rw- continuous, rs- continuous, rα- continuous, wα continuous, gα-continuous, rgα- continuous, gwα- continuous, g*wα-continuous, then f is rgwα-continuous.

Proof: (i) Let F be a closed set in Y. Since F is r-continuous, then $f^{-1}(F)$ is r-closed in X. Since every r-closed set is rgwα-closed by Lemma 2.5, then $f^{-1}(F)$ is rgwα-closed in X. Hence f is rgwα-continuous.

Similarly we can prove (ii).

The converse need not be true as seen from the following example.

Example 3.6: Let X=Y={a,b,c,d,e}, τ ={X, ϕ,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}} and σ ={Y, ϕ, {a},{d},{e}, {a,d}, {a,e},{d,e},{a,d,e }. Let map f: X→Y defined by f(a)=c , f(b)= a , f(c)=b, f(d)=d , f(e)=e, then f is rgwα- continuous but not r-continuous, w-continuous, β-continuous, rs-continuous and rα-continuous, rw-continuous, wα- continuous, gα-continuous, rgα- continuous, gwα- continuous, g*wα-continuous as closed set F= {b,c,d,e} in Y, then $f^{-1}(F)$={a,c,d.e} in X which is rgωα-closed but not r-closed, w-closed, β-closed, rs-closed and rα-closed set in X. and closed set F={b,c,d}in Y $f^{-1}(F)$ ={a,b,d} which is not rw-closed, wα-closed, gα- closed, rgα- closed, gwα- closed, g*wα- closed.

Theorem 3.7: If a map f: X→Y is rgwα-continuous, then it is gβ- continuous but not conversely.

Proof: Let f: X→Y is rgwα- continuous. Let F be any closed set in Y. Then the inverse image $f^{-1}(F)$ is rgwα-closed set in X. Since every rgwα-closed set is gβ-closed set by lemma 2.5, $f^{-1}(F)$ is gβ-closed set in X. Therefore f is gβ- continuous.

The converse need not be true as seen from the following example.

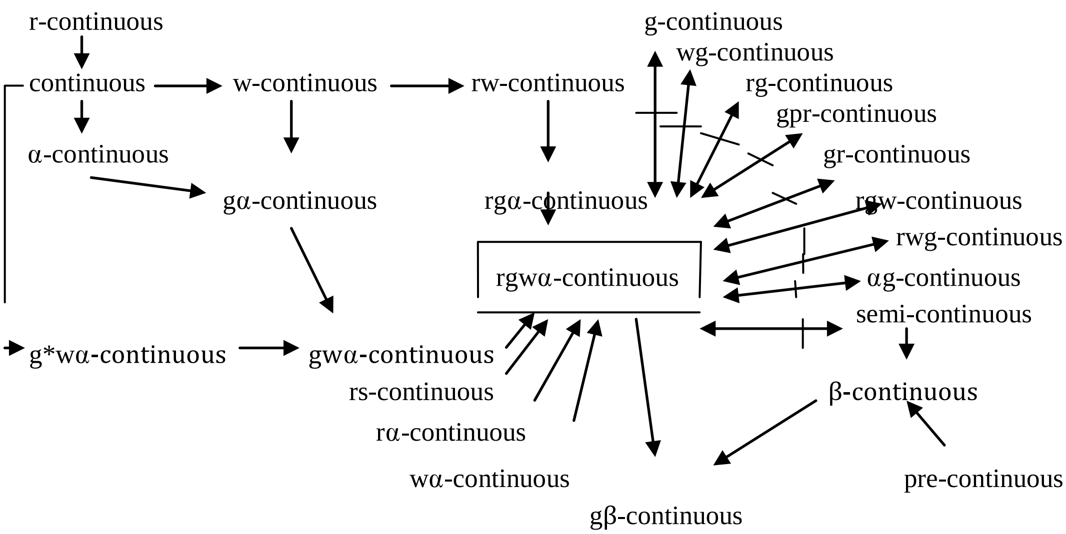
Example 3.8: Let X=Y= {a,b,c}, τ={X, ϕ,{a},{b,c}} and σ ={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b , f(b)= a , f(c)=c, then f is gβ- continuous but not rgwα-continuous as a closed set F={b,c} in Y,$f^{-1}(F)$= f^{-1}{b,c}={a,c} which is not rgwα-closed set.

Remark 3.9: The following examples show that rgwα-continuous maps are independent of g-continuous, wg-continuous, rg-continuous, gr-continuous, gpr-continuous, rgw-continuous, rwg-continuous and αg continuous maps.

Example 3.9: Let X=Y= {a,b,c}, τ={X, ϕ,{a},{b,c}} and σ ={Y, ϕ, {a}}. Let map f: X→Y defined by f(a)=b , f(b)= a , f(c)=c, then f is g-continuous, wg-continuous, rg-continuous, gr-continuous, gpr-continuous, rgw-continuous, rwg-continuous and αg continuous but not rgwα-continuous function, as a closed set F={b,c} in Y $f^{-1}(F)= f^{-1}\{b,c\}$ ={a,c} is not rgwα-closed set.

Example 3.10: Let X= {a,b,c,d}and Y={a,b,c}, τ={X, ϕ,{a},{b},{a,b},{a,b,c}} and σ ={Y, ϕ, {a},{b,c}}. Let map f: X→Y defined by f(a)=b , f(b)= a, f(c)=a, f(d)=c then f is rgwα-continuous but not g-continuous, wg-continuous, rg-continuous, gr-continuous, gpr-continuous, rgw-continuous, rwg-continuous and αg continuous, as a closed set F={a} closed set in Y $f^{-1}(F)$={b} which is not g-closed, wg-closed, rg-closed, gr-closd, gpr-closed, rgw-closed, rwg-closed and αg closed set.

Remark 3.11: From the above discussion and known results we have the following implications



By A →B we mean A implies B but not conversely
and A↮B means A and B are independent of each other.

Theorem 3.12: Let f: X→Y be a map. Then the following statements are equivalent:

(i)f is rgwα-continuous.

(ii) the inverse image of each open set in Y is rgwα-open in X.

Proof: Assume that f:X→Y is rgwα-continuous. Let G be open in Y. The G^c is closed in Y. Since f is rgwα-continuous, $f^{-1}(G^c)$ is rgwα-closed in X. But $f^{-1}(G^c) = X– f^{-1}(G^c)$. Thus $f^{-1}(G^c)$ is rgwα-open in X.
Converserly, Assume that the inverse image of each open set in Y is rgwα-open in X. Let F be any closed set in Y. By assumption $f^{-1}(F^c)$ is rgwα-open in X. But $f^{-1}(F^c) = X– f^{-1}(F)$. Thus $X– f^{-1}(F)$ is rgwα-open in X and so $f^{-1}(F)$ is rgwα-closed in X. Therefore f is rgwα-continuous. Hence (i) and (ii) are equivalent.

Theorem 3.13: If f: (X,τ)→(Y,σ) is map. Then the following holds.

1) f is rgwα-continuous and contra w-irresolute map then f is α-continuous

2) f is rgwα-continuous and contra wα-irresolute map then f is rα-continuous.

3) f is rα-continuous and contra w-irresolute map then f is rgwα-continuous

4) f is g-continuous and contra irresolute map then f is rgwα-continuous.

Proof:
1) Let V be w-closed set of Y, As every w-closed set is closed, V is closed set in Y. Since f is rgwα-continuous and contra w-irresolute map, $f^{-1}(V)$ is rgwα-closed and w-open in X, Now by Lemma 2.6, $f^{-1}(V)$ is α-closed in X. Thus f is α-continuous.

2) Similarly using Lemma 2.6 we can prove this.
3) Let V be closed set of Y. Since f is rα-continuous and contra w-irresolute map, $f^{-1}(V)$ is rα-closed and w-open in X, Now by Lemma 2.6, $f^{-1}(V)$ is rgwα-closed in X. Thus f is rgwα-continuous.
4) Similarly using Lemma 2.6 we can prove this.

Theorem 3.14: Let A be a subset of a topological space X. Then x ∈ rgwαcl(A) if and only if for any rgwα-open set U containing x, A∩U≠ ϕ.

Proof: Let x ∈ rgwαcl(A) and suppose that, there is a rgwα-open set U in X such that x ∈ U and A∩U= ϕ implies that A ⊂ U^c which is rgwα-closed in X implies rgwαcl(A) ⊆ rgwαcl(U^c) = U^C. Since x ∈ U implies that x ∉ U^c implies that x ∉ rgwαcl(A), this is a contradiction.

Converserly, Suppose that, for any rgwα-open set U containing x, A∩U≠ ϕ. To prove that x ∈ rgwαcl(A). Suppose that x ∉ rgwαcl(A), then there is a rgwα-closed set F in X such that x ∉ F and A ⊆ F. Since x ∉ F implies that x ∈ F^c which is rgwα-open in X. Since A ⊆ F implies that A ∩ F^c = ϕ, this is a contradiction. Thus x ∈ rgwαcl(A).

Theorem 3.15: Let f: X→Y be a function from a topological space X into a topological space Y. If f: X→Y is rgwα-continuous, then f(rgwαcl(A)) ⊆ cl(f(A)) for every subset A of X.
Proof: Since f(A) ⊆ cl(f(A)) implies that A ⊆ f^{-1}(cl(f(A))). Since cl(f(A)) is a closed set in Y and f is rgwα-continuous, then by definition f^{-1}(cl(f(A))) is a rgwα-closed set in X containing A. Hence rgwαcl(A)⊆ f^{-1}(cl(f(A))). Therefore f(rgwαcl(A)) ⊆ cl(f(A)).
The converse of the above theorem need not be true as seen from the following example.

Example 3.16: Let X=Y={a,b,c}, τ = {X, ϕ, {a},{b,c}} σ ={Y, ϕ,{a}}, Let map f: X→Y defined by , f(a)=b , f(b)=a, f(c)=c. For every subset of X, f(rgwαcl(A)) ⊆ cl(f(A)) holds . But f is not rgwα-continuous since closed set V= {b,c} in Y, f^{-1}(V)={a,c} which is not rgwα-closed set in X.

Theorem 3.17: Let f: X→Y be a function from a topological space X into a topological space Y. Then the following statements are equivalent:

(i) For each point x in X and each open set V in Y with f(x) ∈ V, there is a rgwα-open set U in X such that x ∈ U and f(U) ⊆ V.
(ii) For each subset A of X, f(rgwαcl(A)) ⊆ cl(f(A)).
(iii) For each subset B of Y, rgwαcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).
(iv) For each subset B of Y, f^{-1}(int(B)) ⊆ rgwαint(f^{-1}(B)).
Proof: (i) → (ii) Suppose that (i) holds and let y ∈ f(rgwαcl(A)) and let V be any open set of Y. Since y ∈ f(rgwαcl(A)) implies that there exists x ∈ rgwαcl(A) such that f(x) = y. Since f(x) ∈ V, then by (i) there exists a rgwα-open set U in X such that x∈U and f(U)⊆V. Since x∈f(rgwαcl (A)), then by theorem 3.14 U∩A≠ ϕ. ϕ ≠ f(U∩A) ⊆f(U)∩f(A) ⊆V∩f(A), then V∩f(A) ≠ ϕ .Therefore we have y = f(x) ∈ cl(f((A)). Hence f(rgwαcl(A)) ⊆ cl(f(A)).
(ii)→ (i) Let if (ii) holds and let x ∈ X and V be any open set in Y containing f(x). Let A = $f^{-1}(V^c)$ this implies that x ∉ A. Since f(rgwαcl(A)) ⊆ cl(f(A)) ⊆ V^c this implies that rgwαcl(A) ⊆ $f^{-1}(V^c)$ = A. Since x ∉ A implies that x ∉ rgwαcl(A) and by theorem 3.14 there exists a rgwα-open set U containing x such that U∩A= ϕ ,then U⊆A^c and hence f(U) ⊆ f(A^c) ⊆ V.
(ii)→ (iii) Suppose that (ii) holds and Let B be any subset of Y. Replacing A by f^{-1}(B) we get from (ii) f(rgwαcl(f^{-1}(B))) ⊆ cl(f(f^{-1}(B))) ⊆ cl(B). Hence rgwαcl (f^{-1}(B)) ⊆ f^{-1}(cl(B)).
(iii) → (ii) Suppose that (iii) holds, let B = f(A) where A is a subset of X. Then we get from (iii), rgwαcl(f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))) implies rgwαcl(A)⊆ f^{-1}(cl(f(A))). Therefore f(rgwαcl(A)) ⊆ cl(f(A)).
(iii)→ (iv) Suppose that (iii) holds. Let B ⊆Y, then Y–B ⊆Y. By (iii) , rgwαcl(f^{-1}(Y–B)) ⊆ f^{-1}(cl(Y–B)) this implies X– rgwαint(f^{-1}(B)) ⊆X– f^{-1}(int(B)) . Therefore f^{-1}(int(B)) ⊆ rgwαint(f^{-1}(B)).
(iv) → (iii) Suppose that (iv) holds Let B ⊆Y, then Y–B ⊆Y. By (iv), f^{-1}(int(Y–B)) ⊆ rgwαint(f^{-1}(Y–B)) this implies that X– f^{-1}(cl(B)) ⊆ X–rgwαcl(f^{-1}(B)). Therefore rgwαcl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

Definition 3.18: Let (X, τ) be topological space and $\tau_{rg\omega\alpha}$ = {V⊆X : rgwαcl(V^c) = V^c }, $\tau_{rg\omega\alpha}$ is toplogy on X.
Definition 3.19: 1) A space (X, τ) is called $T_{rgw\alpha}$-space if every rgwα-closed is closed.
2) A space (X, τ) is called $_{\alpha}T_{rgw\alpha}$ -space if every rgwα-closed set is α-closed set.

Theorem 3.20: Let f: X→Y be a function. Let (X,τ) and (Y,σ) be any two spaces such that $\tau_{rgw\alpha}$ is a topology on X. Then the following statements are equivalent:

(i)For every subset A of X, f(rgwαcl(A)) ⊆ cl(f(A)) holds,

(ii) f:(X, $\tau_{rg\omega\alpha}$)→(Y,σ) is continuous.

Proof: Suppose (i) holds. Let A be closed in Y. By hypothesis f(rgwαcl(f^{-1}(A))) ⊆ cl(f(f^{-1}(A))) ⊆ cl(A) = A. i.e. rgwαcl(f^{-1}(A))⊆f^{-1}(A). Also f^{-1}(A)⊆ rgwαcl(f^{-1}(A)). Hence rgwαcl(f^{-1}(A))= f^{-1}(A). This implies f^{-1}(A)∈ $\tau_{rg\omega\alpha}$. Thus f^{-1}(A) is closed in (X, $\tau_{rg\omega\alpha}$) and so f is continuous. This proves (ii).

Suppose (ii) holds. For every subset A of X, cl(f(A)) is closed in Y. Since f:(X, $\tau_{rgw\alpha}$)→(Y,σ) is continuous, f^{-1}(cl(A)) is closed in (X, $\tau_{rgw\alpha}$) that implies by definition 3.22 rgwαcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A))). Now we have, A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))) and by rgwα-closure, rgwαcl(A)⊆ rgwαcl(f^{-1}(cl(f(A)))= f^{-1}(cl(f(A)). Therefore f(rgwαcl(A))⊆ cl(f(A)). This proves (i).

Remark 3.21: The Composition of two rgwα-continuous maps need not be rgwα-continuous map and this can be shown by the following example.

Example 3.22 : Let X=Y=Z={a,b,c}, τ = {X, ϕ,{a},{b},{a,b}}, σ ={Y, ϕ,{a}, {c}, {a,c}}, η ={Z, ϕ,{a},{a,b}, {a,c}} and a maps f : X→Y is defined as f(a)=b, f(b)=c, f(c)=a, and g : Y→Z is defined as g(a)=b, g(b)=a, g(c)=c, Both f and g are rgwα-continuous maps. But g∘f not rgwα-continuous map, since closed set V={b,c} in Z, $(f\circ g)^{-1}$(V)= f^{-1}(g^{-1}(V)) =f^{-1}{b,c} ={a,b} which is not rgwα-closed set in X.

Theorem 3.23: Let f: X→Y is rgwα-continuous function and g: Y→Z is continuous function then g∘f: X→Z is rgwα-continuous.

Proof: Let g be continuous function and V be any open set in Z then g^{-1}(V) is open in Y. Since f is rgwα-continuous, f^{-1}(g^{-1}(V)) = $(g\circ f)^{-1}$(V) is rgwα-open in X. Hence g∘f is rgwα-continuous.

Theorem 3.24: Let f: X→Y is rgwα-continuous function and g:Y→Z is rgwα-continuous function and Y is $\tau_{rgw\alpha}$-space , then g∘f: X→Z is rgwα-continuous.

Proof: Let g be rgwα-continuous function and V be any open set in Z then g^{-1}(V) is rgwα-open in Y and Y is $T_{rgw\alpha}$-space, then g^{-1} (V) is open in Y. Since f is rgwα-continuous, f^{-1}(g^{-1}(V)) = $(g\circ f)^{-1}$(V) is rgwα-open in X. Hence g∘f is rgwα-continuous.

Theorem 3.25: If a map f: X→Y is completely-continuous, then it is rgwα- continuous.

Proof: Suppose that a map f: (X, τ)→(Y, σ) is completely-continuous. Let F closed set in Y. Then f^{-1}(F) is regular closed in X and hence f^{-1}(F) is is rgwα-closed in X. Thus f is rgwα-continuous.

Theorem 3.26: If a map f: X→Y is α-irresolute, then it is rgwα- continuous.

Proof: Suppose that a map f: (X,τ)→(Y,σ) is α-irresolute. Let V be an open set in Y. Then V is α-open in Y. Since f is α-irresolute, f^{-1}(V) is α-open and hence rgwα-open in X. Thus f is rgwα-continuous.

4. Perfectly rgwα-Continuous and rgwα*-Continuous Functions.

Definition 4.1: A function f from a topological space X into a topological space Y is called perfectly regular generalized weakly α- continuous (briefly perfectly rgwα-Continuous) if f^{-1}(V) is clopen (closed and open) set in X for every rgwα-open set V in Y.

Theorem 4.2: If a map f: X→Y is continuous, then the following holds.

(i) If f is perfectly rgwα-continuous, then f is rgwα-continuous.

(ii) If f is perfectly rgwα-continuous, then f is gβ-continuous.

Proof:

(i) Let F be a open set in Y, as every open is rgwα-open in Y, since F is perfectly rgwα-continuous, then f^{-1}(F) is both closed and open in X, as every open is rgwα-open , f^{-1}(F) is rgwα-open in X. Hence f is rgwα-continuous.

(ii) Let F be a open set in Y, as every open is rgwα-open in Y, since F is perfectly rgwα-continuous, then f^{-1}(F) is both closed and open in X, as every open is rgwα-open that implies is gβ-open, then f^{-1}(F) is gβ-open in X. Hence f is gβ-continuous.

Definition 4.3: A function f from a topological space X into a topological space Y is called regular generalized weakly α*- continuous (briefly rgwα*–continuous) if f^{-1}(V) is rgwα-closed set in X for every α-closed set V in Y.

Theorem 4.4: If A map f: (X,τ)→(Y,σ) be function,

(i) f is rgwα-irresolute then it is rgwα*-continuous.

(ii) f is rgwα*-continuous then it is rgwα-continuous.

Proof:

(i) Let f: X→Y be rgwα-irresolute. Let F be any α-closed set in Y. Then F is rgwα-closed in Y. Since f is rgwα-irresolute, the inverse image f^{-1}(F) is rgwα-closed set in X. Therefore f is rgwα*-continuous.

(ii) Let f: X→Y be rgwα*-continuous. Let F be any closed set in Y. Then F is α-closed in Y. Since f is rgwα*-continuous, the inverse image f^{-1}(F) is rgwα-closed set in X. Therefore f is rgwα-continuous.

Theorem 4.5: If a bijection f: (X, τ)→(Y, σ) is wα*-open , rgwα*-continuous, then it is rgwα-irresolute.
Proof: Let A be rgwα-closed in Y. Let $f^{-1}(A) \subseteq U$ where U is wα-open set in X, Since f is wα*-open map, f(U) is wα-open set in Y. A ⊆ f(U) implies rαcl(A) ⊆ f(U). That is, $f^{-1}(r\alpha cl(A)) \subseteq U$. Since f is rgwα*-continuous, $r\alpha cl(f^{-1}(\alpha cl(A))) \subseteq U$. and so $r\alpha cl(f^{-1}(A)) \subseteq U$ This shows $f^{-1}(A)$ is rgwα-closed set in X. Hence f is rgwα-irresolute.

Theorem 4.6: If f: (X, τ) →(Y, σ) is rgwα-continuous and wα*-closed and if A is rgwα-open (or rgwα-closed) subset of (Y, σ) and (Y, σ) is α-space, then $f^{-1}(A)$ is rgwα-open (or rgwα-closed) in (X, τ).
Proof: Let A be a rgwα-open set in (Y, σ) and G be any wα-closed set in (X, τ) such that $G \subseteq f^{-1}(A)$. Then f(G)⊆ A. By hypothesis f(G) is wα-closed and A is rgwα-open in (Y, σ). Therefore f(G) ⊆ rαInt(A) by and so $G \subseteq f^{-1}(r\alpha Int(A))$. Since f is rgwα-continuous, rαInt(A) is α-open in (Y, σ) and (Y, σ) is α-space, so rαInt(A) is open in (Y, σ). Therefore $f^{-1}(r\alpha Int(A))$ is rgwα-open in (X, τ). Thus $G \subseteq r\alpha Int(f^{-1}(r\alpha Int(A))) \subseteq r\alpha Int(f^{-1}(A))$; that is, $G \subseteq r\alpha Int(f^{-1}(A))$, $f^{-1}(A)$ is rgwα-open in (X, τ). By taking the complements we can show that if A is wα-closed in (Y, σ), $f^{-1}(A)$ is rgwα-closed in (X, τ).

Theorem 4.7: Let (X, τ) be a discrete topological space and (Y, σ) be any topological space. Let f: (X, τ) → (Y, σ) be a map. Then the following statements are equivalent: (i) f is strongly rgwα-continuous. (ii) f is perfectly rgwα-continuous.
Proof: (i)=>(ii) Let U be any rgwα-open set in (Y, σ). By hypothesis $f^{-1}(U)$ is open in (X, τ). Since(X, τ) is a discrete space,$f^{-1}(U)$ is also closed in (X, τ). $f^{-1}(U)$ is both open and closed in (X, τ). Hence f is perfectly rgwα-continuous. (ii)=>(i) Let U be any rgwα-open set in (Y, σ). Then $f^{-1}(U)$ is both open and closed in (X, τ). Hence f is strongly rgwα-continuous.

5. rgwα-Irresolute and Strongly rgwα-Continuous Functions.

Definition 5.1: A function f from a topological space X into a topological space Y is called regular generalized weakly α- irresolute (breifly rgwα-irresolute) map if $f^{-1}(V)$ is rgwα-closed set in X for every rgwα-closed set V in Y.

Definition 5.2: A function f from a topological space X into a topological space Y is called strongly regular generalized weakly α- continuous (strongly rgwα-continuous) map if $f^{-1}(V)$ is closed set in X for every rgwα-closed set V in Y.

Theorem 5.3: If A map f: (X, τ) →(Y, σ) is rgwα-irresolute, then it is rgwα-continuous but not conversely.
Proof: Let f: X→Y be rgwα-irresolute. Let F be any closed set in Y. Then F is rgwα-closed in Y. Since f is rgwα-irresolute, the inverse image $f^{-1}(F)$ is rgwα-closed set in X. Therefore f is rgwα-continuous. The converse of the above theorem need not be true as seen from the following example.

Example 5.4 : X={a,b,c,d,e}, Y={a,b,c,d} τ = {X, ϕ,{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e}} σ ={Y, ϕ,{a},{b}, {a,b},{a,b,c}}, Let map f: X→Y defined by , f(a)=b , f(b)=c , f(c)=d , f(d)=a, f(e)=d then f is rgwα-continuous but f is not rgwα-irresolute, as rgwα-closed set F= {a,b} in Y, then $f^{-1}(F)$={a,d} in X , which is not rgwα-closed set in X.

Theorem 5.5: If A map f: (X, τ)→(Y, σ) is rgwα-irresolute, if and only if the inverse image $f^{-1}(V)$ is rgwα-open set in X for every rgwα-open set V in Y.
Proof: Assume that f: X→Y is rgwα-irresolute. Let G be rgwα-open in Y. The G^c is rgwα-closed in Y. Since f is rgwα- irresolute, $f^{-1}(G^c)$ is rgwα-closed in X. But $f^{-1}(G) = X - f^{-1}(G)$. Thus $f^{-1}(G)$ is rgwα-open in X.
Converserly, Assume that the inverse image of each open set in Y is rgwα-open in X. Let F be any rgwα-closed set in Y. By assumption $f^{-1}(F^c)$ is rgwα-open in X. But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is rgwα-open in X and so $f^{-1}(F)$ is rgwα-closed in X. Therefore f is rgwα-irresolute.

Theorem 5.6: If A map f: (X, τ)→(Y, σ) is rgwα-irresolute, then for every subset A of X ,f(rgwαcl(A)⊂ αcl(f(A)). **Proof:** If A⊂ X then consider αcl(f(A)) which is rgwα-closed in Y. since f is rgwα-irresolute , $f^{-1}(\alpha cl(f(A)))$ is rgwα-closed in X. Furthermore $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\alpha cl(f(A)))$. Therefore by rgwα-closure, $rgw\alpha cl(A) \subseteq f^{-1}(\alpha cl(f(A)))$, consequently , $f(rgw\alpha cl(A) \subseteq f(f^{-1}(\alpha cl(f(A)))) \subseteq \alpha clf((A))$.

Theorem 5.7: Let f: (X, τ) →(Y, σ) and g: (Y, σ) →(Z, η) be any two functions. Then
(i) g o f: (X, τ)→(Z, η) is rgwα-continuous if g is r-continuous and f is rgwα- irresolute.
(ii)g o f : (X, τ)→(Z, η) is rgwα-irresolute if g is rgwα-irresolute and f is rgwα-irresolute.
(iii) g o f : (X, τ)→(Z, η) is rgwα-continuous if g is continuous and f is rgwα-irresolute.

Proof: (i) Let U be a open set in (Z, η). Since g is r-continuous, $g^{-1}(U)$ is r-open set in (Y, σ). Since every r-open is rgwα-open then $g^{-1}(U)$ is rgwα-open in Y, since f is rgwα-irresolute $f^{-1}(g^{-1}(U))$ is an rgwα-open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an rgwα-open set in (X, τ) and hence gof is rgwα-continuous.

(ii) Let U be a rgwα-open set in (Z, η). Since g is rgwα-irresolute, $g^{-1}(U)$ is rgwα-open set in (Y, σ). Since f is rgwα-irresolute, $f^{-1}(g^{-1}(U))$ is an rgwα-open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an rgwα-open set in (X, τ) and hence gof is rgwα- irresolute.

(iii) Let U be a open set in (Z, η). Since g is continuous, $g^{-1}(U)$ is open set in (Y, σ). As every open set is rgwα-open, $g^{-1}(U)$ is rgwα-open set in (Y, σ). Since f is rgwα-irresolute $f^{-1}(g^{-1}(U))$ is an rgwα-open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an rgwα-open set in (X, τ) and hence gof is rgwα-continuous.

Theorem 5.8: Let f: (X, τ) →(Y, σ) is strongly rgwα-continuous then it is continuous.

Proof: Assume that f: (X, τ) →(Y, σ) is strongly rgwα-continuous, Let F be closed set in Y. As every closed is rgwα-closed, F is rgwα-closed in Y. since f is strongly rgwα-continuous then $f^{-1}(F)$ is closed set in X. Therefore f is continuous.

Theorem 5.9: Let f: (X, τ) →(Y, σ) is strongly rgwα-continuous then it is strongly α-continuous but not conversely.

Proof: Assume that f: (X, τ) →(Y, σ) is strongly rgwα-continuous, Let F be α-closed set in Y. As every α-closed is rgwα-closed, F is rgwα-closed in Y. since f is strongly rgwα-continuous then $f^{-1}(F)$ is closed set in X. Therefore f is strongly α-continuous.

The converse of the above theorem 5.9 need not be true as seen from the following example

Example 5.10: Let X=Y={a,b,c,d}, τ ={X, ϕ,{a},{b},{a,b},{a,b,c}} and σ ={Y, ϕ,{a},{b},{a,b},{a,b,c}}. Let map f: X→Y defined by f(a)=b , f(b)=a,f(c)=d,f(d)=c , then f is strongly α-continuous but not continuous and not strongly rgwα-continuous, as closed set F={b,c,d} in Y, then $f^{-1}(F)$={a,c,d} in X which is not closed set in X.

Theorem 5.11: Let f: (X, τ)→(Y, σ) is strongly rgwα-continuous if and only if $f^{-1}(G)$ is open set in X for every rgwα-open set G in Y.

Proof: Assume that f: X→Y is strongly rgwα-continuous. Let G be rgwα-open in Y. The G^c is rgwα-closed in Y. Since f is strongly rgwα-continuous, $f^{-1}(G^c)$ is closed in X. But $f^{-1}(G^c) = X–f^{-1}(G)$. Thus $f^{-1}(G)$ is open in X. Conversely, Assume that the inverse image of each open set in Y is rgwα-open in X. Let F be any rgwα-closed set in Y. By assumption F^c is rgwα-open in X. But $f^{-1}(F^c) = X–f^{-1}(F)$. Thus $X– f^{-1}(F)$ is open in X and so $f^{-1}(F)$ is closed in X. Therefore f is strongly rgwα-continuous.

Theorem 5.12: Let f: (X, τ) →(Y, σ) is strongly continuous then it is strongly rgwα-continuous.

Proof: Assume that f: X→Y is strongly continuous. Let G be rgwα-open in Y and also it is any subset of Y since f is strongly continuous, $f^{-1}(G)$ is open (and also closed) in X. $f^{-1}(G)$ is open in X Therefore f is strongly rgwα-continuous.

Theorem 5.13: Let f: (X, τ) →(Y, σ) is strongly rgwα-continuous then it is rgwα-continuous.

Proof: Let G be open in Y, every open is rgwα-open, G is rgwα-open in Y, since f is strongly rgwα-continuous, $f^{-1}(G)$ is open in X and therefore $f^{-1}(G)$ is rgwα-open in X. Hence f is rgwα-continuous.

Theorem 5.14: In discrete space**,** a map f: (X, τ) →(Y, σ) is strongly rgwα-continuous then it is strongly continuous. **Proof:** F any subset of Y, in discrete space, Every subset F in Y is both open and closed, then subset F is both rgwα-open or rgwα-closed, i) let F is rgwα-closed in Y, since f is strongly rgwα-continuous, then $f^{-1}(F)$ is closed in X. ii) let F is rgwα-open in Y, since f is strongly rgwα-continuous, then $f^{-1}(F)$ is open in X. Therefore $f^{-1}(F)$ is closed and open in X. Hence f is strongly continuous.

Theorem 5.15: Let f: (X, τ) →(Y, σ) and g: (Y, σ) → (Z, η) be any two functions. Then

(i) g o f : (X, τ)→(Z, η) is strongly rgwα-continuous if g is strongly rgwα-continuous and f is strongly rgwα-continuous.

(ii) g o f : (X, τ)→(Z, η) is strongly rgwα-continuous if g is strongly rgwα-continuous and f is continuous.

(iii) g o f : (X, τ)→(Z, η) is rgwα-irresolute if g is strongly rgwα-continuous and f is rgwα-continuous.

(iv) g o f : (X, τ)→(Z, η) is continuous if g is rgwα-continuous and f is strongly rgwα-continuous

Proof:

(i) Let U be a rgwα-open set in (Z, η). Since g is strongly rgwα-continuous, $g^{-1}(U)$ is open set in (Y, σ). As every open set is rgwα-open, $g_{-1}(U)$ is rgwα-open set in (Y, σ). Since f is strongly rgwα-continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly rgwα-continuous.

(ii) Let U be a rgwα-open set in (Z, η). Since g is strongly rgwα-continuous, $g^{-1}(U)$ is open set in (Y, σ). Since f is continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly rgwα-continuous.

(iii) Let U be a rgwα-open set in (Z, η). Since g is strongly rgwα-continuous, $g^{-1}(U)$ is open set in (Y, σ). Since f is rgwα-continuous $f^{-1}(g^{-1}(U))$ is an rgwα-open set in (X, τ). Thus $(gof)^{-1}$ (U) = $f^{-1}(g^{-1}(U))$ is an rgwα-open set in (X, τ) and hence gof is rgwα-irresolute

(iv) Let U be open set in (Z, η). Since g is rgwα-continuous, $g^{-1}(U)$ is rgwα-open set in (Y, σ). Since f is strongly rgwα-continuous f^{-1} (g^{-1} (U)) is an open set in (X, τ). Thus $(gof)^{-1}(U)$ = $f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is continuous.

Theorem 5.16: Let f: (X, τ) →(Y, σ) and g: (Y, σ) →(Z, η) be any two functions. Then

1. g o f : (X, τ)→(Z, η) is strongly rgwα-continuous if g is perfectly rgwα-continuous and f is continuous.
2. g o f : (X, τ)→(Z, η) is perfectly rgwα-continuous if g is strongly rgwα-continuous and f is perfectly rgwα-continuous.

Proof:

1. Let U be a rgwα-open set in (Z, η). Since g is perfectly rgwα-continuous, $g^{-1}(U)$ is clopen set in (Y, σ). g^{-1}(U) is open set in (Y, σ). Since f is continuous $f^{-1}(g^{-1}(U))$ is an open set in (X, τ). Thus $(gof)^{-1}(U)$ = $f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is strongly rgwα-continuous.
2. Let U be a rgwα-open set in (Z, η). Since g is strongly rgwα-continuous, $g^{-1}(U)$ is open set in (Y, σ). g^{-1}(U) is open set in (Y, σ). Since f is perfectly rgwα-continuous , $f^{-1}(g^{-1}(U))$ is an clopen set in (X, τ). Thus $(gof)^{-1}(U)$ = $f^{-1}(g^{-1}(U))$ is an clopen set in (X, τ) and hence gof is perfectly rgwα-continuous.

Theorem 5.17: If A map f: (X, τ) →(Y, σ) is strongly rgwα-continuous and A is open subset of X then the restriction f/A: A→Y is strongly rgwα-continuous.

Proof: Let V be any rgwα-open set of Y, since f is strongly rgwα-continuous , then $f^{-1}(V)$ is open in X. since A is open in X, $(f/A)^{-1}$ (V)=A∩ $f^{-1}(V)$ is open in A. hence f/A is strongly rgwα-continuous.

Theorem: 5.18 Let (X, τ) be any topological space and (Y, σ) be a $T_{rgw\alpha}$ -space and f: (X, τ) →(Y, σ) be a map. Then the following are equivalent: (i) f is strongly rgwα-continuous. (ii) f is continuous.

Proof: (i) =>(ii) Let U be any open set in (Y, σ). Since every open set is rgwα-open, U is rgwα-open in (Y, σ). Then $f^{-1}(U)$ is open in (X, τ). Hence f is continuous. (ii) =>(i) Let U be any rgwα-open set in (Y, σ). Since (Y, σ) is a $T_{rgw\alpha}$ -space, U is open in (Y, σ). Since f is continuous. Then $f^{-1}(U)$ is open in (X, τ). Hence f is strongly rgwα-continuous.

Theorem 5.19: Let (X, τ) be a discrete topological space and (Y, σ) be any topological space. Let f: (X, τ) → (Y, σ) be a map. Then the following statements are equivalent: (i) f is strongly rgwα-continuous. (ii) f is perfectly rgwα-continuous.

Proof: (i)=>(ii) Let U be any rgwα-open set in (Y, σ).By hypothesis $f^{-1}(U)$ is open in (X, τ). Since(X, τ) is a discrete space, $f^{-1}(U)$ is also closed in (X, τ). f^{-1} (U) is both open and closed in (X, τ). Hence f is perfectly rgwα-continuous. **(ii)=> (i)** Let U be any rgwα-open set in (Y, σ). Then $f^{-1}(U)$ is both open and closed in (X, τ). Hence f is strongly rgwα-continuous.

Theorem 5.20: Let f: (X, τ) →(Y, σ) be a map. Both (X, τ) and (Y, σ) are $\tau_{rgw\alpha}$-spaces. Then the following are equivalent:

(i) f is rgwα-irresolute.

(ii) f is strongly rgwα-continuous

(iii) f is continuous.

(iv) f is rgwα-continuous.

Proof: Straight forward.

Theorem 5.21: Let X and Y be $_{\alpha}\tau_{rgw\alpha}$-spaces, then for a function f: (X, τ) → (Y, σ), the following are equivalent: (i) f is α-irresolute. (ii) f is rgwα-irresolute.

Proof: (i)=> (ii): Let f: (X, τ) →(Y, σ) be a α-irresolute. Let V be a rgwα-closed set in Y. As Y $_{\alpha}\tau_{rgw\alpha}$-space, then V be a α-closed set in Y. Since f is α -irresolute, f^{-1} (V) is α -closed in X. But every α-closed set is rgwα-closed in X and hence f^{-1} (V) is a rgwα-closed in X. Therefore, f is rgwα-irresolute.

(ii)=> (i): Let f: (X, τ) → (Y, σ) is a rgwα-irresolute. Let V be a α-closed set in Y. But every α-closed set is rgwα-closed set and hence V is rgwα-closed set in Y and f is rgwα-irresolute implies f^{-1} (V) is rgwα-closed in X. But X is $_{\alpha}\tau_{rgw\alpha}$-space and hence f^{-1} (V) is α-closed set in X. Thus, f is α-irresolute.

6. Conclusion.

In this paper we have introduced and studied the properties of rgwα-continuous and rgwα-irresolute maps. Our future extension is rgwα- continuous and rgwα-irresolute in Fuzzy Topological Spaces.

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