

Mean Time to Recruitment for a Multi Grade Man Power System with Single Threshold, Single Source of Depletion when Inter Policy Decisions form a Geometric Process.

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Abstract: In this paper, for a marketing organization consisting of multi grades subject to the depletion of manpower (wastages) due to policy decisions with high or low attrition rate, is considered. An important system characteristic namely the mean time to recruitment is obtained for a suitable policy of recruitment when (i) wastages are independent and identically distributed exponential random variable and (ii) threshold for each grade has single components with exponential distribution, and (iii) the inter-policy decisions form a Geometric process.

Keywords: Loss of manpowers (wastage), Geometric process, Policy of recruitment, Threshold with single components, Two types of policy decisions with high or low attrition rate, Hyper exponential, Mean time to recruitment.

I. INTRODUCTION

Exits of personal which is in other words known as wastage, is an important aspects in the study of manpower planning. Many models have been discussed using different types of wastages and also different types of distribution for the loss of man powers, the thresholds and inter decision times. Such models are seen in [1] and [2]. In [3], [4], [5] and [6] the authors have obtained the mean time to recruitment in a two grade manpower system based on order statistics by assuming different distribution for thresholds. In [8] for a two grade manpower system with two types of decisions when the wastages form a geometric process is obtained.

The problem of time to recruitment is studied by several authors for the organizations consisting of single grade/two grade/ three grades. More specifically for a two grade system, in all the earlier work, the threshold for the organization is minimum or maximum or sum of the thresholds for the loss of manpower in each grades, no attempt has been made so far to design a comprehensive recruitment policy for a system with two or three grades. In [10] a new

design for a comprehensive univariate CUM recruitment policy of manpower system is used with n grades in order to bring results proved independently for maximum, minimum model as a special case. In all previous work, the problem of time to recruitment is studied for only an organization consisting of at most three grades. In this paper an organization with n -grades is considered and the mean time to recruitment are obtained using an appropriate univariate CUM policy of recruitment (i.e) "The organization survives iff atleast r , ($1 \leq r \leq n$) out of n -grades survives in the sense that threshold crossing has not take place in these grades." when the inter decision time form an geometric process.

II. MODEL DESCRIPTION AND ANALYSIS

An organization with n -grades which takes policy decisions at random epoch is considered. At every decision making epoch, a random number of persons quit the organization. There is an associated loss of man hours to the organization if the person quits. The loss of man hours forms a sequence of independent and identically distributed random variables. Each grade has its own thresholds. The thresholds for the n -grades are independent and identically distributed exponential random variables. The inter decision times forms a Geometric process. The loss of manpower process, inter decision time and the thresholds are statistically independent..

Let X_i , $i=1,2,3,\dots$ is a continuous random variable denoting the amount of depletion caused to the system due to the exit of persons corresponding to the i^{th} decision, t_i , time of occurrence of the i^{th} decision. $G_k(\cdot)$: Cumulative distribution function of $\sum_{i=1}^n x_i$. Let U_i , Independent and identically distributed random variables denoting the time between $(i-1)^{\text{th}}$ and i^{th} decision making epoch, $i=1,2,\dots$ $f(\cdot)$, the Probability density function of U_i , $i=1,2,\dots$ $f_k(\cdot)$ k -fold convolution of $f(\cdot)$, $\tilde{f}(\cdot)$: Laplace transform of $f(\cdot)$. Let $V_k(t)$ be the Probability that

there are exactly k decision making epochs in $(0, t]$. Y_j a continuous random variable denoting the threshold level for the j^{th} grade. Y be a continuous random variable denoting the threshold level for the organization. $H(\cdot)$ the distribution function of Y . Let T_j be the time taken for threshold crossing in the j^{th} grade, $j=1,2,\dots,n$. T time for recruitment in the organization. $E(T)$: Mean time for recruitment.

MAIN RESULTS

The survival function of the time to recruitment is given by

$$P(T > t) = \sum_{k=0}^{\infty} P(\text{Exactly } k \text{ decision in epoch}(0, t] \text{ and the}$$

threshold level Y is not crossed by the total loss of manhours in these k decisions in atleast r grades)

$$i.e P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=r}^k X_i < Y\right) \quad (1)$$

By the law of total probability

$$\begin{aligned} P\left(\sum_{i=r}^k X_i < Y\right) &= \int_0^{\infty} P\left[Y > \sum_{i=r}^k X_i \mid \sum_{i=r}^k X_i = x\right] g_k(x) dx \quad (2) \\ &= \int_0^{\infty} g_k(x) [1 - H(x)] dx \\ &= \int_0^{\infty} g_k(x) \sum_{i=r}^n nC_i [1 - H(x)]^i [H(x)]^{n-i} dx \\ &= \int_0^{\infty} g_k(x) \sum_{i=r}^n nC_i [e^{-\theta x}]^i [1 - e^{-\theta x}]^{n-i} dx \\ &= \sum_{i=r}^n nC_i \int_0^{\infty} g_k(x) e^{-i\theta x} [1 - e^{-\theta x}]^{n-i} dx \end{aligned}$$

Using binomial expansion

$$= \sum_{i=r}^n nC_i \int_0^{\infty} g_k(x) e^{-i\theta x} [1 - (n-i)C_1 e^{-\theta x} + (n-i)C_2 e^{-2\theta x} + \dots (-1)^{n-i} e^{-(n-i)\theta x}] dx$$

$$= \sum_{i=r}^n nC_i \int_0^{\infty} g_k(x) [e^{-i\theta x} - (n-i)C_1 e^{-(i+1)\theta x} + (n-i)C_2 e^{-(i+2)\theta x} + \dots (-1)^{n-i} e^{-n\theta x}] dx$$

By using convolution theorem on Laplace transform

$$= \sum_{i=r}^n nC_i [[\bar{g}(i\theta)]^k - (n-i)C_1 [\bar{g}((i+1)\theta)]^k + (n-i)C_2 e^{-(i+2)\theta x} + \dots (-1)^{n-i} e^{-n\theta x}] dx \quad (3)$$

From renewal theory $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$ (4)

Substituting (3) and (4) in (1) we get,

$$\begin{aligned} P(T > t) &= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \times \\ &\sum_{i=r}^n nC_i [[\bar{g}(i\theta)]^k - (n-i)C_1 [\bar{g}((i+1)\theta)]^k + (n-i)C_2 [\bar{g}((i+2)\theta)]^k + \dots (-1)^{n-i} [\bar{g}(n\theta)]^k] \quad (5) \end{aligned}$$

From equation (5), the distribution function of the time for recruitment is given by

$$\begin{aligned} L(t) &= 1 - P(T > t) \\ &= 1 - \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \\ &\times \sum_{i=r}^n nC_i [[\bar{g}(i\theta)]^k - (n-i)C_1 [\bar{g}((i+1)\theta)]^k + (n-i)C_2 [\bar{g}((i+2)\theta)]^k + \dots (-1)^{n-i} [\bar{g}(n\theta)]^k] \quad (6) \end{aligned}$$

Differentiating with respect to t

$$\begin{aligned} l(t) &= - \sum_{k=0}^{\infty} [f_k(t) - f_{k+1}(t)] \sum_{i=r}^n nC_i [[\bar{g}(i\theta)]^k - (n-i)C_1 [\bar{g}((i+1)\theta)]^k + (n-i)C_2 [\bar{g}((i+2)\theta)]^k + \dots (-1)^{n-i} [\bar{g}(n\theta)]^k] \quad (7) \end{aligned}$$

Taking Laplace transform on both sides,

$$\begin{aligned} \bar{l}(s) &= - \sum_{k=0}^{\infty} [\bar{f}_k(t) - \bar{f}_{k+1}(t)] \sum_{i=r}^n nC_i [\bar{g}(i\theta)]^k \\ &\quad - (n-i)C_1 [\bar{g}((i+1)\theta)]^k \\ &\quad + (n-i)C_2 [\bar{g}((i+2)\theta)]^k \\ &\quad + \dots (-1)^{n-i} [\bar{g}(n\theta)]^k \end{aligned} \quad (8)$$

The mean time to recruitment is

$$\begin{aligned} E(T) &= - \left[\frac{d}{ds} (\bar{l}(s)) \right]_{s=0} \\ &= - \sum_{k=0}^{\infty} \left[\left[\frac{d}{ds} (\bar{f}_k(s)) \right]_{s=0} - \left[\frac{d}{ds} (\bar{f}_{k+1}(s)) \right]_{s=0} \right] \\ &\quad \times \sum_{i=r}^n nC_i [\bar{g}(i\theta)]^k - (n-i)C_1 [\bar{g}((i+1)\theta)]^k \\ &\quad + (n-i)C_2 [\bar{g}((i+2)\theta)]^k \\ &\quad + (-1)^{n-i} [\bar{g}(n\theta)]^k \end{aligned} \quad (9)$$

Since probability density function of loss of manpower $g(x)$ is an exponential with parameter μ ,

$$\bar{g}(s) = \frac{\mu}{\mu+s} \quad (10) \text{ Assume}$$

me that the inter decision times $U_i, i = 1, 2, 3 \dots$ form a geometric process with rate $b, (b > 0)$. It is assumed that the probability density function of U_1 is hyper exponential density function

$$f(t) = p\lambda_h e^{-\lambda_h t} + q\lambda_l e^{-\lambda_l t}, p + q = 1.$$

$$\bar{f}(s) = \frac{p\lambda_h}{\lambda_h + s} + \frac{q\lambda_l}{\lambda_l + s}$$

$$\bar{f}(0) = \frac{p\lambda_h}{\lambda_h} + \frac{q\lambda_l}{\lambda_l} = p + q = 1$$

$$\frac{d}{ds} (\bar{f}(s)) = \frac{-p\lambda_l}{(\lambda_h + s)^2} + \frac{-q\lambda_l}{(\lambda_l + s)^2}$$

$$\begin{aligned} \left(\frac{d}{ds} (\bar{f}(s)) \right)_{s=0} &= - \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \end{aligned} \quad (11)$$

$$\bar{f}_k(s) = \prod_{i=1}^k \bar{f}\left(\frac{s}{b^{i-1}}\right)$$

$$\begin{aligned} \frac{d}{ds} (\bar{f}_k(s)) &= \frac{d}{ds} \left(\prod_{i=1}^k \bar{f}\left(\frac{s}{b^{i-1}}\right) \right) \\ &= \frac{d}{ds} \left(\bar{f}(s) \times \bar{f}\left(\frac{s}{b}\right) \times \bar{f}\left(\frac{s}{b^2}\right) \times \dots \times \bar{f}\left(\frac{s}{b^{k-1}}\right) \right) \end{aligned}$$

$$\begin{aligned} &= \frac{d}{ds} (\bar{f}(s)) \prod_{i=2}^k \bar{f}\left(\frac{s}{b^{i-1}}\right) \\ &\quad + \frac{1}{b} \frac{d}{ds} (\bar{f}(s)) \prod_{\substack{i=1 \\ i \neq 2}}^k \bar{f}\left(\frac{s}{b^{i-1}}\right) \\ &\quad + \frac{1}{b^2} \frac{d}{ds} (\bar{f}(s)) \prod_{\substack{i=1 \\ i \neq 3}}^k \bar{f}\left(\frac{s}{b^{i-1}}\right) + \dots \\ &\quad + \frac{1}{b^{k-1}} \frac{d}{ds} (\bar{f}(s)) \prod_{i=1}^{k-1} \bar{f}\left(\frac{s}{b^{i-1}}\right) \end{aligned}$$

$$\begin{aligned} \left(\frac{d}{ds} (\bar{f}_k(s)) \right)_{s=0} &= \left(\frac{d}{ds} \left(\prod_{i=1}^k \bar{f}\left(\frac{s}{b^{i-1}}\right) \right) \right)_{s=0} \\ &= \left(\frac{d}{ds} (\bar{f}(s)) \right)_{s=0} + \frac{1}{b} \left(\frac{d}{ds} (\bar{f}(s)) \right)_{s=0} \\ &\quad + \frac{1}{b^2} \left(\frac{d}{ds} (\bar{f}(s)) \right)_{s=0} + \dots \\ &\quad + \frac{1}{b^{k-1}} \left(\frac{d}{ds} (\bar{f}(s)) \right)_{s=0} \\ &= \left(1 + \frac{1}{b} + \frac{1}{b^2} + \frac{1}{b^3} + \dots + \frac{1}{b^{k-1}} \right) \left(\frac{d}{ds} (\bar{f}(s)) \right)_{s=0} \end{aligned}$$

$$\begin{aligned} &= \sum_{i=1}^k \frac{1}{b^{i-1}} \times \left(\frac{d}{ds} (\bar{f}(s)) \right)_{s=0} \\ &= - \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \sum_{i=1}^k \frac{1}{b^{i-1}} \end{aligned} \quad (12)$$

Consider

$$\begin{aligned} &\sum_{k=0}^{\infty} \left[\left[\frac{d}{ds} (\bar{f}_k(s)) \right]_{s=0} - \left[\frac{d}{ds} (\bar{f}_{k+1}(s)) \right]_{s=0} \right] \\ &= \sum_{k=0}^{\infty} - \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h \lambda_l} \right) \left[\sum_{i=1}^k \frac{1}{b^{i-1}} - \sum_{i=1}^{k+1} \frac{1}{b^{i-1}} \right] \end{aligned}$$

$$= \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \sum_{k=0}^{\infty} \frac{1}{b^k} \quad (13)$$

Substituting this in (9)

$$\begin{aligned} E(T) &= \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \sum_{k=0}^{\infty} \frac{1}{b^k} \sum_{i=r}^n nC_i \left[[\bar{g}(i\theta)]^k \right. \\ &\quad - (n-i)C_1 [\bar{g}((i+1)\theta)]^k \\ &\quad + (n-i)C_2 [\bar{g}((i+2)\theta)]^k \\ &\quad + \dots \\ &\quad \left. + (-1)^{n-i} [\bar{g}(n\theta)]^k \right] \\ &= \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \sum_{i=r}^n nC_i \left[\sum_{k=0}^{\infty} \left[\frac{\bar{g}(i\theta)}{b} \right]^k \right. \\ &\quad - (n-i)C_1 \sum_{k=0}^{\infty} \left[\frac{\bar{g}((i+1)\theta)}{b} \right]^k \\ &\quad + (n-i)C_2 \sum_{k=0}^{\infty} \left[\frac{\bar{g}((i+2)\theta)}{b} \right]^k \\ &\quad + \dots \\ &\quad \left. + (-1)^{n-i} \sum_{k=0}^{\infty} \left[\frac{\bar{g}(n\theta)}{b} \right]^k \right] \\ &= \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \sum_{i=r}^n nC_i \left[\left[1 - \frac{\bar{g}(i\theta)}{b} \right]^{-1} \right. \\ &\quad - (n-i)C_1 \left[1 - \frac{\bar{g}((i+1)\theta)}{b} \right]^{-1} \\ &\quad + (n-i)C_2 \left[1 - \frac{\bar{g}((i+2)\theta)}{b} \right]^{-1} \\ &\quad + \dots \\ &\quad \left. + (-1)^{n-i} \left[1 - \frac{\bar{g}(n\theta)}{b} \right]^{-1} \right] \end{aligned}$$

Substituting equation (13) in the above equation

$$\begin{aligned} E(T) &= \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \sum_{i=r}^n nC_i \left[\left[\frac{b}{b - \frac{\mu}{\mu+i\theta}} \right] \right. \\ &\quad - (n-i)C_1 \left[\frac{b}{b - \frac{\mu}{\mu+(i+1)\theta}} \right] \\ &\quad + \dots \\ &\quad \left. + (-1)^{n-i} \left[\frac{b}{b - \frac{\mu}{\mu+n\theta}} \right] \right] \\ E(T) &= b \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \sum_{i=r}^n nC_i \left[\left[\frac{\mu + i\theta}{bi\theta + \mu(b-1)} \right] \right. \\ &\quad - (n-i)C_1 \left[\frac{\mu + (i+1)\theta}{b(i+1)\theta + \mu(b-1)} \right] + \dots \\ &\quad \left. + (-1)^{n-i} \left[\frac{\mu + n\theta}{bn\theta + \mu(b-1)} \right] \right] \quad (14) \end{aligned}$$

The mean time for recruitment can be explicitly calculated using (14) for different possible combinations of the values of n and r.

III NUMERICAL ILLUSTRATION

The behavior of the performance measure due to the change in parameter is analyzed numerically for different values of n and r.

Case(i) n=3,r=1

From equation (14) the mean time for recruitment is given by

$$E(T) = b \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \left\{ \frac{3(\mu + \theta)}{b\theta + \mu(b-1)} - \frac{3(\mu + 2\theta)}{2b\theta + \mu(b-1)} + \frac{(\mu + 3\theta)}{3b\theta + \mu(b-1)} \right\}$$

Case(ii) n=3,r=2

From equation (14) the mean time for recruitment is given by

$$E(T) = b \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \left\{ \frac{3(\mu + 2\theta)}{2b\theta + \mu(b-1)} - \frac{2(\mu + 3\theta)}{3b\theta + \mu(b-1)} \right\}$$

Case(iii) n=3,r=3

From equation (14) the mean time for recruitment is given by

$$E(T) = b \left(\frac{p\lambda_l + q\lambda_h}{\lambda_h\lambda_l} \right) \left\{ \frac{(\mu + 3\theta)}{3b\theta + \mu(b-1)} \right\}$$

The influence of parameters on the performance measures namely the mean time for recruitment is studied numerically. In the following table, these performance measures are calculated by varying the parameter ' μ ', ' θ ', and ' b ' and the other parameters $p=0.3$, $\lambda_h=0.5$ and $\lambda_l=0.6$ fixed.

TABLE I:

EFFECTS OF ' μ ', ' θ ' AND ' b ' ON THE PERFORMANCE MEASURE.

μ	θ	b	Case(i) E(T)	Case(ii) E(T)	Case(iii) E(T)
0.2	0.3	1.5	2.8776	2.3393	2.0103
0.2	0.3	2.0	2.5112	2.1705	1.9433
0.2	0.3	2.5	2.3259	2.0784	1.9052
0.2	0.3	3.0	2.2142	2.0205	1.8806
0.2	0.3	3.5	2.2139	1.9808	1.8635
0.2	0.4	1.5	2.6515	2.2101	1.9526
0.2	0.5	1.5	2.5016	2.1285	1.9170
0.2	0.6	1.5	2.3950	2.0722	1.8929
0.2	0.7	1.5	2.3153	2.0310	1.8754
0.2	0.8	1.5	2.2535	1.9597	1.8622
0.1	0.3	2.0	2.2091	1.9984	1.8592
0.2	0.3	2.0	2.5112	2.1705	1.9433
0.3	0.3	2.0	2.7257	2.3219	2.0190
0.4	0.3	2.0	2.8829	2.4492	2.0879
0.5	0.3	2.0	3.0010	2.5574	2.1507

IV CONCLUSIONS

From table I, we conclude as follows

1. As the rate of geometric process ' b ' for inter decision time alone increases, the mean time to recruitment decreases on the average.
2. As ' θ ' alone increases, the mean threshold level decreases and hence the time taken for threshold crossing i.e mean time to recruitment decreases.

3. As μ alone increases, the mean loss of manpower decreases. In fact when the loss of manpower decreases on the average, time taken for threshold crossing and hence the time to recruitment increases on the average.

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