ON SOFT δ-CONTINUOUS FUNCTIONS

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ABSTRACT. In this paper, we introduce a new class of functions called soft δ-continuous functions. We obtain several characterizations and some of their properties. Also, we investigate its relationship with other types of functions.

1. Introduction

The concept of soft sets was first introduced by Molodtsov [11] in 1999 as a general mathematical tool for dealing with uncertain objects. In [11, 12], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on.

After presentation of the operations of soft sets [10], the properties and applications of soft set theory have been studied increasingly [3, 7, 12]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 2, 4, 8, 9, 10, 12]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [5].

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Recently, in 2011, Shabir and Naz [14] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as soft open and soft closed sets, soft subspace, soft interior, soft closure, soft neighborhood of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Hussain and Ahmad [6] investigated the properties of soft open, soft closed, soft interior, soft closure, soft neighborhood of a point. They also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces.

In this paper, we introduce a new class of functions called soft δ-continuous functions. We obtain several characterizations and some of their properties. Also, we investigate its relationship with other types of functions.

2. Preliminaries

In this section, we present some basic definitions and resuts which are needed in further study of this paper which may found in earlier studies. Throughout this paper, X refers to an initial universe, E is a set of parameters, $\wp(X)$ is the power set of X, and A⊂E

Definition 2.1. [11] *A soft set F_A over the universe X is defined by the set of ordered pairs*

$F_A=\{(e,\ F_A(e)) : e\in E,\ F_A(e)\in\wp(X)\}$

where $F_A : E\to\wp(X)$, such that $F_A(e)\neq\emptyset$, if $e\in A\subset E$ and $F_A(e)=\emptyset$ if $e\notin A$. The family of all soft sets over X is denoted by SS(X).

Definition 2.2. [10] *The soft set $F_\emptyset$ over a common universe set X is said to be null soft set, denoted by $\emptyset$. Here $F_\emptyset(e)=\emptyset$, $\forall e\in E$.*

Definition 2.3. [10] *A soft set F_A over X is called an absolute soft set, denoted by $\tilde{A}$, if $e\in A$, $F_A(e)=X$.*

Definition 2.4. [10] *Let F_A, G_B be soft sets over a common universe set X. Then F_A is a soft subset of G_B, denoted $F_A\subset G_B$ if $F_A(e)\subset G_B(e)$, $\forall e\in E$.*

Definition 2.5. [10] *Let F_A, G_B be soft sets over a common universe set X. The union of F_A and G_B, is a soft set H_C defined by $H_C(e)=F_A(e)\cup G_B(e)$, $\forall e\in E$, where $C=A\cup B$.*

That is, $H_C=F_A\cup G_B$.

Definition 2.6. [10] *Let F_A, G_B be soft sets over a common universe set X. The intersection of F_A and G_B, is a soft set H_C defined by $H_C(e)=F_A(e)\cap G_B(e)$, $\forall e\in E$, where $C=A\cap B$.*

That is, $H_C = F_A \cap G_B$.

Definition 2.7. [14] *The complement of the soft set* F_A *over* X*, denoted by* F_A^c *is defined by* $F_A^c(e) = X - F_A(e)$, $\forall e \in E$.

Definition 2.8. [14] *Let* F_A *be a soft set over* X *and* $x \in X$*. We say that* $x \in F_A$ *if* $x \in F_A(e)$, $\forall e \in A$*. For any* $x \in X$, $x \notin F_A$ *if* $x \notin F_A(e)$ *for some* $e \in A$.

Definition 2.9. [16] *The soft set* $F_A \in SS(X)$ *is called a soft point in* $SS(X)$ *if there exist* $x \in X$ *and* $e \in E$ *such that* $F(e) = \{x\}$ *and* $F(e^c) = \emptyset$ *for each* $e^c \in E - \{e\}$ *and the soft point* F_A *is denoted by* x_ε.

Definition 2.10. [14] *A soft topology* τ *is a family of soft sets over* X *satisfying the following properties.*

(1) $\emptyset$, $\tilde{X}$ *belong to* τ.
(2) *The union of any number of soft sets in* τ *belongs to* τ.
(3) *The intersection of any two soft sets in* τ *belongs to* τ.

The triplet (X, τ, E) *is called a soft topological space.*

Definition 2.11. [13] *Let* (X, τ, E) *be a soft topological space over* X*. Then*

(1) *The members of* τ *are called soft open sets in* X.
(2) *A soft set* F_A *over* X *is said to be a soft closed set in* X *if* $F_A^c \in \tau$.
(3) *A soft set* F_A *is said to be a soft neighborhood of a point* $x \in X$ *if* $x \in F_A$ *and* F_A *is soft open in* (X, τ, E)
(4) *The soft interior of a soft set* F_A *is the union of all soft open subsets of* F_A*. The soft interior of* F_A *is denoted by* $int(F_A)$.
(5) *The soft closure of* F_A *is the intersection of all soft closed super sets of* F_A*. The soft closure of* F_A *is denoted by* $cl(F_A)$ *or* $\overline{F_A}$.

Definition 2.12. [15] *A soft set* F_A *in a soft topological space* (X, τ, E) *is said to be a soft regular open (resp. soft regular closed) if* $F_A = int(cl(F_A))$ *(resp.* $F_A = cl(int(F_A))$*).*

3. soft δ-open sets

Definition 3.1. *Let* F_A *be a soft subset of soft topological space* (X, τ, E)*. Then*

(1) x_ε *is called a soft* δ*-cluster point of* F_A *if* $F_A \cap int(cl(U_A)) \neq \emptyset$ *for every soft open set* U_A *containing* x_ε.
(2) *The family of all soft* δ*-cluster point of* F_A *is called the soft* δ*-closure of* F_A *and is denoted by* $cl_\delta(F_A)$.

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(3) *A soft subset F_A is said to be soft δ-closed if $cl_\delta(F_A)=F_A$. The complement of a soft δ-closed set of X is said to be soft δ-open.*

Lemma 3.2. *Let F_A be a soft subset of soft topological space (X, τ, E). Then, the following properties hold:*

(1) *$int(cl(F_A))$ is soft regular open,*
(2) *Every soft regular open set is soft δ-open,*
(3) *Every soft δ-open set is the union of a family of soft regular open sets.*
(4) *Every soft δ-open set is soft open.*

Proof. (1) Let F_A be a soft subset of X and G_A=int(cl(F_A)). Then, we have int(cl(G_A))= int(cl(F_A))=G_A. Therefore G_A is soft regular open.

(2) Let F_A be a soft regular open. For each $x_\varepsilon \in F_A$, $(X-F_A)\cap F_A=\emptyset$ and F_A is soft regular open. Hence $x_\varepsilon \notin cl_\delta(X-F_A)$ for each $x_\varepsilon \in F_A$. This shows that $x_\varepsilon \notin (X-F_A)$ implies $x_\varepsilon \notin cl_\delta(X-F_A)$. Therefore, we have $cl_\delta(X-F_A)\subset X-F_A$ which implies $cl_\delta(X-F_A)=X-F_A$ and hence F_A is soft δ-open.

(3) Let F_A be a soft δ-open set. Then $X-F_A$ is soft δ-closed and hence $X-F_A=cl_\delta(X-F_A)$. For each $x_\varepsilon \in F_A$, $x_\varepsilon \notin cl_\delta(X-F_A)$ and there exists a soft open neighborhood O_{x_ε} such that int(cl(O_{x_ε})) $\cap$ $(X-F_A)=\emptyset$. Therefore, we have $x_\varepsilon \in O_{x_\varepsilon} \subset$ int(cl(O_{x_ε}))$\subset F_A$ and hence $F_A=\cup\{$int(cl(O_{x_ε})) : $x_\varepsilon \in F_A\}$. By (1), int(cl(O_{x_ε})) is soft regular open for each $x_\varepsilon \in F_A$.

(4) This follows from Definition 3.1.

Proposition 3.3. *Intersection of two soft regular open sets is soft regular open.*

Proof. Let F_A and G_A be soft regular open. Then, we have int($F_A \cap G_A$)$\subset F_A \cap G_A$ = int(cl(F_A)) $\cap$int(cl(G_A)) = int(cl(F_A)$\cap$cl(G_A))$\supset$int(cl($F_A \cap G_A$))$\supset$int($F_A \cap G_A$) = $F_A \cap G_A$. Therefore, $F_A \cap G_A$ = int(cl($F_A \cap G_A$)).

Lemma 3.4. *Let F_A and G_A be soft subsets of soft topological space (X, τ, E). Then, the following properties hold.*

(1) *$F_A \subset cl_\delta(F_A)$,*
(2) *If $F_A \subset G_A$, then $cl_\delta(F_A)\subset cl_\delta(G_A)$,*
(3) *$cl_\delta(F_A)=\cap\{G_A \in SS(X) : F_A \subset G_A$ and G_A is soft δ-closed$\}$,*
(4) *If $(F_A)_\alpha$ is a soft δ-closed set of X for each $\alpha \in \triangle$, then $\cap\{(F_A)_\alpha : \alpha \in \triangle\}$ is soft δ-closed,*
(5) *$cl_\delta(F_A)$ is soft δ-closed.*

Proof. (1) For any $x_\varepsilon \in F_A$ and any soft open neighborhood U_A of x_ε, we have $\emptyset \neq F_A \cap U_A \subset F_A$ $\cap$int(cl(U_A)) and hence $x_\varepsilon \in cl_\delta(F_A)$.

(2) Suppose that $x_\varepsilon \notin cl_\delta(G_A)$. There exists a soft open neighborhood U_A of x_ε such that $\emptyset = int(cl(U_A)) \cap G_A$ and hence $int(cl(U_A)) \cap F_A = \emptyset$. Therefore, we have $x_\varepsilon \notin cl_\delta(F_A)$.

(3) Suppose that $x_\varepsilon \in cl_\delta(F_A)$. For any soft open neighborhood U_A of x_ε and any soft δ-closed set G_A containing F_A, we have $\emptyset \notin F_A \cap int(cl(U_A)) \subset G_A \cap int(cl(U_A))$ and hence $x_\varepsilon \in cl_\delta(G_A) = G_A$. This shows that $x_\varepsilon \in \cap\{G_A \in SS(X) : F_A \subset G_A$ and G_A is soft δ-closed$\}$. Conversely, suppose that $x_\varepsilon \notin cl_\delta(F_A)$, then there exists a soft open neighborhood U_A of x_ε such that $int(cl(U_A)) \cap F_A = \emptyset$. By Lemma 3.2, $X - int(cl(U_A))$ is a soft δ-closed set which contains F_A and does not contain x_ε. Therefore, $x_\varepsilon \notin \cap\{G_A \in SS(X) : F_A \subset G_A$ and G_A is soft δ-closed$\}$.

(4) For each $\alpha \in \Delta$, $cl_\delta(\cap_{\alpha\in\Delta}(F_A)_\alpha) \subset cl_\delta((F_A)_\alpha) = (F_A)_\alpha$ and hence $cl_\delta(\cap_{\alpha\in\Delta}(F_A)_\alpha) \subset \cap_{\alpha\in\Delta}(F_A)_\alpha$. By (1), we obtain $cl_\delta(\cap_{\alpha\in\Delta}(F_A)_\alpha) = \cap_{\alpha\in\Delta}(F_A)_\alpha$. This shows that $\cap_{\alpha\in\Delta}(F_A)_\alpha$ is soft δ-closed.

(5) This follows immediately from (3) and (4).

Theorem 3.5. *Let (X, τ, E) be a soft topological space and $\tau_\delta = \{F_A \in SS(X) : F_A$ is a soft δ-open set$\}$. Then τ_δ is a soft topology weaker than τ.*

Proof. (1) It is obvious that $\emptyset$, $\tilde{X} \in \tau_\delta$.

(2) Let $(H_A)_\alpha \in \tau_\delta$ for each $\alpha \in \Delta$. Then $(H_A)^c_\alpha$ is soft δ-closed for each $\alpha \in \Delta$. By Lemma 3.4, $\cap_{\alpha\in\Delta}(H_A)^c_\alpha$ is soft δ closed and $\cap_{\alpha\in\Delta}(H_A)^c_\alpha = (\cup_{\alpha\in\Delta}(H_A)_\alpha)^c$. Hence $\cup_{\alpha\in\Delta}(H_A)_\alpha$ is soft δ-open.

(3) Let F_A, $G_A \in \tau_\delta$. By Lemma 3.2, $F_A = \cup_{\alpha_1\in\Delta_1}(F_A)_{\alpha_1}$ and $G_A = \cup_{\alpha_2\in\Delta_2}(G_A)_{\alpha_2}$, where $(F_A)_{\alpha_1}$ and $(G_A)_{\alpha_2}$ are soft regular open sets for each $\alpha_1 \in \Delta_1$ and $\alpha_2 \in \Delta_2$. Thus $F_A \cap G_A = \cup\{(F_A)_{\alpha_1} \cap (G_A)_{\alpha_2} : \alpha_1 \in \Delta_1,\ \alpha_2 \in \Delta_2\}$. Therefore $F_A \cap G_A$ is soft regular open. Hence $F_A \cap G_A$ is a soft δ-open set by Lemma 3.2.

4. soft δ-continuous functions

Definition 4.1. *Let (X, τ, E) and (Y, σ, E) be two soft topological spaces and a mapping $f : (X, \tau, E) \to (Y, \sigma, E)$ is said to be soft δ-continuous if for each $x_\varepsilon \in SS(X)$ and each soft open neighborhood V_A of $f(x_\varepsilon)$, there exists a soft open neighborhood U_A of x_ε such that $f(int(cl(U_A))) \subset int(cl(V_A))$.*

Theorem 4.2. *For a function $f : (X, \tau, E) \to (Y, \sigma, E)$, then the following properties are equivalent.*

1. *f is soft δ-continuous,*
2. *For each $x_\varepsilon \in SS(X)$ and each soft regular open set V_A containing $f(x_\varepsilon)$, there exists a soft regular open set U_A containing x_ε such that $f(U_A) \subset V_A$,*
3. *$f(cl_\delta(F_A)) \subset cl_\delta(f(F_A))$ for every $F_A \in SS(X)$,*
4. *$cl_\delta(f^{-1}(K_A)) \subset f^{-1}(cl_\delta(K_A))$ for every $K_A \in SS(Y)$,*

(5) *For every soft δ-closed set F_A of Y, $f^{-1}(F_A)$ is soft δ-closed in X,*
(6) *For every soft δ-open set W_A of Y, $f^{-1}(W_A)$ is soft δ-open in X,*
(7) *For every soft regular open set W_A of Y, $f^{-1}(W_A)$ is soft δ-open in X,*
(8) *For every soft regular closed set G_A of Y, $f^{-1}(G_A)$ is soft δ-closed in X.*

Proof. (1)⇒(2) This follows immediately from Definition 4.1.

(2)⇒(3) Let $x_\varepsilon \in SS(X)$ and $F_A \in SS(X)$ such that $f(x_\varepsilon) \in f(cl_\delta(F_A))$. Suppose that $f(x_\varepsilon) \notin cl_\delta(f(F_A))$. Then, there exists a soft regular open neighborhood H_A of $f(x_\varepsilon)$ such that $f(F_A) \cap H_A = \emptyset$. By (2), There exists a soft regular open neighborhood U_A of x_ε such that $f(U_A) \subset V_A$. Since $f(F_A) \cap f(U_A) \subset f(F_A) \cap V_A = \emptyset$, $f(F_A) \cap f(U_A) = \emptyset$. Hence, we get that $U_A \cap F_A \subset f^{-1}(f(U_A)) \cap f^{-1}(f(F_A)) = f^{-1}(f(U_A) \cap f(F_A)) = \emptyset$. Hence we have $U_A \cap F_A = \emptyset$ and $x_\varepsilon \notin cl_\delta(F_A)$. This shows that $f(x_\varepsilon) \notin f(cl_\delta(F_A))$. This is a contradiction. Therefore, $f(x_\varepsilon) \in cl_\delta(f(F_A))$.

(3)⇒(4) Let $K_A \in SS(Y)$ such that $F_A = f^{-1}(K_A)$. By (3), $f(cl_\delta(f^{-1}(K_A))) \subset cl_\delta(f(f^{-1}(K_A)))$. Therefore, we have $cl_\delta(f^{-1}(K_A)) \subset f^{-1}(cl_\delta(f(f^{-1}(K_A)))) \subset f^{-1}(cl_\delta(K_A))$. Thus we obtain that $cl_\delta(f^{-1}(K_A)) \subset f^{-1}(cl_\delta(K_A))$.

(4)⇒(5) Let F_A be soft δ-closed set of Y. By (4), $cl_\delta(f^{-1}(F_A)) \subset f^{-1}(cl_\delta(F_A)) = f^{-1}(F_A)$ and always $f^{-1}(F_A) \subset cl_\delta(f^{-1}(F_A))$. Hence we obtain that $cl_\delta(f^{-1}(F_A)) = f^{-1}(F_A)$. This shows that $f^{-1}(F_A)$ is soft δ-closed.

(5)⇒(6) Let W_A be soft δ-open set of Y. Then $\tilde{Y} - W_A$ is soft δ-closed. By (5), $f^{-1}(\tilde{Y} - W_A) = \tilde{X} - f^{-1}(W_A)$ is soft δ-closed. Therefore, $f^{-1}(W_A)$ is soft δ-open.

(6)⇒(7) Let W_A be soft regular open set of Y. Since every soft regualr open set is soft δ-open, W_A is soft δ-open. By (6), $f^{-1}(W_A)$ is soft δ-open.

(7)⇒(8) Let F_A be soft regualr closed set of Y. Then $\tilde{Y} - F_A$ is soft regular open. By (7), $f^{-1}(\tilde{Y} - F_A) = \tilde{X} - f^{-1}(F_A)$ is soft δ-open. Therefore, $f^{-1}(F_A)$ is soft δ-closed.

(8)⇒(1) Let $x_\varepsilon \in SS(X)$ and V_A be soft open set containing $f(x_\varepsilon)$. Now, set $H_A = int(cl(V_A))$, then by Lemma 3.2, $\tilde{Y} - H_A$ is a soft regular closed set. By (8), $f^{-1}(\tilde{Y} - H_A) = \tilde{X} - f^{-1}(H_A)$ is a soft δ-closed set. Thus we have $f^{-1}(H_A)$ is soft δ-open. Since $x_\varepsilon \in f^{-1}(H_A)$, by Lemma 3.2, there exists a soft open neighborhood U_A of x_ε such that $x_\varepsilon \in U_A \subset int(cl(U_A)) \subset f^{-1}(H_A)$. Hence $f(int(cl(U_A))) \subset int(cl(V_A))$. This shows that f is soft δ-continuous function.

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