Stability Results for Iteration Procedures using Integral Type Contraction Conditions

Anil Rajput [1], Abha Tenguria [2], Anjali Ojha[3*]

Professor and HOD, Department of Mathematics, Chandra Shekhar Azad Govt. P. G. College, Sehore, M.P., India
Professor and HOD, Department of Mathematics, Govt. M. L. B. Girls College, Bhopal, M.P., India
Research Scholar, Department of Mathematics, Chandra Shekhar Azad Govt. P. G. College, Sehore, M.P., India

Abstract: *In this paper we obtained stability results for Picard iteration procedure using Integral type contraction conditions in complete metric space.*

Keywords: *Stability, Picard iteration procedure, multi-valued mapping, fixed point.*

1. INTRODUCTION

Initially Urabe [23] did study on iteration procedures while the formal definition of stability of iteration procedures was given by Harder and Hicks. The study of stability of iteration procedures in metric space was first studied by Ostrowski [26].

Many authors (see [2], [3], [4], [6], [7], [9], [10], [14], [15], [18], [20], [21], [22], [30], etc) worked on convergence and stability result various mappings in different spaces .Harde Hicks [2,3] worked on theoretical as we numerical aspect of stability. Several resear ([1], [8], [10], [11], [12], [13], [14], [16], [17], [19], [20], [24], [25]) have done work on integral type inequalities.Harder and Hicks done a lot of work on stability of Picard iteration using various contraction conditions. Rhoades [7] established stability results for Mann, Kirk or Massa iteration procedures while Bosede and Rhoades [5] obtained stability results for both Picard and Mann iterations using general class of functions. Also Rezapour et al [28] obtained stability results for Picard iteration procedure for integral type contractive conditions.

Recently Branciari [1] worked on the following contraction integral condition:
for some $\alpha \in [0, 1)$ such that

$$\int_0^{d(f(x),f(y))} \varphi(t)dt \leq \alpha \int_0^{d(x,y)} \varphi(t)dt$$

for all $x, y \in E$ where $\varphi: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, nonnegative with $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$.

While Rhoades [8] worked on the integral condition:

$$\int_0^{d(f(x),f(y))} \varphi(t)dt \leq k \int_0^{m(x,y)} \varphi(t)dt \quad \forall\, x, y \in E$$

where

$$m(x,y) = max\{\, d(x,y), d\big(x, f(x)\big), d\big(y, f(y)\big), \big(\tfrac{d(x,f(y))+d(y,f(x))}{2}\big\}$$

And

$$\int_0^{d(f(x),f(y))} \varphi(t)dt \leq k \int_0^{M(x,y)} \varphi(t)dt \quad \forall\, x, y \in E$$

where

$$M(x,y) = max\{\, d(x,y), d\big(x, f(x)\big), d\big(y, f(y)\big), d\big(x, f(y)\big)d\big(y, f(x)\big)\}$$

with $k \in [0, 1)$
to establish stability results.

2. PRELIMINARIES

There are several iterative processes in the literature for which the fixed points of operators have been approximated over the years by various authors. Some of them are as follows:

For $x_0 \in X$, the sequence $\{x_n\}_{n=1}^{\infty}$ given by

$$x_{n+1} = Tx_n, \; n = 0,1,2.. \qquad (1)$$

is called the picard iteration.

Definition 2.1 [32] Let E be a Banach space and T a mapping from E to E is a self map of E.

For $x_0 \in E$, the sequence $\{x_n\}_{n=1}^{\infty}$ given by

$$x_{n+1} = (1 - \alpha_n)\, x_n + \alpha_n T x_n, \; n = 0,1,2\ldots \qquad (2)$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1) such that $\sum_{n=0}^{\infty} \alpha_n = \infty$ is called the Mann iteration.

If we put $\alpha_n = 1$ in equation (2) we get the picard iteration.

Definition 2.2 [27] For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1 - \alpha_n)\, x_n + \alpha_n T t_n$$

$t_n = (1-\beta_n)\, x_n + \beta_n T x_n,\ n = 0,1,2\ldots$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in [0, 1) and satisfy $\sum_{n=0}^{\infty} \alpha_n = \infty$ is called the Ishikawa iteration. If we put $\beta_n = 0$ then the ishikawa iteration becomes the Mann iteration.

Definition 2.3 [26] Let (X,d) be a metric space and Y be an arbitrary nonempty set.
$CL(X) = \{A; A$ is a non empty closed subset of $X\}$,
$N(X) = \{A; A$ is a non-empty subset of $X\}$,
For $x \in X$ and $B \in N(X)$ is defined as
$D(a,B) = \inf\{d(a,b): b \in B\}$

$H(A,B) = \max\{\sup D(a,B); a \in A, \sup D(A,b); b \in B\}$
H is called a generalized Hausdorff on $CL(X)$.

Definition 2.4 [29] Let (X,d) be a metric space and $Y \subseteq X$. Let $T: Y \to CL(X), S: Y \to X$ and SY or TY is a complete subspace of X. Let T and S have a common fixed point p. For any $x_0 \in Y$ there exists a sequence $\{Sx_n\}$ generated by $Sx_{n+1} \epsilon\, Tx_n, n = 1,2\ldots$ converges to p. Let $\{Sy_n\} \subseteq X$ and set $\in_n = H(Sy_{n+1}, Ty_n), n = 0,1,2\ldots$ then this iterative procedure is (S,T) stable if $\lim_{n\to\infty} \in_n = 0$ implies that $\lim_{n\to\infty} Sy_n = p$.

Lemma 2.1 [30, 31] Let us consider $\delta \in [0,1)$ to be a real number and $\{\epsilon_n\}$ be a sequence of positive numbers such that $\lim_{n\to\infty} \epsilon_n = 0$. If $\{u_n\}$ a sequence of positive real numbers such that

$$u_{n+1} \le \delta u_n + \epsilon_n, \quad n = 0,1,2\ldots$$

Then, $\lim_{n\to\infty} u_n = 0$.

Lemma 2.2 Let (X,d) be a complete metric space and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty} \subset X$ and $\{a_n\}_{n=0}^{\infty} \subset [0,1)$ are sequences such that

$$\left| d(u_n,v_n) - \int_0^{d(u_n,v_n)} \varphi(t)dv(t) \right| \le a_n$$

With $\lim_{n\to\infty} a_n = 0$. Then,

$$d(u_n,v_n) - a_n \le \int_0^{d(u_n,v_n)} \varphi(t)dv(t) \le d(u_n,v_n) + a_n$$

Where $\varphi: R^+ \to R^+$ a Lebesgue-Stieltjes integrable mapping which is summable, non-negative such that for each $\epsilon > 0, \int_0^{\epsilon} \varphi(t)dv(t) > 0$.

3. MAIN RESULTS

Theorem 3.1: Let $T: Y \to CL(X)$ and $S: Y \to X$ be a mapping satisfying

$$\int_0^{H(Tx,Ty)} \varphi(t)dv(t) \le e^{LD(Sx,Tx)}\left[\psi \int_0^{D(Sx,Tx)} \varphi(t)dv(t) + k \int_0^{d(Sx,Sy)} \varphi(t)dv(t)\right]$$

on a complete metric space (X,d) and $S(Y) \subseteq T(Y)$ such that $S(Y)$ or $T(Y)$ is a complete subspace of X.Let p is a fixed point of T and for $x_0 \in X$, there exists a sequence

$$Sx_{n+1} \in Tx_n,\quad n = 0,1,2,\ldots$$

Let $v, \psi: R^+ \to R^+$ be monotone increasing function such that $\psi(0) = 0$ and $\psi: R^+ \to R^+$ is a Lebesgue-stieltjes integrable mapping which is summable, non-negative and such that for each $\in > 0$, $\int_0^{\in} \varphi(t)dv(t) > 0$, then Picard iteration is T-Stable.

Proof: Let us consider $\lim_{n\to\infty} \in_n = 0$ where $\in_n = H(Sy_{n+1}, Ty_n)$ such that $\{y_n\}_{n=0}^{\infty} \subset X$.

Let us consider $\{a_n\}_{n=0}^{\infty} \subset (0,1)$

Then by using lemma 2.2 and triangular inequality, we get

$$\int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) \le d(Sy_{n+1}, p) + a_n$$

$$\le H(Sy_{n+1}, Ty_n) + D(Ty_n, p) + a_n$$

$$\le [H(Sy_{n+1}, Ty_n) - a_n] + [D(Ty_n, p) - a_n] + 3a_n$$

$$\le \int_0^{d(Sy_{n+1},Ty_n)} \varphi(t)dv(t) + \int_0^{H(Ty_n,Tp)} \varphi(t)dv(t) + 3a_n$$

$$\le \int_0^{\in_n} \varphi(t)dv(t) + 3a_n + e^{LD(Sp,Tp)}$$

$$\left[\psi \int_0^{D(Sp,Tp)} \varphi(t)dv(t) + k \int_0^{d(Sp,Sy_n)} \varphi(t)dv(t)\right]$$

$$\le k e^{LD(Sp,Tp)} \int_0^{d(Sp,Sy_n)} \varphi(t)dv(t) + \int_0^{\in_n} \varphi(t)dv(t) + 3a_n$$

$$\le k e^{Ld(p,p)} \int_0^{d(p,Sy_n)} \varphi(t)dv(t) + \int_0^{\in_n} \varphi(t)dv(t) + 3a_n$$

By lemma 2.1, we have,

$$u_{n+1} \le \delta u_n + \in'_n$$

$$0 \le \delta = ke^{Ld(p,p)} < 1$$

$$u_n = \int_0^{d(p,Sy_n)} \varphi(t)dv(t)$$

$$\in_n' = \int_0^{\in_n} \varphi(t)dv(t) + 3a_n$$

With

$$\lim_{n\to\infty} \in_n' = \lim_{n\to\infty}\left(\int_0^{\in_n} \varphi(t)dv(t) + 3a_n\right) = 0$$

So that by Lemma 2.1 and $\int_0^{\in} \varphi(t)dv(t) > 0$ for each $\in > 0$

We have,

$$\lim_{n\to\infty} \int_0^{d(p,Sy_n)} \varphi(t)dv(t) = 0$$

Thus,

$$\lim_{n\to\infty} d(p,Sy_n) = 0$$

$$\lim_{n\to\infty} Sy_n = p$$

Conversely, let $\lim_{n\to\infty} Sy_n = p$ then by lemma 2.2,

$$\int_0^{\in_n} \varphi(t)dv(t) = \int_0^{H(Sy_{n+1},Ty_n)} \varphi(t)dv(t)$$

$$\le H(Sy_{n+1},Ty_n) + a_n$$

$$\le d(Sy_{n+1},p) + D(p,Ty_n) + a_n$$

$$\le [d(Sy_{n+1},p) - a_n] + [D(p,Ty_n) - a_n] + 3a_n$$

$$\le \int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) + \int_0^{H(Ty_n,Tp)} \varphi(t)dv(t) + 3a_n$$

$$\le \int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) + e^{LD(Sp,Tp)}$$

$$[\psi \int_0^{D(Sp,Tp)} \varphi(t)dv(t) + k\int_0^{d(Sp,Sy_n)} \varphi(t)dv(t)] + 3a_n$$

$$\le \int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) + e^{Ld(p,p)}[\psi \int_0^{d(p,p)} \varphi(t)dv(t) + k\int_0^{d(p,Sy_n)} \varphi(t)dv(t)] + 3a_n$$

$$\le \int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) + k\int_0^{d(p,Sy_n)} \varphi(t)dv(t)] + 3a_n$$

$$\to 0 \text{ as } n \to \infty$$

Hence,

$$\lim_{n\to\infty} \in_n = 0.$$

Corollary 3.1: Let $T: X \to X$ be a single valued mapping satisfying $\int_0^{d(Tx,Ty)} \varphi(t)dv(t) \le e^{Ld(x,Tx)}[\psi \int_0^{d(x,Tx)} \varphi(t)dv(t) + k\int_0^{d(x,y)} \varphi(t)dv(t)]$ on a complete metric space (X,d). Let p is a fixed point of T and for $x_0 \in X$, there exists a sequence

$$Sx_{n+1} = Tx_n, \quad n = 0,1,2,\ldots$$

Let $v,\psi: R^+ \to R^+$ be monotone increasing function such that $\psi(0) = 0$ and $\psi: R^+ \to R^+$ is a Lebesgue-stieltjes integrable mapping which is summable, non-negative and such that for each $\in > 0$, $\int_0^{\in} \varphi(t)dv(t) > 0$, then Picard iteration is T-Stable.

Proof: We will prove similarly as in theorem 3.1.

Theorem 3.2: Let $T: Y \to CL(X)$ and $S: Y \to X$ be a mapping satisfying

$$\int_0^{H(Tx,Ty)} \varphi(t)dv(t) \le \alpha \int_0^{\max(d(Sx,Sy),D(Sx,Ty),D(Sy,Ty))} \varphi(t)dv(t) + (1-\alpha)[a\int_0^{D(Sx,Ty)} \varphi(t)dv(t) + b\int_0^{d(Sx,Sy)} \varphi(t)dv(t)]$$

for all $x,y \in X, 0 \le \alpha < 1, a,b \ge 0, a+b < 1$ on a complete metric space (X,d) and $S(Y) \subseteq T(Y)$ such that $S(Y)$ *or* $T(Y)$ is a complete subspace of X. Let p is a fixed point of T,S and for $x_0 \in X$, there exists a sequence

$$x_{n+1} \in Tx_n \quad , \quad n = 0,1,2,\ldots$$

Let $v,\psi: R^+ \to R^+$ be monotone increasing function such that $\psi(0) = 0$ and $\psi: R^+ \to R^+$ a Lebesgue-stieltjes integrable mapping which is summable, non-negative and such that for each

$\in > 0$, $\int_0^{\in} \varphi(t)dv(t) > 0$, then the Picard iteration is T-Stable.

Proof: Let us consider $\lim_{n\to\infty} \in_n = 0$ where $\in_n = H(y_{n+1}, Ty_n)$ such that $\{y_n\}_{n=0}^{\infty} \subset X$.
Let us consider $\{a_n\}_{n=0}^{\infty} \subset (0,1)$
Then by using lemma 2.2 and triangular inequality, we get,

$$\int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) \le d(Sy_{n+1},p) + a_n$$

$$\le H(Sy_{n+1},Ty_n) + D(Ty_n,p) + a_n$$

$$\le [H(Sy_{n+1},Ty_n) - a_n] + [D(Ty_n,p) - a_n] + 3a_n$$

$$\le \int_0^{H(Sy_{n+1},Ty_n)} \varphi(t)dv(t) + \int_0^{H(Ty_n,Tp)} \varphi(t)dv(t) + 3a_n$$

$$\le \alpha \int_0^{\max(d(Sy_n,Sp),D(Sy_n,Tp),D(Sp,Tp))} \varphi(t)dv(t)$$

$$(1-\alpha)[a \int_0^{D(Sy_n,Tp)} \varphi(t)dv(t) + b \int_0^{d(Sy_n,Sp)} \varphi(t)dv(t)]$$

$$+ \int_0^{\in_n} \varphi(t)dv(t) + 3a_n.$$

$$\le \int_0^{\in_n} \varphi(t)dv(t) + 3a_n +$$

$$\alpha \int_0^{\max(d(Sy_n,p),d(Sy_n,p),d(p,p))} \varphi(t)dv(t) +$$

$$(1-\alpha)[a \int_0^{d(Sy_n,p)} \varphi(t)dv(t) + b \int_0^{d(Sy_n,p)} \varphi(t)dv(t)]$$

$$\le \int_0^{\in_n} \varphi(t)dv(t) + 3a_n + \alpha \int_0^{d(Sy_n,p)} \varphi(t)dv(t) + (1-\alpha)(a+b) \int_0^{d(Sy_n,p)} \varphi(t)dv(t)$$

$$\le \int_0^{\in_n} \varphi(t)dv(t) + 3a_n$$

$$+(\alpha + (1-\alpha)(a+b)) \int_0^{d(Sy_n,p)} \varphi(t)dv(t)$$

By lemma 2.1, we have,

$$u_{n+1} \le \delta u_n + \in'_n$$

$$0 \le \delta = (\alpha + (1-\alpha)(a+b)) < 1$$

$$u_n = \int_0^{d(p,Sy_n)} \varphi(t)dv(t)$$

$$\in'_n = \int_0^{\in_n} \varphi(t)dv(t) + 3a_n$$

With,

$$\lim_{n\to\infty} \in'_n = \lim_{n\to\infty}\left(\int_0^{\in_n} \varphi(t)dv(t) + 3a_n\right) = 0$$

So that by Lemma 2.1 and $\int_0^{\in} \varphi(t)dv(t) > 0$ for each $\in > 0$, We have,

$$\lim_{n\to\infty} \int_0^{d(p,Sy_n)} \varphi(t)dv(t) = 0$$

Thus,

$$\lim_{n\to\infty} d(p,Sy_n) = 0$$

$$\lim_{n\to\infty} Sy_n = p.$$

Conversely,

Let $\lim_{n\to\infty} Sy_n = p$

then by lemma 2.2,

$$\int_0^{\in_n} \varphi(t)dv(t) = \int_0^{H(Sy_{n+1},Ty_n)} \varphi(t)dv(t)$$

$$\le H(Sy_{n+1},Ty_n) + a_n$$

$$\le d(Sy_{n+1},p) + D(p,Ty_n) + a_n$$

$$\le [d(Sy_{n+1},p) - a_n] + [D(p,Ty_n) - a_n] + 3a_n$$

$$\le \int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) + \int_0^{H(Ty_n,Tp)} \varphi(t)dv(t) + 3a_n$$

$$\le \int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) + 3a_n$$

$$+ \alpha \int_0^{\max(d(Sy_n,Sp),D(Sy_n,Tp),D(Sp,Tp))} \varphi(t)dv(t) +$$

$$(1-\alpha)[a \int_0^{D(Sy_n,Tp)} \varphi(t)dv(t) + b \int_0^{d(Sy_n,Sp)} \varphi(t)dv(t)]$$

$\leq$
$\int_0^{d(Sy_{n+1},p)} \varphi(t)dv(t) + 3a_n +$
$\alpha \int_0^{d(Sy_n,p)} \varphi(t)dv(t) +$

$(1-\alpha)[a \int_0^{d(Sy_n,p)} \varphi(t)dv(t) +$
$b \int_0^{d(Sy_n,p)} \varphi(t)dv(t)]$

$$\to 0 \; as \; n \to \infty$$

Thus,

$$\lim_{n\to\infty} \in_n = 0.$$

Corollary 3.2: Let $T: X \to X$ be a single-valued mapping satisfying

$$\int_0^{d(Tx,Ty)} \varphi(t)dv(t) \leq \alpha \int_0^{\max(d(x,y),d(x,Ty),d(y,Ty)} \varphi(t)dv(t) + (1-\alpha)[a \int_0^{d(x,Ty)} \varphi(t)dv(t) + b \int_0^{d(x,y)} \varphi(t)dv(t)]$$

for all $x, y \in X, 0 \leq \alpha < 1, a, b \geq 0, a + b < 1$ on a complete metric space (X, d). Let p is a fixed point of T
and for $x_0 \in X$, there exists a sequence

$$x_{n+1} = Tx_n \; , \; n = 0,1,2,\ldots$$

Let $v, \psi: R^+ \to R^+$ be monotone increasing function such that $\psi(0) = 0$ and $\psi: R^+ \to R^+$ a Lebesgue-stieltjes integrable mapping which is summable, non-negative and such that for each $\in > 0$, $\int_0^{\in} \varphi(t)dv(t) > 0$, then the Picard iteration is T-Stable.

Proof: We will prove similarly as in theorem 3.2.

REFERENCES

[1] A. Branciari , "A fixed point theorem for mappings satisfying a general contractive condition of integral type", Int. J. Math. Math. Sci., no 9,521-536. 1.1, 2002.

[2] A. M. Harder and T. L. Hicks ,"A Stable iteration procedure for nonexpansive mappings", Math. Japonica, vol. 33, pp. 687-692, 1988.

[3] A.M. Harder and T.L. Hicks, "Stability results for fixed point iteration procedures", Math. Japon, 33, pp. 693-706, 1988.

[4] A . Ostrowski, "The round of stability of iterations", ZAMM Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, 47, pp. 77-81, 1967.

[5] A. O. Bosede and B. E. Rhoades, "Stability of Picard and Mann iteration for a general class of functions", Journal of Advanced Mathematical Studies, vol. 3, no. 2, pp. 23–25, 2010.

[6] A. R. Khan , F. Gürsoy and V. Kumar, "Stability and data dependence results for the Jungck Khan iterative scheme",Turk. J. Math., 40, pp. 631-640, 2016.

[7] B. E. Rhoades, "Fixed point theorems and stability results for fixed point iteration procedures", Indian Journal of Pure and Applied Mathematics, vol. 21, no.1, pp. 1–9, 1990.

[8] B. E. Rhoades, "Two fixed point theorems for mappings satisfying a general contractive condition of integral type", Int. J. Math. Sci. 63, pp 4007-4013, 2003.

[9] C.O. Imoru and M.O. Olatinwo, "On the stability of Picard and Mann iteration processes", Carpathian J. Math, 19, pp. 155-160, 2003.

[10] D. Achour and M. T. Belaib, "Some stability theorems for some iteration processes using contractive condition of integral type", Abstract and Applied Analysis, vol. 3, pp. 108-115, 2011.

[11] D. Dey, A. Ganguly and M. Saha, "Fixed point theorem for under general contractive condition of integral type", Int. J. Math. Math. Sci., no 9, 521-536. 1.1, 2002.

[12] E. Karapinar, P. Shahi and K Tas., "Generalized $\alpha - \psi$ contractive type mappings of integral type and related fixed point theorems", Journal of Inequalities and Applications, pp.160, 2014.

[13] F. Khojasteh, Z. Goodarzi and A. Razani, "Some fixed point theorems of integral type con-traction in cone metric spaces", Fixed Point Theory and Applications, Vol. 2010, Article ID 189684, 201013 pages.

[14] H. Akewe and G. A. Okeke ,"Stability results for ultistep iteration satisfying a general con-tractive condition of integral type in a norm space", J. of NAMP, Vol. 20, pp. 5-12, 2012.

[15] K. Dogan and V. Karakaya, "On the convergence and stability results for a new general iterative process", Scienti.c world journal. Article ID.852475, pp. 8, 2014.

[16] M. O. Olatinwo, "Some common fixed point theorems for self-mappings satisfying two con-tractive condition of integral type in a uniform space",Cent. Eur. J. Math., inf., 6(2), pp. 335-341, 2008.

[17] M. O. Olatinwo, "A result for approximating fixed points of generalized weak contraction of the integral-type by using Picard iteration", Revista Colombiana de Matematicas, 42(2), pp. 145-151, 2008.

[18] M. O. Olatinwo, "Some stability results for Picard and Mann iteration processes using con-tractive condition of integral type", Creative Math. and inf.,19, pp. 57-64, 2010.

[19] M. O. Olatinwo , "On some fixed point theorems and error estimates involving integral type contractive conditions", General Mathematics, Vol 18(2), pp. 47-57, 2010.

[20] M. Olatinwo, O. Owojori and C. Imoru, "Some stability results on Krasnolslseskij and Ishikawa fixed point iteration procedures", Journal of Mathematics and Statistics, 2, pp. 360-362, 2006.

[21] M. O. Osilike, "On stability results for fixed point iteration procedures", Journal of the Nigerian Mathematical Society, vol. 14-15, pp. 17-29, 1996.

[22] M. O. Osilike, "Stability results for the Ishikawa fixed point iteration procedures", Indian Journal of Pure and Applied Mathematics, vol. 26, no 10, pp. 937-945, 1995.

[23] M. Urabe, "Convergence of numerical iteration in solution of equations", J Sci Hirishima Univ Sér A. 19, pp. 479–489, 1956.

[24] P. Vijayaraju, B. E. Rhoades, and R. Mohanraj, "A fixed point theorem for a pair of maps satisfying a general contractive condition of integral type", Int. J. Math. Sci.,15, pp. 2359-2364, 2005.

[25] R. Bhardwaj, "Some common fixed point theorem in metric space using integral type map-pings", IOSR Journal of engineering, Vol. 2, pp:187-190, 2012.

[26] S. B. Nadler Jr., "Multivalued contraction mapping", Pac. J. Math. 30 , 475–488, 1969.

[27] S. Ishikawa, "Fixed Point by a New Iteration Method", Proc. Amer. Math. Soc. **44** , No. 1, 147-150, 1974.

[28] Sh . Rezapour, R. H. Haghi and B. E. Rhoades, "Some results about *T*-stability and almost *T*-stability", Fixed Point Theory, vol. 12, no. 1, pp. 179–18, 2011.

[29] S. L. Singh and Bhagwati Prasad, "Some coincidence theorem and stability of iterative procedures", computers and Mathematics with Applications 55, pp. 2512-2520, 2008.

[30] V. Berinde, "On the stability of some fixed point procedures", Bul. ¸Stiint. Univ.Baia Mare, Ser. B, Matematica-Informatica, 18, No1, 7-14, 2002.

[31] V. Berinde, "Iterative approximation of fixed points", Second edition, Springer-Verlag Berlin Heidelberg, New York, 2007.

[32] W. R. Mann, "Mean Value Methods in Iteration", Proc. Amer. Math. Soc. **44**, 506-510, 1953.

Declaration: “We declare that there is no conflict of interest regarding the publication of this paper.”
(Anil Rajput, Abha Tenguria and Anjali Ojha)