Properties of Sb* Closed Sets in Topology

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Abstract— *In 1990, s.P Arya et al ., [1] introduced the concept of generalized semi closed sets . In 1987 P.Bhattacharya et.al[2] have introduced the notion of semi generalized closed sets in topological spaces . A.Poongothai and R.Parimelazhagan [3] have already introducred the notion of Sb*-closed sets in topological spaces . R.Parimelazhagan and V.Subramania pillai [4] have already introduced the notion of Sg*-closed sets in topological spaces . In this note I continue the study of these two class of sets in connection with studying the further properties of Strongly b star closed functions .Also I introduce Sb*-limit points and Sb*-derived sets, and also to introduce the new notion like Sb*- regular spaces and Sb*- normal. Spaces*

Keywords — *sb* closed sets, Sb*-limit points , Sb*-derived sets , Sb*-regular, Sb*-normal , Sb*-neighbourhoods*

I. Introduction

In 1990 Leveine [5] introduced the notion of generalized closed (briefly g-closed set. In 1963 , semi open sets and semi continuous functions were introduced and investigated by N.Leveine [6]. In 1987 , Bhattacharya and Lahiri [7] used semi open sets to define and investigate the notion of semi generalized closed sets . Now I found the various papers in the field of generalized open sets and generalized closed sets . I used these sets to introduce and study the notion like Sb*-limit points , Sb*-derived sets, Sb*- regular spaces and Sb*-normal. Spaces

II. PRELIMINARIES

Throughout this paper a space (x, τ) means a topological spaces on which no separations are assumed unless explicitly stated. Let A be a subset of a space (x, τ).

Definition:2.1 A subset (X, τ) is said to be

(1) Semi-pre closed (β -closed)[6] set if int(cl(int(A))) $\subseteq$ A
(2) Semi generalized closed set if scl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is semi-open in X
(3) g-closed[6] set if cl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is open in X
(4) w-closed[5] set if cl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is semi-open in X
(5) α -closed[4] set if cl(int(cl(A))) $\subseteq$ A
(6) wg-closed[5] set if cl(int(A) $\subseteq$ U , whenever A $\subseteq$ U and U is open in X
(7) g*-closed[6] set if if cl(A) $\subseteq$ U , whenever A $\subseteq$ U and U is g-open in X
(8) b-closed[9] set if (cl(int(A $\cup$ int(cl(A))) $\supseteq$ A
(9) b**-closed[2] set if A $\subseteq$ (int(cl(int(A))) $\cup$ (cl(int(cl(A)))
(10) Sb*-closed[2] set if cl(int(A) $\subseteq$ U , whenever A $\subseteq$ U and U is b-open in X

The complements of the above mentioned closed sets are their respective open sets

Definition:2.2

(1) The semi closure of A , denoted by scl(A) is the intersection of all semi-closed sets containing A
(2) The semi interior of A , denoted by scl(A) is the union of all semi-open sets contained in A

Definition:2.3

(1) The Strongly b star(briefly Sb*) closure of A , denoted by Sb*cl(A) is the intersection of all Sb*-closed sets containing A
(2) The Strongly b star(briefly Sb*) interior of A , denoted by Sb*Int(A) is the union of all Sb*-open sets contained in A

Definition:2.4

(1) A space X is said to be generalized s-regular (written as gs-regular) space if for each gs-closed set F and a point x $\notin$ F, there exist disjoint semi open sets U and V such that x $\in$ U and F $\subset$ V
(2) A space X is said to be generalized s-normal (written as gs-normal) space if for each pair of gs-closed sets A and B , there exist disjoint semi open sets U and V such that A $\subseteq$ U and B $\subseteq$ V

Definition:2.5 A mapf:X $\rightarrow$ Y is said to be

(1) Continuous function if $f^{-1}(V)$ is closed in X for every closed set V in Y
(2) b-continuous function if $f^{-1}(V)$ is b-closed in X for every closed set V in Y
(3) g-continuous function if $f^{-1}(V)$ is g-closed in X for every closed set V in Y
(4) α -continuous function if $f^{-1}(V)$ is α -closed in X for every closed set V in Y

(5) w-continuous function if $f^{-1}(V)$ is w-closed in X for every closed set V in Y
(6) g*-continuous function if $f^{-1}(V)$ is g*-closed in X for every closed set V in Y
(7) Sb*-continuous function if $f^{-1}(V)$ is Sb*-closed in X for every closed set V in Y
(8) Sg-continuous function if $f^{-1}(V)$ is Sg-closed in X for every closed set V in
(9) gs-continuous function if $f^{-1}(V)$ is gs-closed in X for every closed set V in Y

III- Sb*CLOSED FUNCTIONS AND SOME PROPERTIES OF Sb*NEIGHBOURHOODS

In this section I introduce the notions of strongly b* limit points and strongly b*-derived sets and also to study the characterizations of strongly b* closed functions

Definition:3.1

A function $f:X \rightarrow Y$ is said to be Strongly b*closed (=Sb* closed) if for each closed set F of X , f(F) is a Sb*-closed set in Y

Example:3.2

Let X=Y={a.b.c.d} , τ ={X, ϕ ,{b},{b,c},{a,b,d}}, σ ={Y, ϕ ,{a},{b},{a,b}, {a,b,c}}. Define a function f: $X \rightarrow Y$ as f(a)=a , f(b)=f(c)=d,f(d)=b. Now take A={a,b,c} .Then A={a,d} is Sb*-closed .

Remark:3.3

A map f: $X \rightarrow Y$ is Sb*-closed map iff for each $P \subset Y$ and for each open set U containing $f^{-1}(P)$ there is an Sb*-open set V of Y such that $P \subseteq V$ and $f^{-1}(V) \subseteq U$

Theorem:3.4

If a function f: $X \rightarrow Y$ is continuous and Sb*-closed and A is b-closed of X , then f(A) is Sb*-closed

Proof:

Let f(A) $\subset V$, where V is an open set of Y . Then $A \subset f^{-1}(V)$. Since f is continuous . $f^{-1}(V)$ is an open set in X . Since A is b-closed (cl(int(A)) $\cup$ int(cl(A)) $\subseteq$ $f^{-1}(V)$. Therefore f(cl(int(A)) $\cup$ int(cl(A)) $\subseteq$ V. Since f is Sb*-closed, Then f(cl(int(A)) $\cup$ int(cl(A)) is also Sb*-closed . Thus cl(int(A))(f(cl(int(A)) $\cup$ int(cl(A))) $\subseteq$ V. Since f(A) $\subseteq$ f(cl(int(A)) $\cup$ int(cl(A))) $\Rightarrow$ cl(int(f(A)) $\subset$ cl(int(f (cl(int(A)) $\cup$ int(cl(A)) $\subset$ V. Thus cl(int(f(A))) $\subset$ V is Sb*-closed $\Rightarrow$ f(A) is Sb*-closed

Remark:3.5

(1) α g-closed and Sb*-closed are independent to each other
(2) g-closed and Sb*-closed are independent to each other
(3) semi closed and Sb*-closed are independent to each other
(4) sg-closed and Sb*-closed are independent to each other

The above remarks are already shown in [3]

Remark:3.6

(1) closed maps and Sb*maps are independent to each other
(2) The composition of two Sb*-closed maps need not be a Sb*-closed map

Definition:3.7

The Strongly b* closure of a subset A of a space X is the intersection of all Strongly b* closets containing A and is denoted by Sb*cl(A)

Remark:3.8

Every closed set is Sb*-closed[3] (i.e) A=Sb*cl(A) . But every Sb* closed set need not be a closed set

Example: 3.9

Let X={a.b.c} , τ ={X, ϕ ,{b}} . Here A={a} Sb*-closed but not a closed set

Definition:3.10

A point x of a space X is called Strongly b* limit point (briefly Sb*-limit point) of a subset A of X , if for each Sb*-open set U containing x , A $\cap$ (U-{x}) $\neq \phi$

Definition:3.11

The set of all Strongly b*-limit points of a given set A is called Sb*-derived set of A and it is denoted by Sb*D(A)

Theorem :3.12

The intersection of two Sb*-neighbourhood of a point is also Sb*-neighbourhood of that point

Proof:

Let x_0 be any point on R . Let M and N be the two Sb*-neighbourhoods of x_0. So that there exists $\varepsilon_1>0$, $\varepsilon_2>0$ such that $(x_0-\varepsilon_1, x_0+\varepsilon_1) \subset M$ and $(x_0-\varepsilon_2 , x_0+\varepsilon_2) \subset N$ where $x_0 \in M$ and N . Choose ε is minimum then $(x_0-\varepsilon , x_0+\varepsilon) \subset (x_0-\varepsilon_1 , x_0+\varepsilon_1) \subset M$. Simillary $(x_0-\varepsilon , x_0+\varepsilon) \subset (x_0-\varepsilon_2 , x_0+\varepsilon_2) \subset N \Rightarrow (x_0-\varepsilon , x_0+\varepsilon) \subset M \cap N$ and $x_0 \in M \cap N$. There exist an open interval $(x_0-\varepsilon , x_0+\varepsilon) \subset x_0$ and which is contained in $M \cap N$. Therefore $M \cap N$ is also a Sb*-neighbourhood of the point x_0

Theorem :3.13

If P and Q are subsets of R, then the following properties are hold

(1) $P \subset Q \Rightarrow Sb^*D(P) \subset Sb^*D(Q)$
(2) $Sb^*D(P \cup Q)=Sb^*D(P) \cup Sb^*D(Q)$
(3) $Sb^*D(P \cap Q)=Sb^*D(P) \cap Sb^*D(Q)$
(4) $x \in Sb^*D(P) \Rightarrow x \in Sb^*D[P-\{x\}]$

Proof:

(i) Let $x \in Sb^*D(P)$ where x is a limit point of P . By using the definition of Sb* limit point of P ., we say that x is a Sb*limit point of Q. Hence $x \in Sb^*D(Q)$. Since $x \in Sb^*D(P) \Rightarrow x \in Sb^*D(P) \subset Sb^*D(Q) \Rightarrow Sb^*D(P) \subset Sb^*D(Q)$

(ii) $Sb^*D(P \cup Q) =Sb^*D(P) \cup Sb^*D(Q)$ by(i) we know that $P \subset P \cup Q$ and $Q \subset P \cup Q \Rightarrow Sb^*D(P) \subset Sb^*D(P \cup Q)$ and $Sb^*D(Q) \subset Sb^*D(P \cup Q)$. Therefore $Sb^*D(P) \cup Sb^*D(Q) \subset Sb^*D(P \cup Q) \rightarrow (1)$. Now we have to prove that $Sb^*D(P \cup Q) =Sb^*D(P) \cup Sb^*D(Q)$. For that we take $x \notin Sb^*D(P) \cup Sb^*D(Q) \Rightarrow x \notin Sb^*D(P \cup Q)$ of P and nor a Sb*D limit poin . (i.e) x is neither a Sb* limit poin t of Q. Let M and N be the Sb* neighbourhoods of x such that $M \cap N$ is a Sb* neighbourhood of x contains no points of P and Q other than x . Hence no point of $P \cup Q \Rightarrow x \notin Sb^*D(P \cup Q) \Rightarrow x \notin Sb^*D(P) \cup Sb^*D(Q) \Rightarrow Sb^*D(P \cup Q) \subset Sb^*D(P) \cup Sb^*D(Q) \rightarrow (2)$. From (1) and (2) we have $Sb^*D(P \cup Q)=Sb^*D(P) \cup Sb^*D(Q)$

(iii) $Sb^*D(P \cap Q)=Sb^*D(P) \cap Sb^*D(Q)$. Simillarly by the above proof (2) we get the result .

(iv) . Let $x \in Sb^*D(P)$. Now by using the definition of Sb* limit point , every neighbourhood of x contains atleast one point of P-{x} $\Rightarrow$ x is a Sb* limit point of P-{x}. Therefore $x \in Sb^*D(P) \Rightarrow x \in Sb^*D[P-\{x\}]$

Theorem:3.14

If $f:X \rightarrow Y$ is Sb* closed function , then $Sb^*cl(A)) \subset f(cl(A))$ for every subset A of X

Proof:

Let $A \subset X$. Since f is Sb* closed function , then f(cl(A)) is Sb*closed containing f(A) . Therefore $Sb^*cl(f(A)) \subset f(cl(A)) \Rightarrow Sb^*cl(A)) \subset f(cl(A))$

Definition :3.15

The Strongly b* star interior of a subset A of a space X is the union of all Sb*-open sets contained in A and is denoted by Sb*Int(A)

Remark:3.16 Every open set is Sb* open[3] (i.e) A=Sb*Int(A) . But every Sb* open set need not be an open set

Theorem:3.17

Let X be a topological space and A be a subset of X . Then A is Sb*open iff A contains a Sb* open neighbourhood of each of its points

Proof:

Necessary Part: A is a Sb*-open set . Let $x \in A$ Therefore $x \in A \subset A$. Hence A is a Sb* neighbourhood of x $\Rightarrow$ A contains a Sb* neighbourhood of each of its points

Sufficient part : Let $x \in A$ there exists a neighbourhood N_x of x such that $x \in N_x \subset A$. Now by the definition of Sb*neighbourhood of x , there exists a Sb* open set G_x such that $x \in G_x \subset N_x \subset A$. Since $x \in A$ there exist Sb*open set such that $x \in G_x$, $x \in \cup \{G_x: x \in A\} \Rightarrow A \subset \cup \{G_x: x \in A\} \rightarrow (1)$. Now take $y \in G_x$ for some $x \in A \Rightarrow y \in A \Rightarrow \cup \{G_x: x \in A\} \rightarrow (2)$. From (1) and (2) $A= \cup \{G_x: x \in A\}$. we know that arbitrary union of Sb*open sets is also a Sb* open set . A is a Sb* open set

Theorem:3.18

If A is Sb* closed subset of X and $x \in X-A$ then there exist a Sb* neighbourhood N of x such that $N \cap A=\phi$

Proof:

Let A is Sb* closed . Then X-A is Sb* open set . By the above theorem(3.17) N be a neighbourhood of x such that $N \subset X-A$. So it is clear that $N \cap A=\phi$

IV-Sb* REGULAR AND Sb* NORMAL SPACES

In this section I define Sb*-Regular Spaces and Sb*- Normal Spaces

Definition:4.1

A space X is said to be Strongly b star regular (written as Sb*Regular) space if for each Sb* closed set F and a point $x \notin F$ there exist disjoint open sets U and V such that $x \in F$ and $F \subset V$

Theorem:4.2

If a function f: X $\rightarrow$Y is continuous and Y is Sb*Regular then f is Sb* continuous and assume that arbitrary union of Sb*open seis is Sb* open

Proof:

Let x be an arbitrary point of X . Since Y is Sb* Regular there exist a Sb* closed set F $\Rightarrow x \notin F$ there exist an open set U in Y containing f(x) and V in Y containing f(x) such that $F \subset V$ and $x \in U$ in Y $\Rightarrow cl(int(F)) \subset V$. Since f is continuous , $f^{-1}(F)$ is Sb* closed in X $\Rightarrow$ f is Sb* continuous

Theorem:4.3

If f:X $\longrightarrow$ Y is an open , continuous, Sb* closed function from a regular space X onto a space Y , then Y is b-regular

Proof:

Let C be a closed subset of X . Since f is continuous $f^{-1}(c)$ is a closed subset of Y . And a point $x \notin C$ (i.e) $x \in X\text{-}C \Rightarrow x \notin f^{-1}$ (C). As X is regular space , and since f is open , so there exist dissjont open sets A and B of X such t hat $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint open sets in Y (i.e) $C \subset A$ and $x \in B \Rightarrow C \subset f^{-1}(A)$ and $x \in f^{-1}$ (B). Since f is Sb* closed by the theorem (3.4) there exist Sb*open sets A and B in Y such that $A \supset C$, $x \in B$ (i.e) $f^{-1}(A) \supset C$ and $x \in f^{-1}$ (B) . Since A and B are disjoint then $A \cap B = \phi$. Hence Y is b-Regular.

Definition :4.4

A topological space X is said to be b-Normal if for each pair of non empty disjoint closed sets A and B there exist a disjoint b-open sets $A \subseteq U$ and $B \subseteq V$, $U \cap V = \phi$

Definition :4.5

A topological space X is said to be Sb* Normal if for each pair of non empty disjoint b-closed sets A and B there exist a disjoint b-open sets $A \subseteq U$ and $B \subseteq V$, $U \cap V = \phi$

Theorem:4.6

If f:X $\longrightarrow$ Y is a continuous, Sb* closed function from a normal space X onto a space Y , then Y is b-normal

Proof:

Let P and Q be disjoint closed sets of a space Y . Since f is continuous , $f^{-1}(A)$ and f^{-1} (B) are disjoint closed sets of X . As X is normal space , so there exist disjoint open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is Sb* closed then by the theorem (3.4) there exist Sb*open sets A and B in Y such that$P \subset A$, $Q \subset B$, $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since U and V are disjoint , then A and B are disjoint $\Rightarrow A \cap B = \phi$. Hence Y is b-Normal.

Theorem:4.7

Every Sb*-normal is b-normal

Proof:

Let X be a Sb* normal space . Let A and B be two disjoint b- closed sets . Since X is Sb*-normal there exists disjoint b-open sets U and V such that $A \subseteq U$ and $B \subseteq V$. we know that b-closed set is closed then A and B are closed sets .n Hence X is b-normal

Theorem 4.8

For a space X , the following are equivalent:

1) X is Sb* - regular.

2) For each $x \in X$ and every Sb* - open set U containing x , there exists an open set G such that $x \in V \subseteq bcl(V) \subseteq U$.

3) For every Sb* - closed set F, the intersection of all b- closed neighbourhoods of F is exactly F.

4) For every set A and a Sb* - open set B such that $A \cap B \neq \varphi$, there exists a b- open set G such that $A \cap V \neq \varphi$ and $bcl(V) \subseteq B$

5) For every non empty set A and any Sb* -closed set B satisfying $A \cap B = \varphi$, there exist disjoint b - open sets V and W such that $A \cap V \neq \varphi$ and $B \subseteq M$.

Proof :

(1) → (2): Let $x \in U$ and U is Sb* - open in X. Therefore, $x \notin X - U$ and X - U is Sb* - closed in X . Since X is Sb* - regular , there exist disjoint b - open sets V and W such that $x \in V$ and $X - U \subseteq W$. Then$V \subseteq X - W \subseteq U$. Since W is bi -open , therefore bcl (X - W) = X - W. Hence $x \in G \subseteq bcl(V) \subseteq U$.

(2) → (3): Let F be a Sb* - closed subset of X and $x \notin F$. Then X - F is a Sb* - open set containing x. Since by (2) it is clear that $x \in V \subseteq bcl(V) \subseteq X - F$. Hence, $F \subseteq X - bcl(V) \subseteq X - V$ and $x \notin X - V$. Thus X - V is a b - closed neighbourhood of F which does not contain x. Hence, the intersection of all b- closed neighbourhoods of F is exactly F.

(3) → (4): Let A be a non empty subset of X and B be a Sb* - open set such that $A \cap B \neq \phi$. Let $x \in A \cap B$. Then X - B is a Sb* - closed such that $x \notin X - B$.. Since by (3), Let take M be a b- closed neighbourhood of X - B, such that $x \notin M$. there exists a b- open set U such that $X - B \subseteq U \subseteq M$. Now Take V = X - M. Then V is a b- open set containing x. Also $A \cap V \neq \varphi$. Now, $bcl(V)=bcl(X-M) \subseteq X-U \subseteq B$. Hence $bcl(V) \subseteq B$.

(4) →(5): Let $A \cap B = \varphi$, here A is non-empty and B is Sb*-closed , then $A \cap X\text{-}B \neq \varphi$ where X-B is Sb*-open set . By (4), there exists a b-open set V such that $A \cap V \neq \varphi$ and $V \subseteq bcl(V) \subseteq X\text{-}B$. Now take $W=X\text{-}bcl(V) \Rightarrow B \subseteq W$,where V and W are b-open sets such that $V \cap W = \varphi$

(5) $\longrightarrow$ (1): Let F be a Sb*-Closed subset of X and $x \notin F$. Then {x} and f are disjoint (5). There exist disjoint b-open sets V and W such that $\{x\} \cap V \neq \varphi$ and $F \subseteq W \Rightarrow x \in V$ and $F \subseteq W$. X is Sb*-regular

V-CONCLUSION

The notion of Sb*-normal , Sb*-regular and Sb*-neighbourhoods in topological spaces has been extended and applied for Strongly generalized star closed sets and obtain some characterizations . These notions can be applied for investigating many other properties

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