# "Solow Model" and Its' Linkage with "Harrod-Domar"

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Abstract: In this paper we aim to link Solow Model and HarrodDomar Model. We know that if we Relax "Innada Condition" of Solow Model then unique steady state may not exist. In our paper we

will observe that the growth equation of "HarrodDomar" i.e.  $\frac{s}{\theta} = \delta + n$  is associated with multiple steady state of per capital. We have done this linking Solow Model and HarrodDomar.

Keywords: HarrodDomar, Solow Model, Innada Condition, Steady State, Production function.

### INTRODUCTION

Following are some Assumptions for the Solow Model.

1. Aggregation Production function is continuous in time

Y(t) = F[K(t), L(t)]

Growth in output Y is only possible from the Growth in input (K,L)

- 2. NO National / International trade.
- 3. All the factors are fully employed.
- 4. Labour is Homogeneous and Labour force grows at the rate of *n*.

i.e.  $\frac{L}{L} = n$ 

- 5. Growth of K capital from investment is  $\dot{K} = I(t) \delta K(t)$ where  $\delta$  is depreciation rate.
- 6. Per Capita output is given by

$$y = f(k) = \frac{F(K, L)}{L}$$

### Assumptions on Production function

The Production function must satisfy the following 3 properties:

< 0,

#### 1. Positive and Diminishing Marginal Products

$$Y = F[K(t), L(t)]$$
$$\frac{\partial F}{\partial K} > 0, \qquad \frac{\partial^2 F}{\partial K^2}$$

$$\frac{\partial F}{\partial L} > 0, \qquad \qquad \frac{\partial^2 F}{\partial l^2} < 0,$$

- 2. Constant Return to Scale  $F(CK, CL) = CF(K, L) \quad \forall C > 0$
- 3. Innada Condition

$$\lim_{k \to \infty} f'(k) \to 0$$
$$\lim_{k \to 0} f'(k) \to \infty$$

# Per Capita Accumulation Equation

Consider the production function

$$Y = AK^{\alpha}L^{1-\alpha}; \quad 0 < \alpha < 1$$

where A is constant and L Labour force grows at the rate of n.

i.e. 
$$\frac{l}{L} = n$$
  
Now,  $\dot{K} = I(t) - \delta K(t)$   
 $I(t) = S(t)$ , Since its's a closed economy.  
 $S(t) = sY(t)$   
so, $K\dot{t}(t) = sY(t) - \delta K(t)$   
 $\frac{\dot{K}(t)}{K(t)} = \frac{sY(t)}{K(t)} - \delta$ 

Dividing s Y(t) and K(t) by L(t), we obtain

$$\frac{\dot{K}(t)}{K(t)} = \frac{s y(t)}{k(t)} - \delta \qquad (1)$$
Now,  $Dk(t) = D\left[\frac{K(t)}{L(t)}\right]$ 

$$= \frac{L(t)\dot{K}(t) - K(t)\dot{L}(t)}{L^{2}(t)}$$
 $\dot{k}(t) = \frac{K(t)}{L(t)} - \frac{K(t)L(t)}{L(t)L(t)}$ 

$$= \frac{K(t)}{K(t)L(t)} - \frac{K(t)L(t)}{L(t)L(t)}$$

$$= \frac{K(t)}{L(t)} \left[\frac{K(t)}{K(t)} - \frac{L(t)}{L(t)}\right]$$
 $\dot{k}(t) = k(t) \left[\frac{\dot{K}(t)}{K(t)} - n\right]$ 

$$\frac{k(t)}{k(t)} + n = \frac{K(t)}{K(t)} \qquad (2)$$

$$\frac{k(t)}{k(t)} + n = \frac{K(t)}{K(t)} \quad (2$$

Put (2) in (1), we get

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$$\frac{k(t)}{k(t)} + n = \frac{sy(t)}{k(t)} - \delta$$

$$\dot{k}(t) = sy(t) - (\delta + n)k(t)$$
(3)

Hence Equation (3) is per capita accumulation equation

Now, 
$$Y(t) = AK(t)^{\alpha}L(t)^{1-\alpha}$$
  
 $y(t) = \frac{Y(t)}{L(t)} = Ak(t)^{\alpha}$   
 $\Rightarrow \qquad \dot{k}(t) = sA[k(t)]^{\alpha} - (\delta + n)k(t)]$ 

This is per capita accumulation equation as per our production function

$$Y(t) = AK(t)^{\alpha}L(t)^{1-\alpha}$$



For Steady State

$$\dot{k} = 0$$

$$sA[k(t)]^{\alpha} = (\delta + n)(k(t))$$

$$\frac{sA}{\delta + n} = [k(t)]^{1-\alpha}$$

$$k^* = k(t) = \left(\frac{sA}{\delta + n}\right)^{\frac{1}{1-\alpha}}; \text{ steady state per capita "capital"}$$

Here,

$$f(k) = \frac{Y(t)}{L(t)}$$
$$= \frac{AK(t)^{\alpha} L(t)^{1-\alpha}}{L(t)}$$
$$= A(k(t))^{\alpha}$$
$$f'(k) = \alpha A(k(t))^{\alpha-1}$$

(1)  $\lim_{k\to\infty} A\alpha k^{\alpha-1} \to 0$ 

(2) 
$$\lim_{k\to 0} A\alpha k^{\alpha-1} \to \infty$$

Hence, "Innada Conditions" are satisfied.

 $\therefore$  there must exist a steady state given by

$$k^* = \left(\frac{sA}{\delta+n}\right)^{\frac{1}{1-\alpha}}.$$

Note : Does the "steady state" exists for following Production functions

(1) 
$$Y = AK$$

then 
$$y(t) = \frac{Y(t)}{L(t)} = \frac{AK(t)}{L(t)} = Ak(t)$$

Now, per capita accumulation equation is given by

$$\dot{k} = sy(t) - (\delta + n)k(t)$$

$$\dot{k} = sAk(t) - (\delta + n)k(t)$$
(1)

Now, f(k) = y = Ak

$$f(k) = A$$
$$\lim_{k \to \infty} f'(k) = \lim_{k \to \infty} A = A \neq 0$$

So, it does not satisfy Innada Conditions.

Now, 
$$\dot{k} = [sA - (\delta + n)]k(t)$$
  
$$\lim_{k \to \infty} \dot{k} = \infty; \text{ if } sA > \delta + n$$



"No Steady state " exist Per Capita Capital grows perpetually forever.

(2) 
$$Y = AK + BK^{\alpha}L^{1-\alpha}, \qquad 0 < \alpha < 1$$
$$y = \frac{Y}{L} = \frac{AK}{L} + \frac{BK^{\alpha}L^{1-\alpha}}{L}$$
$$f(k) = y = Ak + Bk^{\alpha}$$
$$f'(k) = A + \alpha Bk^{\alpha-1}$$
$$\lim_{k \to \infty} f'(k) = A$$
$$\lim_{k \to 0} f'(k) = \infty$$

So "Innada Conditions" are not satisfied

So, it doesn't satisfy "Innada Conditions"

$$\dot{k} = sf(k) - (\delta + n)k(t)$$
$$\dot{k} = s[Ak + Bk^{\alpha}] - (\delta + n)k$$
$$\dot{k} = [s - (\delta + n)]k + sBk^{\alpha}$$

Now,

$$\lim_{k\to\infty} \dot{k} = \infty, \qquad \forall s > (\delta + n)$$

Hence "no steady state" exists.

# HARROD DOMAR MODEL

Main assumption of the "HarodDomar" Model is that capital and Labour are pure complements meaning that they cannot substitute for each other in production. The underlying production function is of the Leontief type.

$$Y(t) = \min\{AK(t), BL(t)\}, A \& B are positive constants.$$

Now,  $\dot{k} = sy - (\delta + n)k$ 

$$\frac{Y(t)}{L(t)} = \min\left\{\frac{AK(t)}{L(t)}, \frac{BL(t)}{L(t)}\right\} = \min\{Ak(t), B\}$$

$$\Rightarrow \qquad y(t) = \min\{Ak(t), B\}$$

$$\dot{k} = s \min\{Ak(t), B\} - (\delta + n)k$$

i.e. 
$$\dot{k} = \min\{sAk(t), sB\} - (\delta + n)k$$



### Now , Case I



$$sB = (\delta + n)k^*$$

$$\frac{sB}{\delta + n} = k^*$$
 Steady state per capita "Capital"

CASE II



$$k^* = [0, \hat{k}]$$

Now, sB = sAk at  $\hat{k}$ 

$$\frac{B}{A} = \hat{k}$$
$$k^* = \left[0, \frac{B}{A}\right]; \qquad A = \frac{\delta + n}{s}$$

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So, 
$$k^* = \left[0, \frac{B}{A}\right] = \left[0, \frac{sB}{\delta + n}\right]$$

This case actually satisfies "HarrodDomar" as

$$sA = \delta + n \qquad (1)$$

Now,  $Y(t)=\min\{AK(t), BL(t)\}$ 

Let 
$$\theta = \frac{K(t)}{Y(t)}$$
 which is capital output ratio

So,  $A = \frac{1}{\theta}$  by recognising production function

Put in (1)  $\frac{s}{\theta} = \delta + n$  (HarrodDomar growth equation)

Note : "'HarrodDomar" Knife edge problem can be explained through this also. "Knife edge". Because it can only happened when both slopes are equal i.e.  $sA = \delta + n$  further,  $A = \frac{1}{2} \Rightarrow \frac{s}{2} = \delta + n$ 

$$A = \frac{1}{\theta} \Longrightarrow \frac{s}{\theta} = \delta + n$$

Note : If we consider case 3 when  $sA < \delta + n$ . Here we will get steady state equivalent to 0.

#### CONCLUSION

1. Since the production function, doesn't satisfy Innoda condition (as described above). Hence unique steady state will not exist always as described by case 2.

2. Case 2 satisfies HarrodDomar Model which states knife edge equilibrium solutions. This happens because it can only take place at a point when  $sA < \delta + n$  i.e. both lines are overlapping. In this case, there are many steady states ranging from  $[0, [0, \hat{k}], \hat{k} = \frac{B}{A}$  such that

 $sA = \delta + n$ ,  $A = 1/\theta$  this solution Matches to that of HarrodDomar.

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