An Extension of Fixed Point Theorems in TVS- Valued Cone Metric Spaces

S.K.Tiwari[1], Leena Mahar[2]

[1]Department of Mathematics, DR. C.V. Raman University, Bilaspur, Chhattisgarh-495113, India
[2]Department of Mathematics (M. Phil Scholar) DR. C.V. Raman University, Bilaspur, Chhattisgarh-495113, India

Abstract: *the purpose of this is to obtain common fixed point theorems for contractive mappings in the setting of topological vector space-valued cone metric spaces. Our results generalize some well- known recent results the literature of [29].*

Key word: *common fixed point, contractive mapping, TVS-Valued cone metric space.*

I. INTRODUCTION:-

In 1906, the French mathematician Maurice Frechet [1, 2] introduced the concept of metric spaces, although the name "metric" is due to Housdorff [1,3]. In 1934, the Serbian mathematician Duro Kurepa[4], a Ph.D student of Frechet, introduced metric spaces in which an ordered vector spaces used as the co domain of a metric instead of the set of the real numbers. In the literature the metric spaces with the vector valued metrics are know under various name such as pseudo metric spaces, k-metric spaces generalized metric spaces , cone valued metric spaces ,cone metric spaces, abstract metric spaces and vector valued metric spaces. Fixed point theory in K-metric space was developed by A.I.Perov in 1964[5, 6]. For more details on fixed point theory in K-metric and K-normed spaces, we refer the reader to [7].

In 2007, Hunge and Zhang [8] reintroduced such metric spaces under the name cone metric spaces without mentioning the previous works. But they went further, defining convergent and Cauchy sequences in terms of interior points of the underlying cone. They also proved some fixed point theorem in such spaces in the same work. Subsequently, several authors have studied fixed point theory of cone metric spaces. Many authors [12-27] studied fixed points results of mapping satisfying contractive type condition in Banach space-valued cone metric spaces. Recently, In 2009 I. Beg et al [9] and in 2010 Du [10] generalized cone metric spaces to topological vector space valued cone metric space and obtained common fixed points of a pair of mapping satisfying generalized contractive type conditions w8ithout assumption of normality in a class of topological vector space-valued cone metric spaces which is biggar than that of studied in [12-27].

In this approach ordered topological vector spaces are used as the co domain of metric instead of Banach spaces. While I. Beg et al used Housdorff TVS; Du used locally convex Housdorff TVS. However, a result in [11] shows that if the underlying cone of an ordered TVS is solid and normal it must be an ordered normed spaces. So, proper generalized from Banach space valued cone metric spaces to TVS-CMS can be obtained only in the case of non normal cones.

Several authors had showed that some fixed point theorem in usual metric spaces and their TVS-CMS counterparts are equivalent. See [2],[9] and [10]. One of the tools used to prove such equivalences is the so called nonlinear secularizations function. This method was used earlier in optimization theory to convert optimization problems in Victorian forms to their scalar forms. This function was introduced into fixed point theory by Du [10]. In sequel, Akbar Azam, Ismat Beg and M.Arshad [29] obtain fixed point in topological vector space- valued cone metric spaces which are generalize of the results [15] and [16].

Akbar Azam and B.E.Rhoades[28] obtained common fixed point of a pair of mappings satiasfying a generalized contractive type of condition in TVS valued cone metric spaces. Also, Zoran Kadelburg, Stojan Radenovi´ and Vladimir Rakoˇcevi´c[30], developed the theory of topological vector space valued cone metric spaces with non normal cones. We prove three general fixed point results in these spaces and deduce as corollaries several extensions of theorems about fixed points and common fixed points, known from the theoryof _normed-valued_ cone metric spaces.

In this paper we have prove common fixed point results for contractive type conditions in the setting of Topological vector space- valued cone metric spaces. Our results generalize some well known recent results in the literature of [29].

II. PRELIMINARIES

Definition 2.1: [9,10]: Let (E ,τ) be a topological vector space. A subset P of E is called a cone if :

1) P is closed , nonempty and nontrivial (i.e., P≠{0});

2) ax +by ∊ P for all x ,y ϵ P and nonnegative real numbers a and b and

3) P∩ (-P) = {0}.

We define a partial ordering ≤ on E with respect to P by x ≤y if and only if y-x ∈P. We write x<y if x≤y and x≠y. Likewise, we write x≪y if y-x ∊ intP, where int P denotes the interior of P. If ambiguity is possible we can use the notation $\leq_P$, $<_P$ and $\ll_P$. The pair (E , P) consisting of a TVS E and a solid cone P of E is called a partially ordered TVS.

Definition 2.4[9, 10]: Let X be a nonempty set. A mapping $d: X \times X \to E$ satisfying

(d1) $d(x, y) \geq 0$

(d2) $d(x, y) = 0 \Leftrightarrow x = y;$

(d4) $d(x, y) \leq d(x, z) + d(z, y)$ $for\ all\ x, y, z \in X,$

is called a TVS-valued cone metric on X. The pair (X, d) is called a TVS-valued cone metric space (written briefly as TVS-CMS).

Definition 2.5: [9, 10] Let (X, d) be a TVS-cone metric space, $x \in X$ and $\{x_n\}$, n=1,2,…….be a sequence in X. Then we say

(i) TVS-converges to x if for every c ∊ int P there exists a natural number N such that d $(x_n, x) \ll c$ for all n≥N.

(ii) $\{x_n\}$ is a TVS-cauchy sequence if for every c∈int P there exists a natural number N such that d$(x_n, x_m) \ll c$ for all n,m≥N.

(iii) (X, d) is a TVS-complete cone metric space if every Cauchy sequence in (X, d) is convergent in (X, d).

(iv) MAIN RESULT

The following theorems extend and generalize the result of 2.1 of [29]:

Theorem 3.1:- Let (X, d) be a complete topological vector space-valued cone metric space . P be a cone and m, n be positive integer. If a mapping $T_1, T_2: X \to X$ satisfies $d(T_1{}^m x, T_2{}^n y) \leq Ad(x, y) + Bd\,(x, T_1{}^m x) + Cd\,(y, T_2{}^n y) + Dd\,(x, T_2{}^n y) + Ed(y, T_1{}^n x)$, For all $x, y \in X$, where A, B, C, D, E are non negative real numbers with $A + B + C + D + E < 1$, Then $T_1{}^m$ &$T_2{}^n$ are an unique common fixed point in X.

Proof:- For $x_0 \in X$ and $K \geq 0$,define

$$\boldsymbol{x_{2k+1}} = T_1{}^m x_{2k} \quad \text{and}$$

$$\boldsymbol{x_{2k+2}} = T_2{}^n x_{2k+1}$$

Then

$$d(\boldsymbol{x_{2k+1}}, \boldsymbol{x_{2k+2}}) = d(T_1{}^m x_{2k}\ , T_2{}^n x_{2k+1})$$

$$\leq Ad(x_{2k}, x_{2k+1}) + Bd(x_{2k}, T_1{}^m x_{2k}) + Cd(x_{2k+1}, T_2{}^n x_{2k+1})$$

$$+Dd(x_{2k}, T_2{}^n x_{2k+1}) + Ed(\boldsymbol{x_{2k+1}}, T_1{}^m x_{2k})$$

$\leq [A+B]d(x_{2k},x_{2k+1})+ Cd(\boldsymbol{x_{2k+1}},\boldsymbol{x_{2k+2}})$
$+ Dd\ (x_{2k}, \boldsymbol{x_{2k+2}})$
$\leq\ [A+B+D]d(x_{2k}, x_{2k+1})\ +\ [C+D]d(\boldsymbol{x_{2k+1}}, \boldsymbol{x_{2k+2}})$.

It implies that

$[1-C-D]d(\boldsymbol{x_{2k+1}},\boldsymbol{x_{2k+2}}) \leq [A+B+D]d(x_{2k}, x_{2k+1})$.

Therefore, $d(\boldsymbol{x_{2k+1}},\boldsymbol{x_{2k+2}})\leq \frac{[A+B+D]}{[1-C-D]} d(x_{2k}, x_{2k+1})$.

That is,

$$d\ (\boldsymbol{x_{2k+1}}, \boldsymbol{x_{2k+2}})\ \leq\ F\ d(x_{2k}, x_{2k+1})$$

Where $F = \{A+B+D\}/\{1-C-D\}$.

Similarly,

$d(x_{2k+2}, x_{2k+3}) = d(\ T_1{}^m x_{2k+2}, T_2{}^n x_{2k+1})$

$\leq Ad(x_{2k+2}, x_{2k+1})+Bd(x_{2k+1},T_1{}^m x_{2k+2})$
$+Cd(x_{2k+1}, T_2{}^n x_{2k+1})$

$+ Dd(x_{2k+2,}, T_2{}^n x_{2k+1})+Ed(x_{2k+1}, T_1{}^m x_{2k+2})$

$\leq Ad(x_{2k+2}, x_{2k+1})+Bd(x_{2k+2},x_{2k+3})+Cd(x_{2k+1}, x_{2k+2},)$

$+\ \ Dd(x_{2k+2}, x_{2k+2})\ + Ed(x_{2k+1}, x_{2k+3}$

$\leq[A+C+E]d\ (x_{2k+2}, x_{2k+1})\ +\ [B+E\ d\ (x_{2k+2},x_{2k+3})$

which implies

$$d\ (x_{2k+2},x_{2k+3}) \leq G\ d(x_{2k+1}, x_{2k+2}),$$

with $G = (A+C+E)/(1-B-E)$

Thus $p\,, q \in N$

$d(x_{2p+1},x_{2q+1})\ \leq\ d(x_{2p+1},x_{2q+2}) + d(x_{2p+2},x_{2q+3}) + d(x_{2p+3,} {}_+x_{2q+4}) +\ldots +d(x_{2p},x_{2q+1})$

$\leq\ (F^{2p+1} + F^{2p+2} + \ldots\ldots+ F^{2p})\ d\ (x_0,x_1)$

$\leq\ (F^{2q}/1-q)\ d\ (x_0,x_1),$

Hence, for $0< n < m$

$$d(x_n,x_m) \leq\ (F^{2q}/1-q)\ \ d\ (x_0,x_1),$$

Fix $0 \ll c$ and choose a symmetric neighborhood V of 0 such that c+V ⊆ intP. Since

$a_n \to 0$ as $n \to \infty$, we deduce that $\{x_n\}$ is a Cauchy sequence. Since X is a complete, there exists u∈X such that $x_n \to u$. Fix $0 \ll c$ and choose $n_0 \in \mathbb{N}$ be such that

$d(u,x_{2k}) \ll c/3K$, $d(x_{2k-1},x_{2k}) \ll c/3K$, $d(u, x_{2k-1}) \ll c/3K$,

$K=\max\{1+D/1-B-E, A+E/1-B-E, C/1-B-E\}$.

Now

$d(u, T_1{}^m\ u)\ \leq d(u,x_{2k}) + d(u, T_1{}^m\ u)$
$\leq d(u,x_{2k})+d(T_1{}^m x_{2k-1}, T_1{}^m\ u)$

$\leq d(u,x_{2k})+A\ d(u, x_{2k-1})+Bd(u,T_1{}^m\ u)$

$+Cd(x_{2k-1}, T_1{}^m x_{2k-1})+D\ d(u,T_1{}^m x_{2k-1})$

$+ Ed(x_{2k-1}, T_1{}^m\ u)$

$\leq d(u,x_{2k}) + A\ d(u, x_{2k-1}) + Bd(u, T_1{}^m\ u)$

$+ Cd(x_{2k-1}, x_{2k}) + D\ d(u, x_{2k}) + Ed(x_{2k-1},u)$

$+\ Ed(u, T_1{}^m\ u)$

$\leq(1+D)d(u,x_{2k})+(A+E)d(u,x_{2k-1})+Cd(x_{2k-1}, x_{2k})$

$+(B+E)d(u, T_1{}^m\ u)$

So,

$d(u, T_1{}^m\ u)\ \leq K\ d(u, x_{2k}) +K\ d(u, x_{2k-1},)$

$+ K\ d(x_{2k-1}, x_{2k})$

$\ll \frac{c}{3} + \frac{c}{3} + \frac{c}{3} = c$

Hence

$$d(u, T_1{}^m\ u)\ \ll c/p$$

for every $p \in \mathbb{N}$. From

$c/p - d(u, T_1^{\ m}\ u) \ \in \mathrm{intP}$

being P closed , as p→ ∞, we deduce $-d(u,T_1^m u) \in P$ and so $d((u,T_1^m u) = 0$. This implies that $u = T_1^{\ m}\ u$.

Similarly, by using the inequality,

$$d(u, T_1^{\ m}\ u) \le d(u, x_{2k-1}) + K\ d(u, x_{2k-1})$$
$$+ d(x_{2k-1}, T_2^{\ n} u),$$

we can show that $u = T_2^{\ n} u$, which in turn implies that u is a common fixed point of

$T_1^{\ m}$, $T_2^{\ n}$ and, that is $u = T_1^{\ m}\ u = T_2^{\ n} u$. Hence complete the proof

Theorem 3.2:- Let (X, d) be a complete topological vector space-valued cone metric space . P be a cone and m, n be positive integer. If a mapping $T_1, T_2 : X \to X$ satisfies

$d(T_1^{\ m} x , T_2^{\ n} y) \ \le\ Ad(x, y) + B[d\ (x, T_2^{\ n} y) + d\ (y, T_1^{\ m} x)] + C[d\ (x, T_1^{\ m} x) + d(y, T_2^{\ n} y)]$

For all $x, y \ \in X$, where $A, B, C \ \in$ [0,1]are non negative real numbers with $A + 2B + 2C < 1$

Then $T_1^{\ m}$ &$T_2^{\ n}$ are a unique common fixed point.

Proof:- For $x_0 \in$X and $k \ge 0$,and define the sequence$\{x_n\}$ in X such that

$$\boldsymbol{x_{2k+1}} = T_1^{\ m}\ x_{2k} \quad \text{and}$$

$$\boldsymbol{x_{2k+2}} = T_2^{\ n}\ x_{2k+1}$$

For all $k =$ 0,1,2, ...

We obtain

$$d(\ \boldsymbol{x_{2k+1}}, \boldsymbol{x_{2k+2}}) \ = \ d(\ T_1^{\ m}\ x_{2k}, T_2^{\ n}\ x_{2k+1}\ )$$
$$\le\ Ad(x_{2k},\ \ x_{2k+1}) + B[d(x_{2k}, T_2^{\ n}\ x_{2k+1})$$
$$+ d(x_{2k+1}, T_1^{\ m}\ x_{2k})]$$
$$+ C[d(x_{2k}, T_1^{\ m}\ \ x_{2k})\ + d(x_{2k+1},\ \ T_2^{\ n}\ x_{2k+1})]$$

$$\le\ Ad(x_{2k},\ \ x_{2k+1}) + \ B[d(x_{2k+1}, x_{2k})\ + d(x_{2k}, (x_{2k+1})]$$
$$+\ \ C[d(x_{2k+1}, x_{2k})\ \ + d(x_{2k}, x_{2k+1})]$$
$$\le\ \ [A + B + C]d(x_{2k+1}\ \ x_{2k}) \ + [B + C]\ d(x_{2k}, x_{2k+1}).$$

It implies that

$$d(\ \boldsymbol{x_{2k+1}}, \boldsymbol{x_{2k+2}}) \le \frac{A+B+C]}{(1-B-C)}(d(x_{2k}, x_{2k+1}).$$

where

$$F = (A + B + C)/(1 - B - C) < 1$$

Similarly,

It can be shown that

Induction, we obtain for each $k \ = \ 0,1,2,, \ldots..$

$$d(x_{2k+2}, x_{2k+3}) \le F d(\boldsymbol{x_{2k+1}}, \boldsymbol{x_{2k+2}})$$

for all

$$d(x_{2k+2}, x_{2k+3}) \ \le F d(\boldsymbol{x_{2k+1}}, \boldsymbol{x_{2k+2}})$$
$$\le \ldots\ldots \le F^{k+1}\ d\ (x_0, x_1\ ).$$

For p<q we have

$$d(x_{2p+1}, x_{2q+1}) \le d(x_{2p+1}, x_{2q+2}) + d(x_{2p+2}, x_{2q+3})$$
$$+ d(x_{2p+3,+} x_{2q+4}) + \ldots. + d(x_{2p}, x_{2q+1})$$
$$\le [F^{2p+1} + \ \ F^{2p+2} \ \ + \ldots.. + F^{2q} \ \ ]\ d\ (x_0, x_1\ ).$$
$$\le [(F^{2p+1}/1 - F\ ]\ d\ (x_0, x_1\ ).$$

By the definition of normal cone

$$d(x_{2p+1}, x_{2q+1}) \le\ [(F^{2p+1}/1 - F\ ]\ \ \mathrm{K}d\ (x_0, x_1\ ).$$

which implies that $d(x_{2p+1}, x_{2q+1}) \to 0$ as $p, q \to \infty$

Hence $\{x_n\}$ is Cauchy sequence since $\{x_n\}$ is a cauchy sequence in $T_1^{\ m} x$ which is complete there exists $u \in$X

Let us prove that

$$u = T_1^{\ m} u$$

Then by the triangle inequality

We have

$$d(T_1^{\ m} u, u) \ \le d(T_1^{\ m} u, T_2^{\ n}\ x_{2k-1},) + d(T_2^{\ n}\ x_{2k-1}, u)$$
$$\le A\ d(u, x_{2k-1},) + B\ [d(u, T_2^{\ n}\ x_{2k-1})$$
$$+ d(x_{2k-1},), T_1^{\ m} u)]$$
$$+ C[\ d(u, T_1^m u) + d(x_{2k-1}), T_2^{\ n}\ x_{2k-1})]$$

By the definition of normal cone

$d(T_1{}^m u, u) \leq K(Ad(u, x_{2k-1}) + B[d(u, T_2{}^n x_{2k-1}) + d(x_{2k-1}, T_1{}^m u)] + C[d(u, T_1{}^m u)]$

$+d(x_{2k-1}, T_2{}^n \ x_{2k-1})] \quad + \ d(T_2{}^n x_{2k-1,} u)$

let $k \to \infty$ we obtain

$d(T_1{}^m u, u) \leq A\, d(u,u) + B\, [d(u,u) + d(u, T_1{}^m u)]$

$$+ C[d(u, T_1{}^m u) + d(u,u)] + d(u,u)$$

$$\leq (B + C)\, d(u, T_1{}^m u).$$

Which is a contradiction

since

$$A + 2B + 2C < 1$$

So,

$$T_1{}^m u = u$$

Therefore

$$u = T_1{}^m u$$

where u is a contradiction point

since another point u* in X, such that

$$T_1{}^m \ u^* = u^*$$

$d(T_1{}^m \ u^*, u^*) \leq d(T_1{}^m \ u^*, T_2{}^n u^*)$

$\leq A\, d(u, u^*) + B\, [d(u, T_2{}^n u^*))$

$+ \ d(u^*, T_1{}^m u) \ ] + C[\, d(u, \ T_1{}^m u) \ + d(u^*, T_2{}^n u^*))]$

$\leq A\, d(u, u^*) + B\, [d(u, u^*)$

$+ \ d(u^*, T_2{}^n u^*))] \ + C[\, d(u, \ u^*) \ + d(u^*, T_2{}^n u^*))]$

$\leq (B + C) d(u, T_2{}^n u^*))$

Which is a contradiction,

since $A + 2B + 2C < 1$

S0

$T_1{}^m \ u^* = \ u^*$ we can show that $u = T_2{}^n u$, which in turn implies that u is a common fixed point of

$T_1{}^m$, $T_2{}^n$ and, that is $u = T_1{}^m \ u = T_2{}^n u$. Hence complete the proof

Theorem 3.4:- Let (X, d) be cone metric space with respect to cone P containing in a real Banach E. let T_1 & T_2 be any two surjection self maps of X satisfying .

$$d(T_1{}^m x, T_2{}^n y) \leq Ad(x, y) + B[d(x,, T_1{}^m x)]$$

$$+C[\, d(y, T_2{}^n y)]$$

For all $x, y \in X$, where $A, B, C \geq 0$ are non negative real numbers with $A + B + C > 1$, Then $T_1{}^m$ & $T_2{}^n$ are a unique common fixed point.

Proof:- For x_0 be an arbitrary point in X. since T_1 & T_2 surjection mapping and there exist point $x_1 \in T_1{}^m (x_0)$ and $x_2 \in T_2{}^n (x_1)$ that is

$$T_1{}^m (x_1) = x_0 \text{ and } T_2{}^n (x_2) = x_1$$

In this way,

We define the sequence $\{x_n\}$ with $\boldsymbol{x_{2k+1}} \in T_1{}^m x_{2n}$ and $x_{2k+2} \in T_2{}^n \boldsymbol{x_{2k+1}}$

$$x_{2n} = T_1{}^m \boldsymbol{x_{2k+1}},$$

$$x_{2n+1} = T_2{}^n \boldsymbol{x_{2k+1}}$$

for $n = 0,1,2, \ldots\ldots$..

Note that , if $x_{2n} = x_{2n+1}$ for some $n \geq 0$ then x_{2n} is fixed point of $T_1{}^m$ and $T_2{}^n$.

Now putting

$$x = x_{2n+1} \text{ and } y = x_{2n+2}$$

we have

$d(T_1{}^m \ x_{2n+1}, \ T_2{}^n \ \boldsymbol{x_{2n+2}}) = A\, d(x_{2n+1}, \boldsymbol{x_{2n+2}}) + B\, d(x_{2n+1}, T_1{}^m x_{2k+1}) + C\, d(x_{2k+2}, T_2{}^n \boldsymbol{x_{2n+2}})$

$d(x_{2n}, \boldsymbol{x_{2n+2}}) \geq A\, d(x_{2n+1}, \ \boldsymbol{x_{2n+2}}) + B\, d(x_{2n+1}, x_{2n}) + C\, d(\boldsymbol{x_{2n+2}}, x_{2k+1})$

$d(x_{2n+1}, \ \boldsymbol{x_{2n+2}}) \leq [1 - A/A - C]\, d(x_{2n}, \boldsymbol{x_{2n+1}})$

where

$h = [1 - A/A - C] < 1$

In general

$$d(x_{2n}, \boldsymbol{x_{2n+1}}) \leq h \ d(x_{2n-1}, x_{2n})$$

$$d(x_{2n}, \boldsymbol{x_{2n+1}}) \leq h^{2n} \ d(x_{2n-1}, x_{2n})$$

So for every positive integer P. We have,

$$d(x_{2n}, \boldsymbol{x_{2n+p}}) \le d(x_{2n}, \boldsymbol{x_{2n+1}}) + d(x_{2n+1}, \boldsymbol{x_{2n+2}}) + \ldots\ldots\ldots + d(x_{2n+p-1}, \boldsymbol{x_{2n+p}})$$

$$\le (h^{2n} + h^{2n+1} + \ldots\ldots\ldots + h^{2n+p+1})\, d(x_0, x_1).$$

$$= h^{2n}(1 + h + h^n + \ldots\ldots\ldots + h^{p+1})\, d(x_0, x_1).$$

$$< [h^{2n}/1 - h]\, d(x_0, x_1).$$

Therefore $\{x_n\}$ is a cauchy sequence which is complete space in X . There exist

$$x^* \in X \text{ such that } x_{2n} \to x^*.$$

since $T_1{}^m$ is surjection map. There exist a point y in X , such that

$$y \in T_1{}^m(x^*)$$

i.e. $\quad x^* = T_1{}^m(y)$

Now consider

$$d(x_{2n}, x^*) = d(T_1{}^m x_{2n+1}, y)$$

$$\le A\, d(x_{2n+1}, y) + B\, d(x_{2n+1}, T_1{}^m x_{2n+1}) + C\, d(y, T_1{}^m y)$$

$$d(x^*, x^*) \ge A\, d(x^*, y) + B\, d(x^*, x^*) + C\, d(y, x^*)$$

$$0 \ge [A + C]\, d(x^*, y)$$

$$d(x^*, y) = 0 \qquad \text{as } [A+C] > 0$$

$$x^* = y$$

Hence x^* is a fixed point of $T_1{}^m$ as $T_1{}^m y = x^* = y$

Now if z be another point of $T_1{}^m$

i.e. $\quad T_1{}^m = z$

$$d(x^*, z) = d(T_1{}^m x^*, T_1{}^m z)$$

$$\ge A\, d(x^*, z) + B\, d(T_1{}^m x^*, T_1{}^m z) + C\, d(z, T_1{}^m x)$$

$$= 0$$

$$d(x^*, z) = 0$$

Thus $\quad x^* = z$

Therefore $\quad T_1{}^m$ has a unique fixed point

Similarly , we can be established that

. $\quad T_2{}^n x^* = x^*$

Hence

$$x^* = x^* = T_2{}^n x^*$$

Thus x* is the common fixed point of T_1 &T_2.

Corollary 3.5 :- Let (X, d) be a cone metric space with respect to a cone P containing in a real Banach space E and T_1, & T_2 be any two surjection self maps of X satisfying

$$d(T_1 x, T_2 y) \ge k\, d(x, y) \ldots\ldots\ldots\ldots\ldots$$

For all $x, y \in X$, where k≥0 . Then T_1 &T_2 have an unique common fixed point.

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