Fractal Dimension by Slope Dimention Method

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Abstract: *An important feature of fractal geometry is that it can measure fractional dimension i.e dimension between integers. In this paper, box counting method has been applied to determine the dimension of ECG of a heart. Slope of the regression line could give the box–counting dimension i.e a Fractal dimension.*

Key words: *Fractal, dimension*

1 Introduction

Felix Hausdorff (1868-1942) and Abraham Biscovitch (1891-1970) gave concept of integral dimension. They found that many curve have dimension between 1 and 2 i.e. they have fractal dimension. The classical Euclidean geometry is not enough to describe complex nature when we are dealing with geometrical objects called fractals then a new branch of geometry, called fractal geometry comes out. Now a days fractal geometry have received a great attention in different fields like medical, economy science etc.

2 Fractal dimension

An important feature of fractal geometry is the characterization of irregular objects at different scale. Many fractal features have been identified, among which the fractal dimension is one of the most important feature[1]. we know From the work of Euclid [2] that a line is 1 dimensional, a square is 2 dimensional, and cube is 3 dimensional. Here we discuss the theory of fractal dimension by the definition of Box counting dimension. Here we can say fractal dimension is non integral dimension of an Object[1].

2.1 Box Counting Dimension

During the measurement of length of a coastline using a fixed sized ruler, we would find that the length increases as the length of the ruler decreases. The ruler is therefore inadequate to describe the complexity of geographical curve [1].

Mandelbrot suggests in [2] that dimension should be considered as a continuous quantity that ranges from 0 to ∞, and in particular dimension is described between 1 and 2 that describes their space filling ability[3]. To illustrate similarity dimension we consider a unit line segment, which is 1 dimensional. if we magnify the line segment twice ,we will have 2 connected line segments both of the same length as the original one see fig 2.3(a). Similarly when we look for a unit square in the plane of 2 dimensional. When we magnify it twice we will have similar square, made up of 4 connected unit squares see fig 2.3(b). Next we take a cube i.e. a 3 dimensional object when we magnify it two times we get a cube made up of eight small cubes of unit length see fig 2.3(c) [4]. Here we find that the number of copies of the original object when magnified two times is two to the power of the dimension.

$$i.e.\, m^D = N \qquad (2.1)$$

Where m is the magnification, D is the dimension,

N is the number of copies of the original object when magnified m

We solve for D in equation 2.1 we get

$$D = \frac{\log N}{\log m}$$

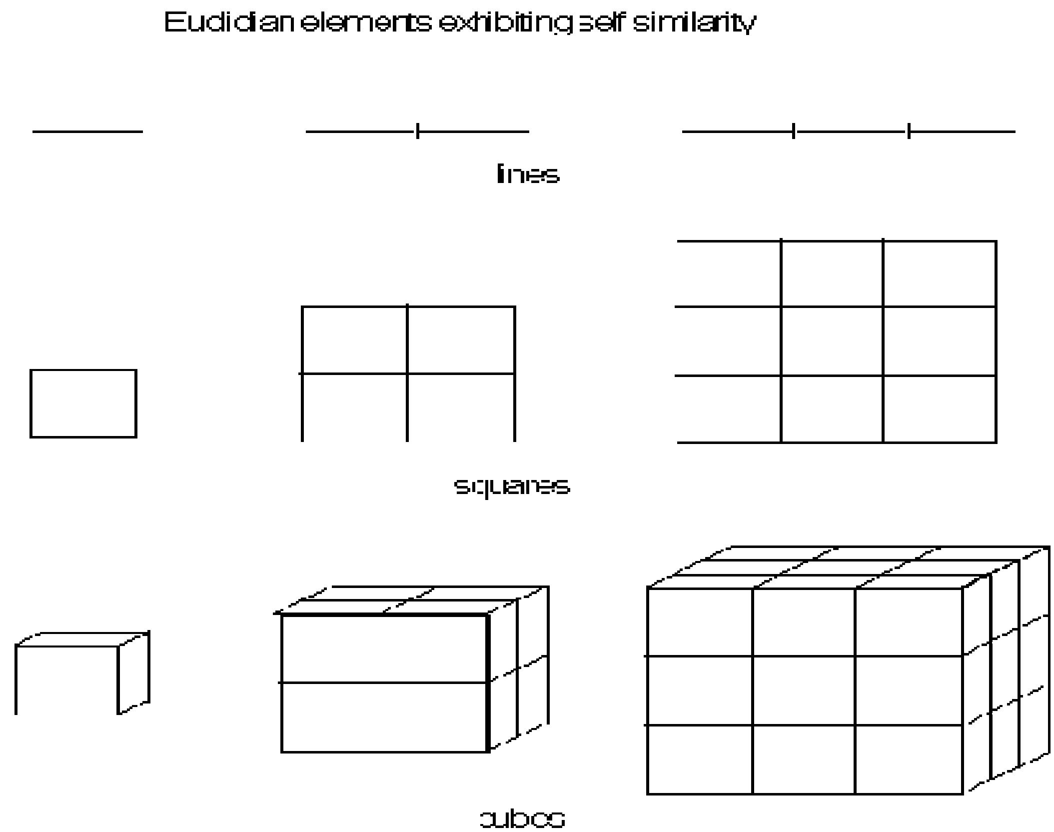

Figure 2.1 Basic construction of lines2.1(a), squares2.1(b) and cubes2.1(c) of unit lengths [1,2,3]

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Table 1: dimensions at different scaling factors			
	D=1(Line)	D=2(Square)	D=3(cube)
S=1/1	1	1	1
S=1/2	2	4	8
S=1/3	3	9	27

This relationship can be shown as follows-

$$i.e. m^D = N$$

$$\therefore D = \frac{\log N}{\log m}$$

Similarity dimension can only be calculated for a small class of strictly self-similar sets.

Steps to calculate the fractal dimension of image in the plane

1. we cover an area by a grid of different mesh sizes
2. we compare the grid sizes and the number of squares containing at least a part of the image.
3. Then we find the ratio of the number of grids to the grid size .

This ratio gives us the dimension of an object. [2], [4] lie approximately on a straight line with slope D_b. This program provides data in different box sizes and box numbers. The data were composed with a well known method, namely "simple linear regression" especially "Least square method".

3 Determination of Regression Equation (slope dimension)

Linear regression is a way to model the relationship between two variables. we might also recognize the equation as the slope formula. The equation has the form Y= α + bX, where Y is the dependent variable (that's the variable that goes on the Y axis), X is the independent variable (i.e. it is plotted on the X axis), b is the slope of the line and a is the y-intercept[5].

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

The equation of a regression line ,that is best fit to data, could be described as- Y=a+bX
Here slope of the regression line i.e b could give the box –counting dimension, and is known as slope dimension.

3.1 Box counting dimension of ECG of heart

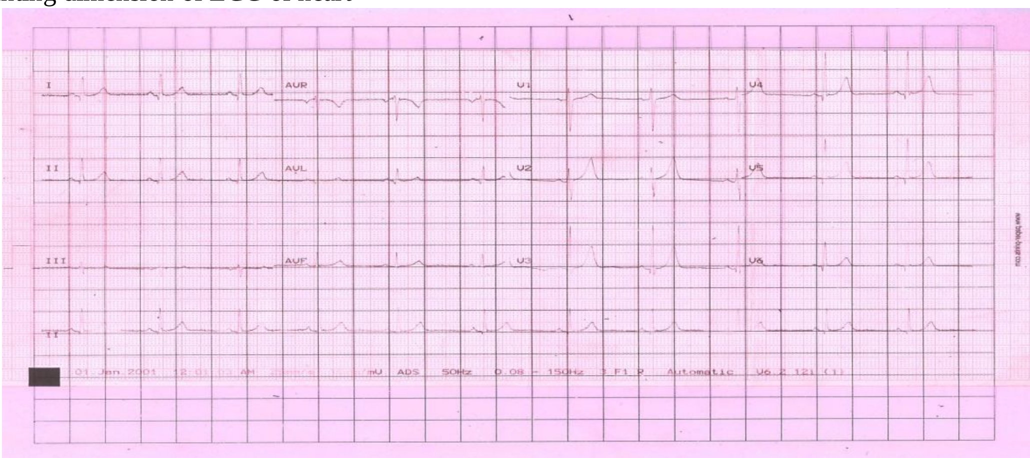


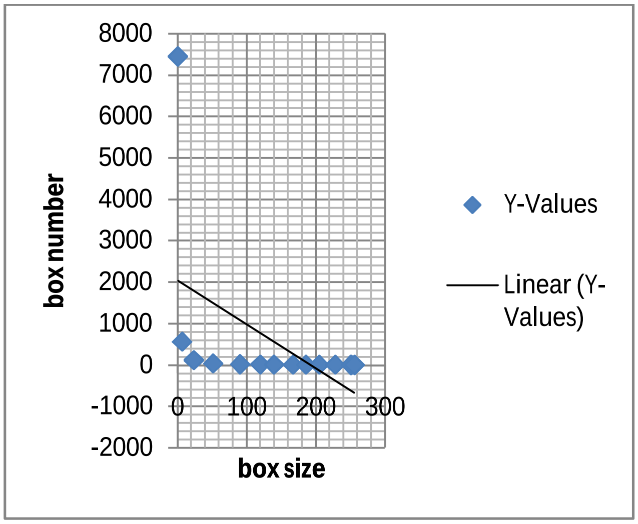
Figure1 : 12 phases of ECG of heart covered by boxes

Table -1 calculation of Slope of ECG data

X=Box size	Y=Box number	XY	X^2
1	7453	7453	1
7	564	3948	49
24	119	2856	576
52	40	2080	2704
91	20	1820	8281
120	12	1440	14400
140	10	1400	19600
167	6	1002	27889
186	6	1116	34596
205	6	1230	42025
228	6	1368	51984
251	4	1004	63001
256	4	1024	65536
∑X=1728	∑Y=8250	∑XY=27741	∑X^2= 330642

α=2042.01 and b=10.5880

Regression equation $y = 2042.01 + 10.5880x$



Graph 1: the regression line graph of ECG

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