

# Certain finite integral involving sequence of functions, a general class of polynomials and multivariable Aleph-functions VI

F.Y. AYANT<sup>1</sup>

<sup>1</sup> Teacher in High School , France

**ABSTRACT**

In the present paper we evaluate a finite integral with involving the product of sequence of functions, product of two multivariable Aleph-functions and general class of polynomials of several variables with general argument. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializing the parameters their in.

Keywords:Multivariable Aleph-function, general class of polynomials, Aleph-function of two variables, I-function of several variables, sequence of functions.

**2010 Mathematics Subject Classification.** 33C99, 33C60, 44A20

## 1.Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [6] , itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

$$\text{We define : } \mathcal{N}(z_1, \dots, z_r) = \mathcal{N}_{p_i, q_i, \tau_i; R; p_i^{(1)}, q_i^{(1)}, \tau_i^{(1)}; R^{(1)}; \dots; p_i^{(r)}, q_i^{(r)}, \tau_i^{(r)}; R^{(r)}}^{0, n; m_1, n_1, \dots, m_r, n_r} \left( \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_r \end{matrix} \right)$$

$$[(a_j; \alpha_j^{(1)}, \dots, \alpha_j^{(r)})_{1, n}] , [\tau_i(a_{ji}; \alpha_{ji}^{(1)}, \dots, \alpha_{ji}^{(r)})_{n+1, p_i}] :$$

$$\dots, [\tau_i(b_{ji}; \beta_{ji}^{(1)}, \dots, \beta_{ji}^{(r)})_{m+1, q_i}] :$$

$$[(c_j^{(1)}; \gamma_j^{(1)})_{1, n_1}, [\tau_i^{(1)}(c_{ji}^{(1)}; \gamma_{ji}^{(1)})_{n_1+1, p_i^{(1)}}]; \dots ; [(c_j^{(r)}; \gamma_j^{(r)})_{1, n_r}, [\tau_i^{(r)}(c_{ji}^{(r)}; \gamma_{ji}^{(r)})_{n_r+1, p_i^{(r)}}]]$$

$$[(d_j^{(1)}; \delta_j^{(1)})_{1, m_1}, [\tau_i^{(1)}(d_{ji}^{(1)}; \delta_{ji}^{(1)})_{m_1+1, q_i^{(1)}}]; \dots ; [(d_j^{(r)}; \delta_j^{(r)})_{1, m_r}, [\tau_i^{(r)}(d_{ji}^{(r)}; \delta_{ji}^{(r)})_{m_r+1, q_i^{(r)}}]]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(s_1, \dots, s_r) \prod_{k=1}^r \theta_k(s_k) y_k^{s_k} ds_1 \dots ds_r \tag{1.1}$$

with  $\omega = \sqrt{-1}$

$$\psi(s_1, \dots, s_r) = \frac{\prod_{j=1}^n \Gamma(1 - a_j + \sum_{k=1}^r \alpha_j^{(k)} s_k)}{\sum_{i=1}^R [\tau_i \prod_{j=n+1}^{p_i} \Gamma(a_{ji} - \sum_{k=1}^r \alpha_{ji}^{(k)} s_k) \prod_{j=1}^{q_i} \Gamma(1 - b_{ji} + \sum_{k=1}^r \beta_{ji}^{(k)} s_k)]} \tag{1.2}$$

$$\text{and } \theta_k(s_k) = \frac{\prod_{j=1}^{m_k} \Gamma(d_j^{(k)} - \delta_j^{(k)} s_k) \prod_{j=1}^{n_k} \Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}} [\tau_{i^{(k)}} \prod_{j=m_k+1}^{q_i^{(k)}} \Gamma(1 - d_{ji^{(k)}}^{(k)} + \delta_{ji^{(k)}}^{(k)} s_k) \prod_{j=n_k+1}^{p_i^{(k)}} \Gamma(c_{ji^{(k)}}^{(k)} - \gamma_{ji^{(k)}}^{(k)} s_k)]} \quad (1.3)$$

Suppose, as usual, that the parameters

$$a_j, j = 1, \dots, p; b_j, j = 1, \dots, q;$$

$$c_j^{(k)}, j = 1, \dots, n_k; c_{ji^{(k)}}^{(k)}, j = n_k + 1, \dots, p_i^{(k)};$$

$$d_j^{(k)}, j = 1, \dots, m_k; d_{ji^{(k)}}^{(k)}, j = m_k + 1, \dots, q_i^{(k)};$$

$$\text{with } k = 1, \dots, r, i = 1, \dots, R, i^{(k)} = 1, \dots, R^{(k)}$$

are complex numbers, and the  $\alpha's, \beta's, \gamma's$  and  $\delta's$  are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^n \alpha_j^{(k)} + \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_i^{(k)}} \gamma_{ji^{(k)}}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_i^{(k)}} \delta_{ji^{(k)}}^{(k)} \leq 0 \quad (1.4)$$

The real numbers  $\tau_i$  are positives for  $i = 1$  to  $R$ ,  $\tau_{i^{(k)}}$  are positives for  $i^{(k)} = 1$  to  $R^{(k)}$

The contour  $L_k$  is in the  $s_k$ -p lane and run from  $\sigma - i\infty$  to  $\sigma + i\infty$  where  $\sigma$  is a real number with loop, if necessary, ensure that the poles of  $\Gamma(d_j^{(k)} - \delta_j^{(k)} s_k)$  with  $j = 1$  to  $m_k$  are separated from those of  $\Gamma(1 - a_j + \sum_{i=1}^r \alpha_j^{(k)} s_k)$  with  $j = 1$  to  $n$  and  $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$  with  $j = 1$  to  $n_k$  to the left of the

contour  $L_k$ . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_k| < \frac{1}{2} A_i^{(k)} \pi, \text{ where}$$

$$A_i^{(k)} = \sum_{j=1}^n \alpha_j^{(k)} - \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_{i^{(k)}} \sum_{j=n_k+1}^{p_i^{(k)}} \gamma_{ji^{(k)}}^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i^{(k)}} \sum_{j=m_k+1}^{q_i^{(k)}} \delta_{ji^{(k)}}^{(k)} > 0, \text{ with } k = 1, \dots, r, i = 1, \dots, R, i^{(k)} = 1, \dots, R^{(k)} \quad (1.5)$$

The complex numbers  $z_i$  are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \dots, z_r) = O(|z_1|^{\alpha_1}, \dots, |z_r|^{\alpha_r}), \max(|z_1|, \dots, |z_r|) \rightarrow 0$$

$$\aleph(z_1, \dots, z_r) = O(|z_1|^{\beta_1}, \dots, |z_r|^{\beta_r}), \min(|z_1|, \dots, |z_r|) \rightarrow \infty$$

where, with  $k = 1, \dots, r : \alpha_k = \min[Re(d_j^{(k)} / \delta_j^{(k)})], j = 1, \dots, m_k$  and

$$\beta_k = \max[\operatorname{Re}((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \dots, n_k$$

Serie representation of Aleph-function of several variables is given by

$$\aleph(y_1, \dots, y_r) = \sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \dots \sum_{g_r=0}^{m_r} \frac{(-)^{G_1+\dots+G_r}}{\delta_{g_1}^{G_1} G_1! \dots \delta_{g_r}^{G_r} G_r!} \psi(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \times \theta_1(\eta_{G_1, g_1}) \dots \theta_r(\eta_{G_r, g_r}) y_1^{-\eta_{G_1, g_1}} \dots y_r^{-\eta_{G_r, g_r}} \tag{1.6}$$

Where  $\psi(\dots), \theta_i(\dots), i = 1, \dots, r$  are given respectively in (1.2), (1.3) and

$$\eta_{G_1, g_1} = \frac{d_{g_1}^{(1)} + G_1}{\delta_{g_1}^{(1)}}, \dots, \eta_{G_r, g_r} = \frac{d_{g_r}^{(r)} + G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions  $\delta_{g_i}^{(i)} [d_j^i + p_i] \neq \delta_{j_i}^{(i)} [d_{g_i}^i + G_i]$  (1.7)

for  $j \neq m_i, m_i = 1, \dots, \eta_{G_i, g_i}; p_i, n_i = 0, 1, 2, \dots, ; y_i \neq 0, i = 1, \dots, r$  (1.8)

Consider the Aleph-function of s variables

$$\aleph(z_1, \dots, z_s) = \aleph_{P_i, Q_i, \mu_i; r': P_i^{(1)}, Q_i^{(1)}, \mu_i^{(1)}; \dots; P_i^{(s)}, Q_i^{(s)}, \mu_i^{(s)}; r^{(s)}}^{0, N: M_1, N_1, \dots, M_s, N_s} \left( \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_s \end{matrix} \right) \left( \begin{matrix} [u_j; \mu_j^{(1)}, \dots, \mu_j^{(r')}]_{1, N} \\ \dots \\ [l_i(u_{ji}; \mu_{ji}^{(1)}, \dots, \mu_{ji}^{(r')})_{N+1, P_i}] \\ \dots \\ [l_i(v_{ji}; \nu_{ji}^{(1)}, \dots, \nu_{ji}^{(r')})_{M+1, Q_i}] \end{matrix} \right) \left( \begin{matrix} [(a_j^{(1)}); \alpha_j^{(1)}]_{1, N_1}, [l_{i(1)}(a_{ji(1)}^{(1)}); \alpha_{ji(1)}^{(1)}]_{N_1+1, P_i^{(1)}}; \dots; [(a_j^{(s)}); \alpha_j^{(s)}]_{1, N_s}, [l_{i(s)}(a_{ji(s)}^{(s)}); \alpha_{ji(s)}^{(s)}]_{N_s+1, P_i^{(s)}} \\ [(b_j^{(1)}); \beta_j^{(1)}]_{1, M_1}, [l_{i(1)}(b_{ji(1)}^{(1)}); \beta_{ji(1)}^{(1)}]_{M_1+1, Q_i^{(1)}}; \dots; [(b_j^{(s)}); \beta_j^{(s)}]_{1, M_s}, [l_{i(s)}(b_{ji(s)}^{(s)}); \beta_{ji(s)}^{(s)}]_{M_s+1, Q_i^{(s)}} \end{matrix} \right) = \frac{1}{(2\pi\omega)^s} \int_{L_1} \dots \int_{L_s} \zeta(t_1, \dots, t_s) \prod_{k=1}^s \phi_k(t_k) z_k^{t_k} dt_1 \dots dt_s \tag{1.9}$$

with  $\omega = \sqrt{-1}$

$$\zeta(t_1, \dots, t_s) = \frac{\prod_{j=1}^N \Gamma(1 - u_j + \sum_{k=1}^s \mu_j^{(k)} t_k)}{\sum_{i=1}^{r'} [l_i \prod_{j=N+1}^{P_i} \Gamma(u_{ji} - \sum_{k=1}^s \mu_{ji}^{(k)} t_k) \prod_{j=1}^{Q_i} \Gamma(1 - v_{ji} + \sum_{k=1}^s \nu_{ji}^{(k)} t_k)]} \tag{1.10}$$

and  $\phi_k(t_k) = \frac{\prod_{j=1}^{M_k} \Gamma(b_j^{(k)} - \beta_j^{(k)} t_k) \prod_{j=1}^{N_k} \Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{r^{(k)}} [l_{i^{(k)}} \prod_{j=M_k+1}^{Q_{i^{(k)}}} \Gamma(1 - b_{ji^{(k)}}^{(k)} + \beta_{ji^{(k)}}^{(k)} t_k) \prod_{j=N_k+1}^{P_{i^{(k)}}} \Gamma(a_{ji^{(k)}}^{(k)} - \alpha_{ji^{(k)}}^{(k)} s_k)]} \tag{1.11}$

Suppose , as usual , that the parameters

$$u_j, j = 1, \dots, P; v_j, j = 1, \dots, Q;$$

$$a_j^{(k)}, j = 1, \dots, N_k; a_{j i^{(k)}}^{(k)}, j = n_k + 1, \dots, P_{i^{(k)}};$$

$$b_{j i^{(k)}}^{(k)}, j = m_k + 1, \dots, Q_{i^{(k)}}; b_j^{(k)}, j = 1, \dots, M_k;$$

$$\text{with } k = 1 \dots, s, i = 1, \dots, r', i^{(k)} = 1, \dots, r^{(k)}$$

are complex numbers , and the  $\alpha' s, \beta' s, \gamma' s$  and  $\delta' s$  are assumed to be positive real numbers for standardization purpose such that

$$U_i^{(k)} = \sum_{j=1}^N \mu_j^{(k)} + \iota_i \sum_{j=N+1}^{P_i} \mu_{j i}^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} + \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{j i^{(k)}}^{(k)} - \iota_i \sum_{j=1}^{Q_i} v_{j i}^{(k)} - \sum_{j=1}^{M_k} \beta_j^{(k)} - \iota_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \beta_{j i^{(k)}}^{(k)} \leq 0 \tag{1.12}$$

The reals numbers  $\tau_i$  are positives for  $i = 1, \dots, r$  ,  $\iota_{i^{(k)}}$  are positives for  $i^{(k)} = 1 \dots r^{(k)}$

The contour  $L_k$  is in the  $t_k$ -p lane and run from  $\sigma - i\infty$  to  $\sigma + i\infty$  where  $\sigma$  is a real number with loop , if necessary ,ensure that the poles of  $\Gamma(b_j^{(k)} - \beta_j^{(k)} t_k)$  with  $j = 1$  to  $M_k$  are separated from those of  $\Gamma(1 - u_j + \sum_{i=1}^s \mu_j^{(k)} t_k)$  with  $j = 1$  to  $N$  and  $\Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} t_k)$  with  $j = 1$  to  $N_k$  to the left of the contour  $L_k$  . The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_k| < \frac{1}{2} B_i^{(k)} \pi, \text{ where}$$

$$B_i^{(k)} = \sum_{j=1}^N \mu_j^{(k)} - \iota_i \sum_{j=N+1}^{P_i} \mu_{j i}^{(k)} - \iota_i \sum_{j=1}^{Q_i} v_{j i}^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} - \iota_{i^{(k)}} \sum_{j=N_k+1}^{P_{i^{(k)}}} \alpha_{j i^{(k)}}^{(k)} + \sum_{j=1}^{M_k} \beta_j^{(k)} - \iota_{i^{(k)}} \sum_{j=M_k+1}^{Q_{i^{(k)}}} \beta_{j i^{(k)}}^{(k)} > 0, \text{ with } k = 1, \dots, s, i = 1, \dots, r, i^{(k)} = 1, \dots, r^{(k)} \tag{1.13}$$

The complex numbers  $z_i$  are not zero. Throughout this document , we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \dots, z_s) = O(|z_1|^{\alpha'_1}, \dots, |z_s|^{\alpha'_s}), \max(|z_1|, \dots, |z_s|) \rightarrow 0$$

$$\aleph(z_1, \dots, z_s) = O(|z_1|^{\beta'_1}, \dots, |z_s|^{\beta'_s}), \min(|z_1|, \dots, |z_s|) \rightarrow \infty$$

where, with  $k = 1, \dots, s, z : \alpha'_k = \min[Re(b_j^{(k)} / \beta_j^{(k)})], j = 1, \dots, M_k$  and

$$\beta'_k = \max[Re((a_j^{(k)} - 1) / \alpha_j^{(k)})], j = 1, \dots, N_k$$

We will use these following notations in this paper

$$U = P_i, Q_i, l_i; r'; V = M_1, N_1; \dots; M_s, N_s \tag{1.15}$$

$$W = P_{i(1)}, Q_{i(1)}, l_{i(1)}; r^{(1)}, \dots, P_{i(r)}, Q_{i(r)}, l_{i(s)}; r^{(s)} \tag{1.16}$$

$$A' = \{(u_j; \mu_j^{(1)}, \dots, \mu_j^{(s)})_{1,N}\}, \{l_i(u_{ji}; \mu_{ji}^{(1)}, \dots, \mu_{ji}^{(s)})_{N+1, P_i}\} \tag{1.17}$$

$$B = \{l_i(v_{ji}; v_{ji}^{(1)}, \dots, v_{ji}^{(s)})_{M+1, Q_i}\} \tag{1.18}$$

$$C = (a_j^{(1)}; \alpha_j^{(1)})_{1, N_1}, l_{i(1)}(a_{ji(1)}^{(1)}; \alpha_{ji(1)}^{(1)})_{N_1+1, P_{i(1)}}, \dots, (a_j^{(s)}; \alpha_j^{(s)})_{1, N_s}, l_{i(s)}(a_{ji(s)}^{(s)}; \alpha_{ji(s)}^{(s)})_{N_s+1, P_{i(s)}} \tag{1.19}$$

$$D = (b_j^{(1)}; \beta_j^{(1)})_{1, M_1}, l_{i(1)}(b_{ji(1)}^{(1)}; \beta_{ji(1)}^{(1)})_{M_1+1, Q_{i(1)}}, \dots, (b_j^{(s)}; \beta_j^{(s)})_{1, M_s}, l_{i(s)}(\beta_{ji(s)}^{(s)}; \beta_{ji(s)}^{(s)})_{M_s+1, Q_{i(s)}} \tag{1.20}$$

The multivariable Aleph-function write :

$$\aleph(z_1, \dots, z_s) = \aleph_{U:W}^{0, N:V} \left( \begin{array}{c|c} z_1 & A : C \\ \cdot & \cdot \\ \cdot & \cdot \\ z_s & B : D \end{array} \right) \tag{1.21}$$

The generalized polynomials defined by Srivastava [10], is given in the following manner :

$$S_{N_1, \dots, N_t}^{M_1, \dots, M_t} [y_1, \dots, y_t] = \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_t=0}^{[N_t/M_t]} \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_t)_{M_t K_t}}{K_t!} A[N_1, K_1; \dots; N_t, K_t] y_1^{K_1} \dots y_t^{K_t} \tag{1.22}$$

Where  $M_1, \dots, M_s$  are arbitrary positive integers and the coefficients  $A[N_1, K_1; \dots; N_t, K_t]$  are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1)_{M_1 K_1}}{K_1!} \dots \frac{(-N_t)_{M_t K_t}}{K_t!} A[N_1, K_1; \dots; N_t, K_t] \tag{1.23}$$

In the document , we note :

$$G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) = \phi(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \theta_1(\eta_{G_1, g_1}) \dots \theta_r(\eta_{G_r, g_r}) \tag{1.24}$$

where  $\phi(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}), \theta_1(\eta_{G_1, g_1}), \dots, \theta_r(\eta_{G_r, g_r})$  are given respectively in (1.2) and (1.3)

## 2. Sequence of functions

Agarwal and Chaubey [1], Salim [5] and several others have studied a general sequence of functions. In the present document we shall study the following useful series formula for a general sequence of functions.

$$R_n^{\alpha, \beta} [x; E, F, g, h; p, q; \gamma; \delta; e^{-sx^r}] = \sum_{w, v, u, t, e, k_1, k_2} \psi(w, v, u, t, e, k_1, k_2) x^R \tag{2.1}$$

where 
$$\sum_{w,v,u,t,e,k_1,k_2} = \sum_{w=0}^{\infty} \sum_{v=0}^n \sum_{u=0}^v \sum_{t=0}^n \sum_{c=0}^t \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \tag{2.2}$$

and the infinite series on the right side (2.1) is absolutely convergent,  $R = ln + qv + pt + rw + k_1r + k_2q$

and 
$$\psi(w, v, u, t, e, k_1, k_2) = \frac{(-)^{t+w+k_2} (-v)_u (-t)_e (\alpha)_t l^n}{w!v!u!t!e!K_n k_1!k_2!} \frac{s^{w+k_1} F^{\gamma n-t}}{(1 - \alpha - t)_e} (\alpha - \gamma n)_e$$

$$(-\beta - \delta n)_v g^{v+k_2} h^{\delta n-v-k_2} (v - \delta n)_{k_2} E^t \left( \frac{pe + rw + \lambda + qn}{l} \right)_n \tag{2.3}$$

where  $K_n$  is a sequence of constants. By suitably specializing the parameters involving in (2.1), a general sequence of function reduced to generalized polynomials set studied by Raizada [4], a class of polynomials introduced by Fujiwara [3] and several others authors.

### 3. Required integral

We have the following integral, see Brychkov ([2], 4.1.1 Eq.40 page 115)

$$\int_{-1}^1 \frac{(1 - x^2)^{s-\frac{1}{2}}}{(1 + 2ax + a^2)^s (1 + 2bx + b^2)^s} dx = \frac{\sqrt{\pi} \Gamma(s + \frac{1}{2})}{\Gamma(s + 1)} {}_2F_1[s, 2s; s + 1; ab] \tag{3.1}$$

where  $Re(s) > -\frac{1}{2}$ .  ${}_2F_1$  is the Gauss hypergeometric function, see Slater ([9], page 1).

### 4. Main integral

Let 
$$X = \frac{(1 - x^2)}{(1 + 2ax + a^2)(1 + 2bx + b^2)} \tag{4.1}$$

We have the following general integral.

$$\int_{-1}^1 \frac{(1 - x^2)^{s'-\frac{1}{2}}}{(1 + 2ax + a^2)^{s'} (1 + 2bx + b^2)^{s'}} R_n^{\alpha,\beta} [zX^A; E, F, g, h; p, q; \gamma; \delta; e^{-s(zX^A)^r}]$$

$$S_{N_1, \dots, N_t}^{M_1, \dots, M_t} \begin{pmatrix} y_1 X^{\gamma_1} \\ \dots \\ y_t X^{\gamma_t} \end{pmatrix} N_{u:w}^{0,n:v} \begin{pmatrix} z_1 X^{\alpha_1} \\ \dots \\ z_r X^{\alpha_r} \end{pmatrix} N_{U:W}^{0,N:V} \begin{pmatrix} Z_1 X^{\eta_1} \\ \dots \\ Z_s X^{\eta_s} \end{pmatrix} dx = \sqrt{\pi} \sum_{n=0}^{\infty} \sum_{w,v,u,t,e,k_1,k_2}$$

$$\sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \dots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_t=0}^{[N_t/M_t]} a_1 \frac{(-)^{G_1+\dots+G_r}}{\delta_{g_1} G_1! \dots \delta_{g_r} G_r!} G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \frac{(ab)^n}{n!}$$

$$\psi(w, v, u, t, e, k_1, k_2) \mathfrak{N}_{U_{33}; W}^{0, N+3; V} \left( \begin{matrix} Z_1 \\ \dots \\ Z_s \end{matrix} \left| \begin{matrix} (\frac{1}{2}-s'-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i; \eta_1, \dots, \eta_s), \\ \dots \\ (-s'-n-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i; \eta_1, \dots, \eta_s), \end{matrix} \right. \right.$$

$$\left. \begin{matrix} (1-s'-n-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i; \eta_1, \dots, \eta_s), \\ \dots \\ (1-s'-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i; \eta_1, \dots, \eta_s), \end{matrix} \right.$$

$$\left. \begin{matrix} (1-n-2(s'-RA+\sum_{i=1}^t K_i \gamma_i + \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i); 2\eta_1, \dots, 2\eta_s), A' : C \\ \dots \\ (1-2(s' +RA+\sum_{i=1}^t K_i \gamma_i + \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i); 2\eta_1, \dots, 2\eta_s), B : D \end{matrix} \right) \quad (4.2)$$

where  $U_{33} = P_i + 3; Q_i + 3; \iota_i; r'$

Provided that

a)  $\min\{A, \gamma_i, \alpha_j, \eta_k, \} > 0, i = 1, \dots, t, j = 1, \dots, r, k = 1, \dots, s,$

b)  $Re[s' + RA + \sum_{i=1}^r \alpha_i \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^s \eta_i \min_{1 \leq j \leq M_i} \frac{b_j^{(i)}}{\beta_j^{(i)}}] > -\frac{1}{2}$

c)  $|arg z_k| < \frac{1}{2} A_i^{(k)} \pi,$  where  $A_i^{(k)}$  is defined by (1.5);  $i = 1, \dots, r$

d)  $|arg Z_k| < \frac{1}{2} B_i^{(k)} \pi,$  where  $B_i^{(k)}$  is defined by (1.13);  $i = 1, \dots, s$

e) The series on the right-hand side of (3.1) is assumed to be absolutely convergent

**Proof**

Expressing the general sequence of functions  $R_n^{\alpha, \beta} [z X^A; E, F, g, h; p, q; \gamma; \delta; e^{-s(z X^A)^t}]$  in serie with the help of equation (2.1), the Aleph-function of r variables in series with the help of equation (1.6), the general class of polynomial of several variables  $S_{N_1, \dots, N_t}^{M_1, \dots, M_t}$  in serie with the help of equation (1.22) and the Aleph-function of s variables in Mellin-Barnes contour integral with the help of equation (1.9), changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) and then evaluating the resulting integral with the help of equation (3.1). Finally, expressing the Gauss hypergeometric function  ${}_2F_1$  in serie, use the relation  $\Gamma(a)(a)_n = \Gamma(a+n)$  with  $Re a > 0$  and interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

5. Multivariable I-function

If  $l_i, l_{i(1)}, \dots, l_{i(s)} \rightarrow 1$ , the Aleph-function of several variables degenerate to the I-function of several variables. The simple integral have been derived in this section for multivariable I-functions defined by Sharma et al [6].

**Corollary 1**

$$\int_{-1}^1 \frac{(1-x^2)^{s'-\frac{1}{2}}}{(1+2ax+a^2)^{s'}(1+2bx+b^2)^{s'}} R_n^{\alpha,\beta}[zX^A; E, F, g, h; p, q; \gamma; \delta; e^{-s(zX^A)^r}]$$

$$S_{N_1, \dots, N_t}^{M_1, \dots, M_t} \left( \begin{matrix} y_1 X^{\gamma_1} \\ \dots \\ y_t X^{\gamma_t} \end{matrix} \right) \mathfrak{N}_{u:w}^{0,n:v} \left( \begin{matrix} z_1 X^{\alpha_1} \\ \dots \\ z_r X^{\alpha_r} \end{matrix} \right) I_{U:W}^{0,N:V} \left( \begin{matrix} Z_1 X^{\eta_1} \\ \dots \\ Z_s X^{\eta_s} \end{matrix} \right) dx = \sqrt{\pi} \sum_{n=0}^{\infty} \sum_{w,v,u,t,e,k_1,k_2}$$

$$\sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \dots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_t=0}^{[N_t/M_t]} a_1 \frac{(-)^{G_1+\dots+G_r}}{\delta_{g_1} G_1! \dots \delta_{g_r} G_r!} G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \frac{(ab)^n}{n!}$$

$$\psi(w, v, u, t, e, k_1, k_2) I_{U_{33}:W}^{0,N+3:V} \left( \begin{matrix} Z_1 & \left| \begin{matrix} (\frac{1}{2}-s'-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i; \eta_1, \dots, \eta_s), \\ \dots \\ \dots \\ Z_s & \left| \begin{matrix} (-s'-n-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i; \eta_1, \dots, \eta_s), \\ \dots \\ \dots \end{matrix} \end{matrix} \right. \right.$$

$$(1-s'-n-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i; \eta_1, \dots, \eta_s),$$

$$\dots$$

$$\dots$$

$$(1-s' -RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i; \eta_1, \dots, \eta_s),$$

$$\left. \begin{matrix} (1-n-2(s'-RA+\sum_{i=1}^t K_i \gamma_i + \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i); 2\eta_1, \dots, 2\eta_s), A' : C \\ \dots \\ \dots \\ (1-2(s' +RA+\sum_{i=1}^t K_i \gamma_i + \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i); 2\eta_1, \dots, 2\eta_s), B : D \end{matrix} \right) \tag{5.2}$$

under the same notations and conditions that (4.1) with  $l_i, l_{i(1)}, \dots, l_{i(s)} \rightarrow 1$

**6. Aleph-function of two variables**

If  $s = 2$ , we obtain the Aleph-function of two variables defined by K.Sharma [8], and we have the following simple integral.

**Corollary 2**

$$\int_{-1}^1 \frac{(1-x^2)^{s'-\frac{1}{2}}}{(1+2ax+a^2)^{s'}(1+2bx+b^2)^{s'}} R_n^{\alpha,\beta}[zX^A; E, F, g, h; p, q; \gamma; \delta; e^{-s(zX^A)^r}]$$



$$S_{N_1, \dots, N_t}^{M_1, \dots, M_t} \left( \begin{matrix} y_1 X^{\gamma_1} \\ \dots \\ y_t X^{\gamma_t} \end{matrix} \right) \aleph_{u:w}^{0, n; v} \left( \begin{matrix} z_1 X^{\alpha_1} \\ \dots \\ z_r X^{\alpha_r} \end{matrix} \right) \aleph_{U:W}^{0, N; V} \left( \begin{matrix} Z_1 X^{\eta_1} \\ \dots \\ Z_2 X^{\eta_2} \end{matrix} \right) dx = \sqrt{\pi} \sum_{n=0}^{\infty} \sum_{w, v, u, t, e, k_1, k_2}$$

$$\sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \dots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_t=0}^{[N_t/M_t]} a_1 \frac{(-)^{G_1+\dots+G_r}}{\delta_{g_1} G_1! \dots \delta_{g_r} G_r!} G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \frac{(ab)^n}{n!}$$

$$\psi(w, v, u, t, e, k_1, k_2) \aleph_{U_{33}:W}^{0, N+3; V} \left( \begin{matrix} Z_1 & \left( \frac{1}{2} - s' - \text{RA} - \sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i; \eta_1, \eta_2 \right), \\ \dots & \dots \\ \dots & \dots \\ Z_2 & \left( -s' - n - \text{RA} - \sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i; \eta_1, \eta_2 \right), \end{matrix} \right)$$

$$\left( 1 - s' - n - \text{RA} - \sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i; \eta_1, \eta_2 \right),$$

⋮

$$\left( 1 - s' - \text{RA} - \sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i; \eta_1, \eta_2 \right),$$

$$\left( 1 - 2(s' + n - \text{RA} + \sum_{i=1}^t K_i \gamma_i + \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i); 2\eta_1, 2\eta_2 \right), A' : C \right) \tag{6.2}$$

$$\left( 1 - 2(s' + \text{RA} + \sum_{i=1}^t K_i \gamma_i + \sum_{i=1}^{\eta} G_{i, g_i} \alpha_i); 2\eta_1, 2\eta_2 \right), B : D \right)$$

under the same notations and conditions that (4.1) with  $s = 2$

### 7. I-function of two variables

If  $\iota_i, \iota'_i, \iota''_i \rightarrow 1$ , then the Aleph-function of two variables degenerate in the I-function of two variables defined by sharma et al [7] and we obtain the same formula with the I-function of two variables.

#### Corollary 3

$$\int_{-1}^1 \frac{(1-x^2)^{s'-\frac{1}{2}}}{(1+2ax+a^2)^{s'}(1+2bx+b^2)^{s'}} R_n^{\alpha, \beta} [zX^A; E, F, g, h; p, q; \gamma; \delta; e^{-s(zX^A)^r}]$$

$$S_{N_1, \dots, N_t}^{M_1, \dots, M_t} \left( \begin{matrix} y_1 X^{\gamma_1} \\ \dots \\ y_t X^{\gamma_t} \end{matrix} \right) \aleph_{u:w}^{0, n; v} \left( \begin{matrix} z_1 X^{\alpha_1} \\ \dots \\ z_r X^{\alpha_r} \end{matrix} \right) I_{U:W}^{0, N; V} \left( \begin{matrix} Z_1 X^{\eta_1} \\ \dots \\ Z_2 X^{\eta_2} \end{matrix} \right) dx = \sqrt{\pi} \sum_{n=0}^{\infty} \sum_{w, v, u, t, e, k_1, k_2}$$

$$\sum_{G_1, \dots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \dots \sum_{g_r=0}^{m_r} \sum_{K_1=0}^{[N_1/M_1]} \dots \sum_{K_t=0}^{[N_t/M_t]} a_1 \frac{(-)^{G_1+\dots+G_r}}{\delta_{g_1} G_1! \dots \delta_{g_r} G_r!} G(\eta_{G_1, g_1}, \dots, \eta_{G_r, g_r}) \frac{(ab)^n}{n!}$$

$$\psi(w, v, u, t, e, k_1, k_2) I_{U_{33}:W}^{0,N+3:V} \left( \begin{array}{c|c} Z_1 & (\frac{1}{2}-s'-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i; \eta_1, \eta_2), \\ \dots & \dots \\ \dots & \dots \\ Z_2 & (-s'-n-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i; \eta_1, \eta_2), \end{array} \right.$$

$$\left. \begin{array}{c} (1-s'-n-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i; \eta_1, \eta_2), \\ \dots \\ \dots \\ (1-s'-RA-\sum_{i=1}^t K_i \gamma_i - \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i; \eta_1, \eta_2), \\ \\ (1-2(s'+n-RA+\sum_{i=1}^t K_i \gamma_i + \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i); 2\eta_1, 2\eta_2), A' : C \\ \dots \\ \dots \\ (1-2(s'+RA+\sum_{i=1}^t K_i \gamma_i + \sum_{i=1}^{\eta} G_{i,g_i} \alpha_i); 2\eta_1, 2\eta_2), B : D \end{array} \right) \tag{7.1}$$

under the same notations and conditions that (4.1) with  $s = 2$  and  $\iota_i, \iota'_i, \iota''_i \rightarrow 1$

### 8. Conclusion

In this paper we have evaluated a finite integral involving the multivariable Aleph-functions, a class of polynomials of several variables and the general of sequence of functions. The integral established in this paper is of very general nature as it contains Multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

### REFERENCES

[1] Agrawal B.D. And Chaubey J.P. Certain derivation of generating relations for generalized polynomials. Indian J. Pure and Appl. Math 10 (1980), page 1155-1157, ibid 11 (1981), page 357-359

[2] Brychkow Y.A. Handbook of Special Functions. Derivatives. Integrals, Series and Other Formulas. CRC. Press. Taylor and Francis Group. Boca. Raton. London. New York. 2008.

[3] Fujiwara I. A unified presentation of classical orthogonal polynomials. Math. Japon. 11 (1966), page 133-148.

[4] Raizada S.K. A study of unified representation of special functions of Mathematics Physics and their use in statistical and boundary value problem. Ph.D. Thesis, Bundelkhand University, Jhansi, India, 1991

[5] Salim T.O. A serie formula of generalized class of polynomials associated with Laplace transform and fractional integral operators. J. Rajasthan. Acad. Phy. Sci. 1(3) (2002), page 167-176.

[6] Sharma C.K. and Ahmad S.S.: On the multivariable I-function. Acta ciencia Indica Math , 1994 vol 20, no2, p 113-116.

[7] C.K. Sharma and P.L. Mishra : On the I-function of two variables and its properties. Acta Ciencia Indica Math , 1991, Vol 17 page 667-672.

[8] Sharma K. On the integral representation and applications of the generalized function of two variables , International Journal of Mathematical Engineering and Sciences , Vol 3 , issue1 ( 2014 ) , page 1-13.

[9] Slater L.J. Generalized hypergeometric functions. Cambridge. University. Press. 1966

[10] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191.

Personal adress : 411 Avenue Joseph Raynaud

Le parc Fleuri , Bat B

83140 , Six-Fours les plages

Tel : 06-83-12-49-68

Department : VAR

Country : FRANCE