Non Linear Stability of L4 in photogravitational CRTBP

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Abstract: *We have investigated the non-linear stability of the triangular libration point L_4 of the Restricted Three Body Problem, when both the primaries are axes symmetric and source of radiation. It is observed that stability of L_4 depends upon the lengths of the semi axes of the primaries and the radiation parameters.*

Key Words: *Restricted Three Body Problem, Axis Symmetric Body, Libration Points, Non Linear Stability, KAM Theorem*

I. Introduction

In 2006 Aggarwal et al. [1] investigated the non-linear stability of the triangular libration point L_4 of the Restricted Three Body Problem under the presence of the third and fourth order resonances, when both the primaries are triaxial and source of radiation. They discussed the non linear stability of L_4 in the absence of resonances. In 2015, Jain and Aggarwal [2] discussed the existence and stability of libration points in restricted problem under the effect of dissipative force. They observed that the non-collinear libration points are unstable for all values of mass parameter. Leontovich [3] investigated the critical case of the stability of the triangular libration points and proved that the triangular libration points in the restricted problem are stable for all permissible mass ratios except for a set of measure zero. Deprit and Deprit [4] calculated the exceptional values for the triangular libration points and proved that the non-linear stability of these points can be answered in the affirmative for all values of the mass ratio in the range of linear stability except at three mass ratios 0.024293..., 0.013516... and 0.010913.... Bhatnagar and Hallan [5] studied the effect of perturbations ε and ε' in the coriolis and the centrifugal forces respectively on the nonlinear stability of the libration points in the restricted problem. They established that, in the non-linear sense, the collinear points are unstable for all mass ratios and the triangular points are stable in the range of linear stability except for three mass ratios. Gyorgyey [6] investigated the non-linear stability of motion around L_5 in the elliptic restricted problem of three bodies numerically with emphasis on the effect of orbital eccentricity of the primaries on shape of the established stability regions. It is shown that with increasing eccentricity, the width of these regions is decreasing. Gozdziewski et al. [7] investigated the non-linear stability of the triangular libration points in the photogravitational restricted three-body problem in the whole range of parameters. Bhatnagar et al. [8] studied the effect of perturbed potentials on the non-linear stability of L_4 and proved that the triangular libration point L_4 is stable in the range of linear stability except for three mass ratios. Bhatnagar and Hallan [9] studied the non-linear stability of an ellipsoidal cluster of stars sharing galactic rotation. They found that the cluster is stable for all densities in the range of linear stability except for those satisfying certain equations where Arnold's theorem is not applicable.

The equilibrium solutions and linear stability of m_3 and m_4 considering one of the primaries as an oblate spheroid have been examined by Aggarwal and Kaur [10]. They concluded that there are no non-collinear equilibrium solutions of the system. Subbarao and Sharma [11] investigated the non-linear stability of L_4 in the restricted three-body problem when the bigger primary is an oblate spheroid. They found that the triangular libration point L_4 is stable in the range of linear stability except for three mass ratios. In a series of paper Sharma Ravinder, Taqvi and Bhatnagar. [12] and [13] have studied the linear stability of the libration points of the planar restricted three body problem when the primaries are triaxial rigid bodies and source of radiations. They have observed that the collinear points are unstable, while the triangular points are stable for the mass parameter $0 \le \mu \le \mu_{crit}$ (the critical mass parameter). It is further seen that the triangular points have long or short periodic elliptical orbits in the same range of μ . Douskos et al. [14] have studied the stability of equilibrium points in the relativistic restricted problem. The result is contrary to recent results of other authors. Jain et al.[15], [16] have studied the

non linear stability of L_4, but they have not taken both the primaries as triaxial and source of radiation. They have normalized the Hamiltonian by Birkhoff's normalization technique using double d'Alemberts series. Whereas we have taken both the primaries as triaxial and source of radiation and have normalized the Hamiltonian by Birkhoff's normalization technique using generating function.

II. Equations of Motion

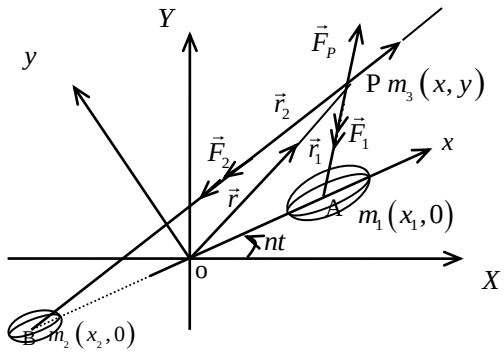

Fig. The configuration of CRTBP

Let there be three masses m_1, m_2, m_3; $(m_1 \geq m_2)$. Let the bodies with masses m_1 and m_2 revolve with the angular velocity n (say) in circular orbits without rotation about their centre of mass O. Let there be an infinitesimal mass m_3 which is moving in the plane of motion of m_1 and m_2 and is being influenced by their motion but not influencing them. We consider both the primaries are source of radiation and triaxial with one of the axis of each as the axis of symmetry and their equatorial plane coincident with the plane of motion. Let the line joining m_1 and m_2 be taken as X-axis and O their center of mass as origin. Let the line passing through O and perpendicular to OX and lying in the plane of motion of m_1 and m_2 be the Y-axis. Let us consider a synodic system of coordinates $O(xyz)$; initially coincident with the inertial system $O(XYZ)$, rotating with the angular velocity n about Z-axis; (the z-axis is coincident with Z-axis). Let initially the principal axes of the primaries be parallel to the synodic axes $O(xyz)$ and their axes of symmetry be perpendicular to the plane of motion. Since the rigid body is revolving without rotation about O with the same angular velocity as that of synodic axes and so principal axes of m_1 and m_2 will remain parallel to them throughout the motion.

We have adopted the notation and terminology of Szebehely[17]. As a consequence the distance between the primaries does not change and is taken equal to one; the sum of the masses of the primaries is also taken as one. The unit of time is chosen so as to make the gravitational constant unity. Using dimensionless variables, the equations of motion of the infinitesimal mass m_3 in the synodic co-ordinate system (x, y) are

$$\begin{aligned} \ddot{x} - 2n\dot{y} &= \Omega_x \ , \\ \ddot{y} + 2n\dot{x} &= \Omega_y \ , \end{aligned} \qquad (1)$$

where

$$\Omega = \frac{n^2}{2}\left((1-\mu)r_1^2 + \mu r_2^2\right) + \left(\frac{1}{r_1} + \frac{A_1}{2r_1^3} + \frac{3A_2 y^2}{2r_1^5} - \frac{P}{r_1}\right)(1-\mu) + \left(\frac{1}{r_2} + \frac{A_1'}{2r_2^3} + \frac{3A_2' y^2}{2r_2^5} - \frac{P'}{r_2}\right)\mu,$$

$$\mu = \frac{m_2}{m_1 + m_2} \leq \frac{1}{2} \quad \Rightarrow m_1 = 1 - \mu,$$

$$r_1^2 = (x-\mu)^2 + y^2, \ r_2^2 = (x+1-\mu)^2 + y^2,$$

$$P = \frac{\text{Radiation pressure due to the bigger primary}}{\text{Gravitational force due to the bigger primary}},$$

$$P' = \frac{\text{Radiation pressure due to the smaller primary}}{\text{Gravitational force due to the smaller primary}},$$

$$A_1 = \frac{2a^2 - c^2 - b^2}{5R^2}, \quad A_1' = \frac{2a'^2 - c'^2 - b'^2}{5R^2},$$

$$A_2 = \frac{b^2 - a^2}{5R^2}, \ A_2' = \frac{b'^2 - a'^2}{5R^2},$$

$$0 < A_1, A_1', A_2, A_2', P, P' << 1,$$

a, b and c are the lengths of the semiaxes of the triaxial body of mass m_1,

a', b' and c' are the lengths of the semiaxes of the triaxial body of mass m_2,

R = dimensional distance between the primaries.

The mean motion n of the primaries is given by

$$n = 1 + \frac{3}{4}A_1 + \frac{3}{4}A_1' \ .$$

It may be observed that n is independent of the Radiation Parameters P and P'.

III. Location of the Librations Point L_4

The co-ordinates (x, y) of the libration point L_4 are given by:

$$x = -\frac{1}{2} + \mu + \alpha_1 A_1 + \alpha_1' A_1' + \beta_1 P + \beta_1' P',$$

$$y=\frac{\sqrt{3}}{2}+\alpha_2 A_1+\alpha_2' A_1'+\beta_2 P+\beta_2' P',$$

where

$$\alpha_1=-\frac{1}{2}, \qquad \alpha_1'=\frac{1}{2},$$

$$\alpha_2=-\frac{1}{2\sqrt{3}}, \qquad \alpha_2'=-\frac{1}{2\sqrt{3}},$$

$$\beta_1=\frac{1}{3}, \qquad \beta_1'=-\frac{1}{3},$$

$$\beta_2=-\frac{1}{3\sqrt{3}}, \qquad \beta_2'=-\frac{1}{3\sqrt{3}}.$$

IV. First order Normalization

The Hamiltonian function is given by

$$H(x,y,p_x,p_y)=\frac{1}{2}\left(p_x^{\,2}+p_y^{\,2}\right)+n\left(yp_x-xp_y\right)$$
$$-\frac{m_1}{r_1}-\frac{m_2}{r_2}-\frac{m_1}{2r_1^3}A_1-\frac{m_2}{2r_2^3}A_1'$$
$$+\frac{m_1}{r_1}P+\frac{m_2}{r_2}P'.$$

Applying the translation given by

$$x\to x-\frac{\gamma}{2}+\alpha_1 A_1+\alpha_1' A_1'+\beta_1 P+\beta_1' P',$$

$$y\to y+\frac{\sqrt{3}}{2}+\alpha_2 A_1+\alpha_2' A_1'+\beta_2 P+\beta_2' P',$$

$$p_x\to p_x-n\left(\frac{\sqrt{3}}{2}+\alpha_2 A_1+\alpha_2' A_1'+\beta_2 P+\beta_2' P'\right),$$

$$p_y\to p_y+n\left(-\frac{\gamma}{2}+\alpha_1 A_1+\alpha_1' A_1'+\beta_1 P+\beta_1' P'\right),$$

and substituting the expansions of $r_1^{-1}, r_2^{-1}, r_1^{-3}, r_2^{-3}, r_1^{-5}$ and r_2^{-5} in power series of x and y, we obtain $H=\sum_{k=0}^{\infty}H_k$, where H_k = the sum of the terms of kth degree homogenous in variables x, y, p_x, p_y.

The linear stability is assured when $\mu<\mu_{c0}$, where

$$\mu=\mu_o-0.285002A_1-0.06278A_1'-1.38127A_2$$
$$-0.10349A_2'-0.00891747\left(P+P'\right)\equiv\mu_{co},$$

and $\mu_o=0.0385208965\ldots$.

If ω_1 and ω_2 are the long and short periodic frequencies they are related to each other as

$$\omega_1^2+\omega_2^{\,2}=1-\frac{3}{2}\gamma A_1+\frac{3}{2}\gamma A_1'-3\left(1+\gamma\right)A_2-3\left(1-\gamma\right)A_2',$$

$$\omega_1^2\omega_2^{\,2}=\frac{9}{16}\left(1-\gamma^2\right)\left(3+13A_1+13A_1'\right)$$
$$+\frac{45}{64}\left(7+4\gamma-3\gamma^2\right)A_2+\frac{45}{64}\left(7-4\gamma-3\gamma^2\right)A_2'$$
$$+\frac{3}{8}\left(1-\gamma^2\right)\left(P+P'\right),$$
$$\left(0<\omega_2<\frac{1}{\sqrt{2}}<\omega_1<1\right).$$

V. Determination of the Normal Co-ordinates

We follow the method of Whittaker [18] to determine the normal co-ordinates. Applying the transformation

$\left(x,y,p_x,p_y\right)\to\left(q_1',q_2',p_1',p_2'\right)$ given by $X=JT$,

where

$$X=\begin{pmatrix}x\\y\\p_x\\p_y\end{pmatrix}, \qquad T=\begin{pmatrix}q_1'\\q_2'\\p_1'\\p_2'\end{pmatrix},$$

J is the square matrix given by

$$J=(J_{ij}).$$

All the values of J_{ij}'s (i, j; 1..4) are given in Aggarwal et al. [1].

Next, we perform the canonical transformation

$$\left(q_1',q_2',p_1',p_2'\right)\to\left(q_1'',q_2'',p_1'',p_2''\right)$$

i.e. $q_1'=\frac{1}{2}q_1''+\frac{1}{\omega_1}ip_1''$, $\quad p_1'=\frac{1}{2}i\omega_1 q_1''+p_1''$,

$$q_2'=-\frac{1}{2}q_2''+\frac{1}{\omega_2}ip_2'',\ p_2'=-\frac{1}{2}i\omega_2 q_2''+p_2''.$$

The new Hamiltonian H becomes,

$$H=i\omega_1 q_1''p_1''+i\omega_2 q_2''p_2''$$
$$+\sum_{\substack{\alpha+\beta=3\\ \alpha=\alpha_1+\alpha_2\\ \beta=\beta_1+\beta_2}}^{\infty}h'_{\alpha_1\alpha_2\beta_1\beta_2}q_1''^{\alpha_1}q_2''^{\alpha_2}p_1''^{\beta_1}p_2''^{\beta_2},$$

where

$$h'_{\alpha_1\alpha_2\beta_1\beta_2}=x_{\alpha_1\alpha_2\beta_1\beta_2}+iy_{\alpha_1\alpha_2\beta_1\beta_2},$$

All the values of $x_{\alpha_1\alpha_2\beta_1\beta_2}$ and $y_{\alpha_1\alpha_2\beta_1\beta_2}$ (of order four) in terms of $h'_{\alpha_1\alpha_2\beta_1\beta_2}$ have been given in Aggarwal et al. [1].

Next we use the Birkhoff's transformation [19] $\left(q_1'',q_2'',p_1'',p_2''\right)\to\left(q_1''',q_2''',p_1''',p_2'''\right)$ defined by the generating function

$$S = q_1'' p_1''' + q_2'' p_2''' + S_3 + S_4,$$

where

$$q_i''' = \frac{\partial S}{\partial p_i'''} = q_i'' + \frac{\partial S_3}{\partial p_i'''} + \frac{\partial S_4}{\partial p_i'''},$$

$$p_i'' = \frac{\partial S}{q_i''} = p_i''' + \frac{\partial S_3}{\partial q_i''} + \frac{\partial S_4}{\partial q_i''}, (i = 1, 2).$$

(Szebehely [17])

Due to the absence of resonances the new Hamiltonian H' will not contain all third order terms and is given by

$$H' = i\omega_1 q_1''' p_1''' + i\omega_2 q_2''' p_2''' - c_{20}(q_1''' p_1''')^2 + c_{11}(q_1''' p_1''')(q_2''' p_2''') - c_{02}(q_2''' p_2''')^2,$$

where

$$c_{20} = -h'_{2020} - \frac{3\omega_1^2}{8}\left(x_{0030}^2 + y_{0030}^2\right) - \frac{3}{2}\left(x_{1020}^2 + y_{1020}^2\right) + \frac{1}{2}\left(x_{1011}^2 + y_{1011}^2\right) - \frac{\omega_1^2}{2\omega_2\left(2\omega_1 - \omega_2\right)}\left(x_{0120}^2 + y_{0120}^2\right) + \frac{\omega_1^2\omega_2}{8\left(2\omega_1 + \omega_2\right)}\left(x_{0021}^2 + y_{0021}^2\right),$$

$$c_{11} = h'_{1111} - \frac{2\omega_1^2}{\omega_1\left(\omega_1 - 2\omega_2\right)}\left(x_{1002}^2 + y_{1002}^2\right) + \frac{\omega_2^2\omega_1}{2\left(2\omega_2 + \omega_1\right)}\left(x_{0012}^2 + y_{0012}^2\right) - \frac{\omega_1^2\omega_2}{2\left(2\omega_1 + \omega_2\right)}\left(x_{0021}^2 + y_{0021}^2\right) - \frac{2\omega_1^2}{\omega_2\left(2\omega_1 - \omega_2\right)}\left(x_{0120}^2 + y_{0120}^2\right) + 2\left(x_{0111}x_{1020} + y_{0111}y_{1020}\right) - \frac{4}{\omega_2}\left(x_{0201}x_{1011} + y_{1011}y_{0201}\right),$$

$$c_{02} = h'_{0202} - \frac{3\omega_2^2}{8}\left(x_{0003}^2 + y_{0003}^2\right) + \frac{6}{\omega_2^2}\left(x_{0201}^2 + y_{0201}^2\right) - \frac{1}{2}\left(x_{0111}^2 + y_{0111}^2\right) - \frac{\omega_2^2}{2\omega_1\left(\omega_1 - 2\omega_2\right)}\left(x_{1002}^2 + y_{1002}^2\right) - \frac{\omega_2^2\omega_1}{8\left(\omega_1 + 2\omega_2\right)}\left(x_{0012}^2 + y_{0012}^2\right).$$

We shall now apply KAM-theorem according to which we shall examine the value of $D = c_{20}\omega_2^2 + c_{11}\omega_1\omega_2 + c_{02}\omega_1^2$.

If $D \neq 0$ for any values of A_1, A_1', A_2, A_2', P and P', the motion will be stable provided $\omega_1 \neq 2\omega_2$ and $\omega_1 \neq 3\omega_2$.

The cases $\omega_1 = 2\omega_2$ and $\omega_1 = 3\omega_2$ coreespond the third and fourth order resonance and have been discussed by Aggarwal et al. [1].

VI. Stability

We have calculated the value of D for different values of A_1, A_1', A_2, A_2', P and P', in the linear stability range $\mu < \mu_{c0}$. (Tables – 1, 2, 3 and 4)

Table – 1

$A_1 = 0.01$, $A_2 = 0.001$, $A_1' = 0.01$, $A_2' = 0.001$, $P = 0.001$, $P' = 0.001$

μ	D
0.010	0.090000
0.011	0.279000
0.012	0.322000
0.013	**0.231000**
0.014	**-0.008889**
0.015	-0.444000
0.016	-1.180000
0.020	**-23.128000**
0.022	**65.648000**
0.025	30.361000

Table – 2

$A_1' = 0.01$, $A_2' = 0.001$, $P = 0.001$, $P' = 0.001$

μ	A_1	A_2	D
0.01	0.002	0.0020	-4.2000
0.01	0.010	0.0020	-3.0494
0.01	**0.020**	**0.0020**	**-0.5950**
0.01	**0.030**	**0.0020**	**3.716**
0.01	0.002	0.0040	13.849
0.01	0.010	0.0040	19.897
0.01	0.002	0.0060	290.796
0.01	**0.010**	**0.0060**	**278.215**
0.01	**0.010**	**0.0084**	**-62.110**
0.01	0.010	0.0085	-236.809

From Table – 1, we observe that for $\mu = 0.013$, the value of D is 0.231>0 and for $\mu = 0.014$, the value of D is -0.00889<0. This suggests that for some μ such that $0.013 < \mu < 0.014$, the value of D =0.

Again from Table – 2 and Table – 3, we observe that sign of D changes at the bold values of the parameters, therefore for some values of parameters the value of D=0. Hence we conclude that except for the cases when D = 0 the Libration point L_4 is stable and when D = 0 the stability of L_4 can not be decided and it needs further investigation.

Table – 3

$A_1 = 0.01, A_2 = 0.001, P = 0.001, P' = 0.001$			
μ	A_1'	A_2'	D
0.01	0.02	0.00	-1.14
0.01	**0.02**	**0.001**	**-0.449**
0.01	**0.02**	**0.002**	**0.228**
0.01	0.02	0.01	5.195
0.01	**0.02**	**0.02**	**10.272**
0.01	**0.01**	**0.00**	**-0.595**
0.01	0.002	0.001	0.09
0.01	0.01	0.01	5.641
0.01	**0.01**	**0.02**	**10.536**
0.01	**0.01**	**0.0085**	**-236.809**

Table – 4

$A_1 = 0.01, A_2 = 0.01, A_1' = 0.01, A_2' = 0.001$			
μ	P	P'	D
0.01	0.01	0.00	-0.0001456
0.01	0.01	0.001	-0.0001469
0.01	0.01	0.005	-0.0001488
0.01	0.01	0.01	-0.0001511
0.01	0.01	0.02	-0.0001559
0.01	0.02	0.001	-0.0001481
0.01	0.02	0.005	-0.00015
0.01	0.02	0.01	-0.0001524
0.01	0.02	0.02	-0.0001572

VII. Conclusion

We have investigated the non-linear stability of the triangular libration point L_4 of the Restricted Three Body Problem, when both the primaries are axes symmetric and source of radiation. It is found through KAM theorem that L_4 is stable or unstable depending upon the values of the parameters, A_1, A_1', A_2, A_2', P and P', where A_1, A_1', A_2, and A_2', depend upon the lengths of the semi axes of the primaries and P and P' are the radiation parameters.

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