Mixed Burst Error Correcting Codes

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Abstract — *In this paper, we construct codes which are an improvement on the previously known block-wise burst error correcting codes in terms of their error correcting capabilities. Along with different bursts in different sub-blocks, the given codes also correct overlapping bursts of a given length in two consecutive sub-blocks of a code word. Such codes are called mixed burst correcting (mbc) codes.*

Keywords — *mixed burst, fixed burst, overlapping burst, error pattern-syndromes, parity check matrix.*

I. INTRODUCTION

Burst is the most common error in many communication systems and block-wise burst error correcting codes are developed to deal with such errors. Correcting burst error in blocks has an additional benefit as one knows the pattern of errors in each sub-block and when we consider error correction in such a system, we correct errors which occur in the same sub-block.

Most of the studies in burst error correcting codes are with respect to the usual definition of burst according to which '*A burst of length b is a vector whose all the non zero components are confined to some b consecutive positions, the first and last of which is non zero*'.

There is another definition of burst due to Chein and Tang [2] with a modification due to Dass [3], known as CTD-burst, according to which

"*A CTD-burst is a vector whose all the non zero components are confined to some b-consecutive positions, the first of which is non zero*".

According to this definition, (10000000) is a burst of length 8 whereas (0001000) will be a burst of length at most 4. This definition has been found very useful in error analysis experiments on telephone lines [1] and in channels where error normally do not occur near the end of a vector particularly when the burst length is large.

Such block wise burst error correcting codes were first introduced by Dass and Tyagi [4]. Recently, Tyagi and Sethi [6], have generalized this idea to three sub-blocks of length n_1, n_2 and $n_3, n_1+n_2+n_3=n$ and named them as $(n_{1b_1}, n_{2b_2}, n_{3b_3})$ linear codes, some of which turn out to be byte oriented [5].

Definition. *An* $(\boldsymbol{n}_{1b_1}, \boldsymbol{n}_{2b_2}, \boldsymbol{n}_{3b_3})$ *code is an* $(\mathbf{n}=\boldsymbol{n}_1+\boldsymbol{n}_2+\boldsymbol{n}_3, \mathbf{k})$ *code that correct all bursts of length* $\boldsymbol{b}_1$ *(fixed) in the first sub-block of length* $\boldsymbol{n}_1$*, all bursts of length* $\boldsymbol{b}_2$ *(fixed) in the second sub-block of length* $\boldsymbol{n}_2$ *and all bursts of length* $\boldsymbol{b}_3$ *(fixed) in the third sub-block of length* $\boldsymbol{n}_3$*.*

In this communication, we modify $(n_{1b_1}, n_{2b_2}, n_{3b_3})$ codes as *mixed burst correcting codes*, (*mbc*-codes) in such a way that along with fixed bursts of length b_1, b_2 and b_3, the modified codes also correct over all burst of length b (fixed) in two consecutive sub-blocks, thereby improving upon their error correcting capabilities. The paper is divided into two sections. In section II, we present necessary condition where as in section III we give sufficient condition.

II. NECESSARY CONDITION

Theorem 1. The number of parity check digits required for an $(n=n_1+n_2+n_3, k)$ linear code that correct all fixed bursts of length b_1, b_2 and b_3 in first n_1, next n_2 and last n_3-components along with the overlapping burst of length $b, b \ge b_1+b_2$ and $b \ge b_2+b_3$ ($b=b_1+b_2$, $b=b_2+b_3$ only when $n_i=b_i$, $i=1,2$ $n_i=b_i$, $i=1,2$), in any two consecutive sub-blocks, is at least

$$q^{n-k} \ge$$
$$1+(q-1)\begin{bmatrix}(n_1-b_1+1)q^{b_1-1} \\ +(n_2-b_2+1)q^{b_2-1}+(n_3-b_3+1)q^{b_3-1}\end{bmatrix}$$
$$+\frac{1}{2}(q-1)^2 q^{b_1+b_2-2}(b-b_1-b_2+1)(b-b_1-b_2+2)$$
$$+\frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2). \tag{1}$$

Proof. The theorem is proved by enumerating

(i) all the error patterns of length b_1 (fixed) in first n_1-components;

(ii) all the error patterns of length b_2 (fixed) in next n_2-components;

(iii) all the error patterns of length b_3 (fixed) in the last n_3-components;

(iv) all bursts of length b (fixed) in the first two sub-blocks of length n_1 and n_2; and

(v) all bursts of length b (fixed) in the last two sub-blocks of length n_2 and n_3.

The number of error pattern in (i) to (iii) comes out to be

$$(q-1)[(n_1-b_1+1)q^{b_1-1}+(n_2-b_2+1)q^{b_2-1} +(n_3-b_3+1)q^{b_3-1}] \quad (2)$$

as shown in Tyagi and Sethi [6].Therefore, we need to calculate number of error patterns in (iv) and (v).

In (iv), since burst error b (fixed) is of the type that part of it that lies in the first n_1-digits is a burst of length b_1 (fixed) and remaining part in next n_2 digits is a burst of length b_2 (fixed), therefore, the starting positions of such a burst in the first n_1 digits can be and $\left(n_1-(b-b_2)+1\right)$ the last starting position can be (n_1-b_1+1) th component. To enumerate the number of such vectors assume that the burst starts from the jth component. Obviously

$$n_1-(b-b_2)\le j\le n_1-b_1+1. \quad (3)$$

This burst may continue up to $(b+j-1)$ th component where

$$b-n_1+j-1\le n_2. \quad (4)$$

The number of bursts of length b_1 (fixed) starting from the j[th] component is

$$(q-1)q^{b_1-1}. \quad (5)$$

where as the number of bursts of length b_2 (fixed) in a vector of length $(b-n_1+j-1)$ is

$$(b-b_2+n_1+j)(q-1)q^{b_2-1}. \quad (6)$$

So, the total number of bursts under category (iv) starting from the j[th] component is

$$\left(b-b_2-n_1+j\right)\left(q-1\right)^2 q^{b_1+b_2-2}. \quad (7)$$

Thus, the total number of bursts under category (iv) for all possible values of j is.

$$(q-1)^2 q^{b_1+b_2-2}\sum_{j=n_1-b+b_2+1}^{n_1-b_1+1}(b-b_2-n_1+j) \quad (8)$$

$$=\frac{1}{2}(q-1)^2 q^{b_1+b_2-2}(b-b_1-b_2+1)(b-b_1-b_2+2). \quad (9)$$

Similarly, the total number of bursts under category (v) is

$$=\frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2). \quad (10)$$

Since all these error vectors in (2), (9) and (10) should have different syndromes for error correction, therefore, the total number of cosets q^{n-k} should be at least as large as the number of error patterns (including the pattern of all zeros) and therefore we must have

$$q^{n-k}\ge 1+(2)+(9)+(10)$$

i.e.

$$q^{n-k}\ge 1+(q-1)\begin{bmatrix}(n_1-b_1+1)q^{b_1-1}\\ +(n_2-b_2+1)q^{b_2-1}+(n_3-b_3+1)q^{b_3-1}\end{bmatrix} +\frac{1}{2}(q-1)^2 q^{b_1+b_2-2}(b-b_1-b_2+1)(b-b_1-b_2+2) +\frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2).$$

Incidentally, it can be shown that the result applies to non-linear codes also.

Discussion. If there is no overlapping burst of length b, the condition reduces to upper bound given by Tyagi and Sethi [6] i.e.

$$q^{n-k}\ge 1+(q-1)[(n_1-b_1+1)q^{b_1-1}+(n_2-b_2+1)q^{b_2-1} +(n_3-b_3+1)q^{b_3-1}].$$

II.2. Sufficient Condition

Theorem 2. Given positive integers b_1, b_2 and b_3; there will always exists an $(n_{1b_1}, n_{2b_2}, n_{3b_3})-(n,k)$ linear code that correct all fixed bursts of length b_1 (fixed), b_2 (fixed) and b_3 (fixed) in the first n_1, next n_2 and last n_3 digits and all the overlapping bursts of length b (fixed) ($b\ge b_1+b_2$ and $b\ge b_2+b_3$, $b=b_1+b_2=b_2+b_3$ only when $n_i=b_i$, $i=1$ to 2) in any two consecutive sub-blocks, satisfying the inequality.

$$\begin{aligned} q^{n-k} &\ge q^{b_3-1}[1+(n_3-2b_3+1)(q-1)q^{b_3-1}] \\ &+q^{b_2-1}+[1+(n_2-2b_2+1)(q-1)q^{b_2-1}+(n_3-b_3+1)(q-1)q^{b_3-1} \\ &+\frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2)] \\ &+q^{b_1-1}\Big[1+k-(n_3-n_2-2b_1+1)(q-1)q^{b_1-1} \\ &+(n_2-b_2+1)(q-1)q^{b_2-1}+(n_3-b_3+1)(q-1)q^{b_3-1} \\ &+\frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2) \\ &+\frac{1}{2}(q-1)^2 q^{b_1+b_2-2}(b-b_1-b_2+1)(b-b_1-b_2+2)\Big]. \end{aligned} \quad (11)$$

Proof. The existence of such a code is shown here by constructing an appropriate $(n-k)\times n$ parity check matrix H for the desired code. If H_1' denotes the number of columns of the parity check matrix H' in the first n_1-digits, H_2' denotes the columns of the parity check matrix H' in the next n_2-digits, and H_3' denotes the columns of the parity check matrix H' in the last n_3-digits, then the matrix H' may be expressed as $H'=[H_3'H_2'H_1']$. Then the required matrix H may be obtained from H' by reversing the order of its columns. i.e. $H'=[H_1'H_2'H_3']$.

Select any non zero (n-k) -tuple as the first column of H' (in H_3'). Subsequent columns are added to H' such that after having selected n_3-1 columns $h_1,h_2,...,h_{n_3-1}$ a column h_{n_3} is added provided that

$$h_{n_3}\neq(u_{n_3-b_3+1}h_{n_3-b_3+1}+\ldots+u_{n_3-1}h_{n_3-1})+(v_ih_i+\ldots+v_{i+b_3-1}h_{i+b_3-1}) \quad (12)$$

where either all v_i are not zero or if v_s is the last non zero coefficient then $b_3\leq s\leq n_3-b_3$.

This construction assures that the code which is the null space of the finally constructed matrix H will be capable of correcting all bursts of length b_3 (fixed) in the third sub-block of length n_3. To choose the v_i is equivalent to enumerating the number of bursts of length b_3 (fixed) in an (n_3-b_3) tuple.

$$(n_3-2b_3+1)(q-1)q^{b_3-1}. \quad (13)$$

Thus, the total number of columns to which h_{n_3} cannot be equal is

$$q^{b_3-1}[1+(n_3-2b_3+1)(q-1)q^{b_3-1}]. \quad (14)$$

Now, we shall add $(n_3+1)^{th},(n_3+2)^{th}\ldots$ columns of H' (in H_2'). We wish to assure that the code so constructed is capable of correcting all bursts of length b_2 (fixed) in the second sub-block of length n_2, along with an overlap burst of length $b(b\geq b_1+b_2)$ in (n_1+n_2) components.

As the first requirement, the general t^{th} column $(t>n_3)$ to be added should not be a linear combination of the immediate proceeding b_2-1 columns $h_{t-b_2+1}....h_{t-1}$ together with any b_2 consecutive amongst $h_{n_3+1},h_{n_3+2},...,h_{t-b_2}$ i.e

$$h_t\neq(u_{t-b_2+1}h_{t-b_2+1}+\ldots+u_{t-1}b_{t-1})+(v_rh_r+\ldots+h_{r+b_2-1}h_{r+b_2-1}). \quad (15)$$

Where h_r amongst $h_{n_3+1,}h_{n_3+2},\ldots,h_{t-b_2}$ and either all the v_r are zero or if v_t is the last non-zero coefficient, then $b_2\leq t\leq t-n_3-b_2$.

The u_t in (15) can obviously be selected in q^{b_2-1} ways.

Using the u_t in (15) is equivalent to choosing the number of bursts of length b_2 (fixed) in a vector of length $t-n_3-b_2$. Their number is

$$(t-n_3-2b_2+1)(q-1)q^{b_2-1}. \quad (16)$$

Second requirement is that t^{th} column should also not be a linear combination of the immediately proceeding b_2-1 columns $h_{t-b_2+1},\cdots,h_{t-1}$ $(t-b_2+1\geq n_3+1)$ together with any b_3 consecutive columns from amongst $h_1,h_{2..1}h_{n_3}$.

i.e.

$$h_t\neq(u_{t-b_2+1}h_{t-b_2+1}+\ldots+u_{t-1}h_{t-1})+(v_ih_i+\ldots+v_{i+b_3-1}h_{i+b_3-1}) \quad (17)$$

where all the v_i are not zero, and if v_s is the last non-zero coefficient, then $b_3<s$. The number of ways in which the coefficient. u_t in (17) can be selected in q^{b_2-1} ways, choosing the coefficient v_i in (17) is equivalent to enumerating the bursts of length b_3 (fixed) in a vector of length n_3. Their number is

$$(n_3-b_3+1)(q-1)q^{b_3-1}. \quad (18)$$

Third requirement is that the t^{th} column should also not be a linear combination of the immediately proceeding b_2-1 columns $h_{t-b_2+1}h_{t-b_2+2}\ldots h_{t-1}$ together with any b_3+b_2 consecutive columns amongst $h_1,h_2,\ldots,h_{n_3-1},h_{n_3},h_{n_3+1},\ldots,h_{n_3+t-1}$. i.e

$$h_1\neq(u_{t-b_2+1}h_{t-b_2+1}+\ldots+u_{t-1}h_{t-1})+(v_jh_j+v_{j+1}h_{j+1}+\ldots+v_jh_{j+b-1}) \quad (19)$$

where $j=n_3-(b-b_2)+1,\ldots,n_3-b_3+1$, and all $v_j's$ are not zero and if v_s is the last non zero coefficient, then $b_1+b_2<s$. The number of ways in which the coefficient u_t in (19) can be selected is $q^{b_2+b_3-2}$. Choosing the coefficient v_j in (19) is equivalent to enumerating the burst of length b (fix) in a vector of length $n_3+n_2,b-b_2\leq n_2,$

Their number is

$$(q-1)^2q^{b_2+b_3-2}\sum_{j=n_3-(b-b_2)+1}^{n_3-b_3+1}(j+(b-b_2)-n_3) \text{ i.e.}$$

$$\frac{1}{2}(q-1)^2q^{b_2+b_3-2}(b-(b_2+b_3)+1)(b-(b_2+b_3)+2). \quad (20)$$

So, the total number of combination to which h_t cannot be equal is (16) + (18) + (20) i.e.

$$q^{b_2-1}\begin{bmatrix}1+(t-n_3-2b_2+1)(q-1)q^{b_2-1}\\+(n_3-b_3+1)(q-1)q^{b_3-1}\end{bmatrix}+\frac{1}{2}(q-1)^2q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2). \quad (21)$$

Taking $t=n_3+n_2$ as the last column of the second sub-block, the equation (21) becomes

$$q^{b_2-1}\begin{bmatrix}1+(n_2-2b_2+1)(q-1)q^{b_2-1}\\+(n_3-b_3+1)(q-1)q^{b_3-1}\end{bmatrix}$$
$$+\frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-b_2-b_3+1)(b-b_2-b_3+2). \quad (22)$$

The first requirement assures that in the code which is the null space of the final constructed matrix H. The syndromes of any two bursts each of which is of length b_2 (fixed) are not equal, the second requirement assures that the syndrome of two bursts, one of which is the burst of length b_2 (fixed) in the sub-block of length n_2 and the other bursts of length b_3 (fixed) in the sub-block of length n_3 are different and the third requirement assures that the syndromes of two bursts, one of which is a burst of length b_2 in the sub-block of length n_2 and the other burst of length b fixed in two consecutive sub-blocks of length n_2 and n_3, are different.

Now we shall start adding $(n_3+n_2+1)^{th}$, $(n_3+n_2+2)^{th}, \ldots,$ columns of H' (in H_1'), we wish to assure that the code so constructed is capable of correcting all bursts of length b_1 (fixed) in the first sub-block of length n_1. For this, we lay down the following requirements.

As the first requirement, the general k^{th} column $(k>n_3+n_2)$ to be added should not be a linear combination of the immediately preceding b_1-1 columns $h_{k-b_1+1},\ldots,h_{k-1},(k-b_1+1\geq n_3+n_2+1)$ together with any b_1 consecutive columns from amongst $h_{n_3+n_2+1},\ldots,h_{k-b}$, i.e

$$h_k \neq (u_{k-b_1+1}h_{k-b_1+1}+\ldots+u_{k-1}h_{k-1}) + (v_r h_r+\ldots+v_{r+b_1-1}h_{r+b_1-1}) \quad (23)$$

where h_r are amongst $h_{n_3+n_2+1}, h_{n_3+n_2+2},\ldots,h_{k-b_1}$, and either all the v_r are zero or if v_k is the last non-zero coefficient, then

$$b_1 \leq k \leq k-(n_3+n_2)-b_1.$$

The u_k in (23) can obviously be selected in q^{b_1-1} ways. Choosing v_r in (23) is equivalent to choosing the number of bursts of length b_1 (fixed) in a vector of length $k-(n_3+n_2)-b_1$. Their number is

$$1+(k-n_3-n_2-2b_1+1)(q-1)q^{b_1-1}. \quad (24)$$

The second requirement is that the k^{th} column should also not be a linear combination of the immediately preceding $h_{k-b_1+1},\ldots,h_{k-1}$ $(k-b_1+1\geq n_3+n_2+1)$ together with any b_2 consecutive columns from amongst $h_{n_3+1},\ldots,h_{n_3+n_2}$ i.e.

$$h_k \neq (u_{k-b_1+1}h_{k-b_1+1}+\ldots+u_{k-1}h_{k-1}) + (v_i h_i+\ldots+v_{i+b_2-1}h_{i+b_2-1}) \quad (25)$$

where all the v_i are not zero and if v_s is the last non zero coefficient, then $b_2 \leq s$. The number of ways in which the coefficient u_k in (25) can be selected is q^{b_2-1}. Choosing the coefficient v_i in (25) is equivalent to enumerating the bursts of length b_2 (fixed) in a vector of length n_2 This number is

$$(n_2-b_2+1)(q-1)q^{b_2-1}. \quad (26)$$

The third requirement is that the k^{th} column should also not be a linear combination of the immediately preceding b_1-1 columns $h_{k-b_1+1},\ldots,h_{k-1}$ $(k-b_1+1\geq n_3+n_2+1)$ together with any b_3 consecutive columns from amongst $h_1,h_2,\ldots,h_{n_3}$ i.e.

$$h_k \neq (u_{k-b_1+1}h_{k-b_1+1}+\ldots+u_{k-1}h_{k-1}) + (v_i h_i+\ldots+v_{i+b_3-1}h_{i+b_3-1}) \quad (27)$$

where all the v_i's are not zero, and if v_s is the last non zero coefficient, then $b_3 \leq s$. The number of ways in which the coefficient u_k in (27). Can be selected is q^{b_3-1}. Choosing the coefficient v_i in (27) is equivalent to enumerating the bursts of length b_3 (fixed) in a vector of length n_3. Their number is

$$(n_3-b_3+1)(q-1)q^{b_3-1}. \quad (28)$$

The fourth requirement is the k^{th} column should also not be a linear combination of the immediately preceding b_1-1 columns $h_{k-b_1+1},\ldots,h_{k-1}$ $(k+b_1-1\geq n_3+n_2+1)$ together with any b_2+b_3 consecutive columns from amongst $h_1,h_2,\ldots h_{n_3+n_2}$. i.e

$$h_k \neq (u_{k-b_1+1}h_{k-b_1+1}+\ldots+u_{k-1}h_{k-1}) + (u_j h_j+\ldots+v_{j-b+1}h_{j-b+1}) \quad (29)$$

where

$j=n_3-(b-b_2)+1,\ n_3-(b-b_2)+2,\ldots,n_3-b_3+1.$

Also all v_i are not zero and if v_s is the last non zero coefficient, then $b_3+b_2\leq s$, the number of ways in which the coefficient u_k in (29) can be selected is $q^{b_3+b_2-2}$. Choosing the burst of length b_3+b_2 (fixed) in a vector of length n_3+n_2. Their number is

$$(q-1)^2 q^{b_2+b_3-2}\sum_{j=n_3-(b-b_2)+1}^{n_3-b_3+1}(j+(b-b_2)-n_3)$$

i.e.

$$\frac{1}{2}(q-1)^2 q^{b_2+b_3-2}(b-(b_2+b_3)+1)(b-(b_2+b_3)+2)\ .$$

(30)

The fifth requirement is that the k^{th} column should also not be a linear combination of the immediately preceding $b_1 - 1$ columns $h_{k-b_1+1}, \ldots, h_{k-1}$ $(k - b_1 + 1 \geq n_3 + n_2 + 1)$ together with any $b_1 + b_2$ consecutive columns from amongst

$$h_{n_3+1}, h_{n_3+2}, \ldots, h_{n_3+n_2}, h_{n_3+n_2+1}, \ldots, h_{n_3+n_2+k-1}$$

i.e.

$$h_k \neq (u_{k-b_1+1} \; h_{k-b_1+1} + \ldots + u_{k-1} h_{k-1}) + (v_j h_j + \ldots + v_{j+b-1} h_{j+b-1}) \quad (31)$$

where

$j = n_2 - (b - b_1) + 1, \; n_2 - (b - b_1) + 2, \ldots, n_2 - b_2 + 1.$

where all v_j's are non zero and if v_s is the last non zero coefficient then $b_1 + b_2 \leq s$. The number of ways in which the coefficient u_k in (31) can be selected is $q^{b_1+b_2-2}$. Choosing the coefficient v_j in (31) is equivalent to enumerating the bursts of length $b_1 + b_2$ fixed in a vector $n_1 + n_2$. Their number is

$$(q-1)^2 q^{b_1+b_2-2} \sum_{j=n_2-(b-b_1)+1}^{n_2-b_2+1} (j + (b - b_2) - n_2)$$

i.e.

$$\frac{1}{2}(q-1)^2 q^{b_1+b_2-2} (b - (b_1 + b_2) + 1)(b - (b_1 + b_2) + 2) \,. \quad (32)$$

So, the total number of combination to which h_k can not to equal is (24)+ (26) + (28) + (30) + (32) i.e.

$$q^{b_1-1} \begin{bmatrix} 1 + (k - n_3 - n_2 - 2b_1 + 1)(q-1) q^{b_1-1} \\ + (n_2 - b_2 + 1)(q-1) q^{b_2-1} \\ + (n_3 - b_3 + 1)(q-1) q^{b_3-1} \end{bmatrix}$$
$$+ \frac{1}{2}(q-1)^2 q^{b_1+b_2-2} (b - b_1 - b_2 + 1)(b - b_1 - b_2 + 2)$$
$$+ \frac{1}{2}(q-1)^2 q^{b_2+b_3-2} (b - b_2 - b_3 + 1)(b - b_2 - b_3 + 2). \quad (33)$$

The first requirement assures that in the code, which is the null space of the final constructed matrix H, the syndromes of any two bursts, each of which is of length b_1 (fixed) are not equal, the second requirement assures that the syndromes of two bursts, one of which is a bursts of length b_1 (fixed) in the sub-block of length n_1 and the other is a burst of length b_2 in the sub-block of length n_2, are different, the third requirement assures that the syndromes of two bursts, one of which is a burst of length b_1 (fixed) in the sub-block of length n_1, and other is a burst of length b_3 (fixed) in the sub-block of length b_3 (fixed) in the sub-block of length n_3, are different, the fourth requirement assures that the two syndromes of two bursts, one of which is a burst of length b_1 (fixed) in the sub-block of length n_1 and the other is a burst of length b (fixed) in two consecutive sub-blocks of length n_2 and n_3 are different, and the fifth requirement assures that the syndromes of two bursts, one of which is a burst of length b_1 (fixed) in the sub-block of length n_1 and the other is the burst of length b in two consecutive sub-blocks of length n_1 and n_2, are different.

At worst of all these linear combination considered in (14), (22) and (33) may be distinct, thus while choosing the n_3^{th} column, we must have

$$q^{n-k} \geq (14)\,; \quad (34)$$

while choosing the $(n_3 + n_2)^{th}$ column, we must have

$$q^{n-k} \geq (22)\,; \quad (35)$$

where as while choosing the n^{th} column $(n_3 + n_2 + n_1)$ we must have

$$q^{n-k} \geq (33)\,. \quad (36)$$

However, the requisite matrix H' can be completed if $q^{n-k} \geq \max\{(34),(35),(36)\}$, which is expression (11).

The required parity check matrix $H = [H_1{}' H_2{}' H_3{}'] = [h_1 h_2, \ldots, h_n]$ is then obtained from $H' = [H_3{}' H_2{}' H_1{}'] = [h_n h_{n-1} h_{n-2}, \ldots, h_2 h_1]$ by reversing its columns altogether i.e. h_j becomes h_{n-j+1}.

DISCUSSION

We present here different possible cases based on the length of the burst and size of the sub-blocks viz.

(1) $b_1 = b_2 = b_3;\; n_1 = n_2 = n_3;$ i.e. the length of bursts as well as sub-blocks is same.

(2) $b_1 = b_2 = b_3;\; n_1 = n_2 \neq n_3;$ i.e. the length of bursts is equal but the size of two sub-blocks is different.

(3) $b_1 = b_2 = b_3;\; n_1 \neq n_2 \neq n_3;$ i.e. the length of bursts is equal but the sub-blocks are of different size.

(4) $b_1 \neq b_2 = b_3,\; n_1 = n_2 = n_3;$ i.e. the length of bursts is same only in two sub-blocks whereas size of sub-blocks is same.

(5) $b_1 \neq b_2 = b_3,\; n_1 \neq n_2 = n_3;$ i.e. the length of two burst as well as sub-blocks is same.

(6) $b_1 \neq b_2 = b_3,\; n_1 \neq n_2 \neq n_3;$ the length of two bursts are same but the size of all sub-blocks are different.

All the cases discussed above have been illustrated by the following examples 1 to 6 respectively.

Example 1. For $n_1 = n_2 = n_3 = N$, $b_1 = b_2 = b_3 = b'$, the given bound in (1) can be expressed as

$$2^{3N-K} \geq 1 + q^{b'-1}3(N - b' + 1) + (q-1)^2 q^{2(b'-1)}(b - 2b' + 1)(b - 2b' + 2) \quad (37)$$

For $N = 2$, $b' = 1$, $b = 3$, we have obtained (6,1)-code that can correct all single errors in all the sub-blocks and a burst of length 3 simultaneously in two consecutive sub-blocks. For this the following matrix

$$H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

may be considered as parity check matrix. It can be verified in the following table that the code is a *mbc*-code.

Table 1

Error Pattern	Syndrome
10 00 00	1 0 0 0 0
01 00 00	0 1 0 0 0
00 10 00	0 0 1 0 0
00 01 00	0 0 0 1 0
00 00 10	0 0 0 1 0
00 00 01	1 1 1 1 1
10 10 00	1 0 1 0 0
01 10 00	0 1 1 0 0
01 01 00	0 1 0 1 0
00 10 10	0 0 1 0 1
00 01 10	0 0 0 1 1
00 01 01	1 1 1 0 1

Case 2. If $b_1 = b_2 = b_3 = b'$, $N = n_1 = n_2 \neq n_3$, then the bound in (1) can be expressed as

$$2^{n-k} \geq 1 + 2(N - b' + 1)(q-1)q^{b'-1} + (n_3 - b_3 + 1)(q-1)q^{b_3 - 1} + (q-1)^2 q^{2(b'-1)}(b - 2b' + 1)(b - 2b' + 2) \quad (38)$$

For $N = 3$, $n_3 = 4$, $b' = 2$, $b = 5$ we have the following parity check matrix for a (10,4) linear code

$$H_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It can be verified from the following error-pattern syndrome table that the code is a *mbc*-code

Table 2

Error Pattern			Syndrome
100	000	0000	000100
110	000	0000	101110
010	000	0000	101010
011	000	0000	110111
000	100	0000	000010
000	110	0000	000011
000	010	0000	000001
000	011	0000	000101
000	000	1000	100000
000	000	1100	110000
000	000	0100	010000
000	000	0110	011000
000	000	0010	001000
000	000	0011	001100
100	100	0000	000110
100	110	0000	000111
110	100	0000	101100
110	110	0000	101101
010	100	0000	101000
010	110	0000	101001
011	100	0000	110101
011	110	0000	110100
010	010	0000	101011
010	011	0000	101111
011	010	0000	110110
011	011	0000	110010
000	100	1000	100010
000	100	1100	110010
000	110	1000	100011
000	110	1100	110011
000	010	1000	100001
000	010	1100	110001
000	011	1000	100101
000	011	1100	110101
000	010	0100	010001
000	010	0110	011001
000	011	0100	010101
000	011	0110	011101

Case 3. For $n_1 \neq n_2 \neq n_3$, $b_1 = b_2 = b_3 = b'$, the inequality (1) can be expressed as

$$2^{n-k} \geq 1 + 2^{b'-1}(n - 3b' + 3) + (q-1)^2 q^{2(b'-1)}(b - 2b' + 1) \cdot (b - 2b' + 2) \quad (39)$$

In this case, for $N = 9$, $b_1 = b_2 = b_3 = 1$, $b = 3$, we have obtained a (9, 4) code that may correct all single errors in all the three sub-blocks together with the bursts of length 3(fix) simultaneously in the vector of length $n_1 + n_2$ and $n_2 + n_3$.

Consider the following parity check matrix

$$H_3 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It can be verified that the code so constructed is a (9, 4) *mbc*-code.

Case 4. If $b_1 \neq b_2 = b_3 = b'$, $n_1 = n_2 = n_3 = N$, then equality (1) can be expressed as

$$q^{n-k} \geq$$
$$1+(q-1)q^{b_1-1}(N-b_1+1)+2(q-1)(N-b'+1)q^{b'-1}$$
$$+\frac{1}{2}(q-1)^2 q^{b_1+b'-2}(b-b_1-b'+1)(b-b_1-b'+2)$$
$$+\frac{1}{2}(q-1)^2 q^{2(b'-1)}(b-2b'+1)(b-2b'+2) \quad (40)$$

For $N=3, b_1=1, b'=2, b=4$, the parity check matrix for the code (9, 4) may be given as

$$H_4 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It can be verified that the code so constructed is a (9,4) *mbc*-code.

Case 5. If $b' = b_1 = b_2 \neq b_3$, $N = n_1 = n_2 \neq n_3$, then the bound given in (1) can be expressed as

$$2^{n-k} \geq 1+2(q-1)(N-b'+1)q^{b'-1}$$
$$+\frac{1}{2}(q-1)^2 q^{2(b'-1)}(b-2b'+1)(b-2b'+2)$$
$$+\frac{1}{2}(q-1)^2 q^{b_3+b'-2}(b-b_3-b'+1)(b-b_3-b'+2) \quad (41)$$

For $N=2$, $n_3=3$, $b'=1$, $b_3=2$, $b=3$, it can be verified from the following parity check matrix

$$H_5 = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

that the code so constructed is a (7,2) *mbc*-code.

Case 6. If $b' = b_1 = b_2 \neq b_3$, $n_1 \neq n_3 \neq n_3$, then the inequality (1) can be expressed as.

$$2^{n-k} \geq 1+(q-1)(n_1-b'+1)q^{b'-1}$$
$$+(q-1)(n_2-b'+1)q^{b'-1}$$
$$+(q-1)(n_3-b_3+1)q^{b_3-1}$$
$$+\frac{1}{2}(q-1)^2 q^{2(b'-1)}(b-2b'+1)(b-2b'+2)$$
$$+\frac{1}{2}(q-1)^2 q^{b_3+b'-2}(b-b_3-b'+1)(b-b_3-b'+2) \quad (42)$$

For $n_1=2$, $n_2=3$, $n_3=4$, $b'=1$, $b_3=2$, $b=3$, the (9,4) code obtained from the following parity check matrix

$$H_6 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is a *mbc*-code.

III. Open Problems and Remarks

In this paper, we have obtained lower and upper bounds on the number of parity-check digits for $(n_{1b_1}, n_{2b_2}, n_{3b_3})$ *mbc*-linear codes, which corrects burst in three different sub-blocks of a codeword. We have shown the existence of linear codes for different values of the parameters $n_1, n_2, n_3, k, b_1, b_2, b_3$, $b \geq b_1 + b_2 = b_2 + b_3$ by constructing appropriate parity check matrices following the synthesis procedure outlined in the proof of Theorem 1. However, the problem needs further investigation to

(1) find the possibilities of the existence of *mbc*-linear codes in non-binary case;
(2) find the possibilities of the existence of *mbc*-optimal codes in binary and non-binary cases.

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